CH5350: Applied Time-Series Analysis

Arun K. Tangirala

Department of Chemical Engineering, IIT Madras

Models for Linear Non-stationary Processes

Classical Time-Series Model

Once a time-series is realized as predictable, the search for a suitable mathematical model is carried out. Classical approaches in the early days rested on the philosophy that a series is made up of three components

$$\boxed{\text{Time-Series} = \text{Trend} + \text{Seasonal Component} + \text{Stationary component}}$$

The trend and seasonal components could be combined into a single component under the banner of *deterministic* component.

Classical Approach ... contd.

Several efficient non-parametric and semi-parametric methods were subsequently developed to realize such a decomposition. The trend usually contains a polynomial type of trend while the seasonal component captures the periodic characteristics, if any.

Extracting the deterministic portions of a series is not trivial, but can be effectively carried out with suitable **regression, smoothing and filtering operations.**

Note: The seasonal component is usually a deterministic periodic signal, and assumed to be uncorrelated with the non-seasonal component.

Modern Approach

In 1970s, a new approach to modelling the seasonal (including the non-stationary and trend components) was introduced.

Unlike the models based on additive approach, **multiplicative models** were postulated. These are more generic in the nature because they take into account the correlation between seasonal and non-seasonal (stationary) components, and also model the integrating (random walk) effects.

The resulting models are known as **seasonal ARIMA (SARIMA) models.**

Non-stationarities

ARMA models are capable of representing almost all classes of (linear) stationary processes. On the other hand, there are several processes that exhibit non-stationarities, of which there are numerous variants.

We shall focus on two popular types, **mean** and **variance** non-stationary processes.

In addition, the series may contain periodicities and seasonalities. We shall study such processes in greater detail later.

Mean non-stationarities

Among the mean non-stationary processes, we have again two types:

1. Deterministic trends (polynomial functions of time)
2. Stochastic trends, esp. random walk type (**integrating** effects)

We shall, be largely concerned with **integrating type non-stationarities**, since these are frequently encountered across different disciplines.

Handling non-stationarities

There are primarily two ways of handling non-stationarities:

1. Eliminating trends by first fitting polynomial models to the series and working with the residuals.
2. Including integrating effects in the model by suitable differencing of the data

It turns out the second class of methods can also handle trends of polynomial types. A single degree of differencing eliminates a trend of first-order.

Exogenous effects

If the non-stationarities are due to external signals (exogenous effects), then those effects have to be removed first by fitting a deterministic model between the exogenous signals and then fitting a time-series model to the residuals. Else the deterministic and time-series models can be jointly estimated as well. **This is the crux of system identification**.

Fitting trends

Trends in series can be eliminated by two different approaches:

1. By fitting a trend using polynomial fits
2. Estimating the trend by the use of suitable filters

Example

Consider developing a model for the following series

$$v[k] = \underbrace{\alpha_0 + \alpha_1 k}_{m[k]} + w[k] \quad \alpha_0, \alpha_1 \in \mathcal{R}$$

where $v[k]$ is a zero-mean stationary process

Then, the linear trend (in time) can be estimated by **fitting a straight line using a least squares method.**

Example . . . contd.

Alternatively,

$$E(v[k]) = E(\alpha_0 + \alpha_1 k + w[k]) + E(w[k]) = \alpha_0 + \alpha_1 k$$

Thus, the deterministic component (trend) is the average of the series $v[k]$. To put this observation into practice, we replace the theoretical average with an estimate.

The filtering approach essentially uses this idea to extract the trend.

Estimating trends using filters

1. Smoothing with a **two-sided finite moving average filter**

$$\hat{m}[k] = \frac{1}{2M+1} \sum_{j=-M}^{M} v[k-j] \tag{1}$$

- This moving average filter assumes linear trend over the interval $[k-M, k+M]$ and that the average of the remaining terms is close to zero
- The filter provides us, therefore, with an estimate of the linear trend

Estimating trends using filters ...contd.

2. A "clever" choice of filter can be used to eliminate polynomial trends. The Spencer 15-point MA filter can be used to estimate polynomial trends of degree 3.

Spencer's filter coefficients:

$$a_j = 0, |j| > 7, \quad a_j = a_{-j}, |j| \leq 7$$

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
$74/320$	$67/320$	$46/320$	$21/320$	$3/320$	$-5/320$	$-6/320$	$-3/320$