IIT KHARAGPUR

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CERTIFICATION COURSES

OPTIMIZATION IN CHEMICAL ENGINEERING

Linear Programming: The Simplex Method

(WEEK – 9: LECTURE – 41)

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Linear Programming: The Simplex Method

Week 9:

- Simplex method
- Use of artificial variables
- Two phase method

- **Today's Topic:**

- Simplex method

Linear Programming Problem: Simplex Method

Maximize: $Z = \boldsymbol{c}^T \boldsymbol{x}$

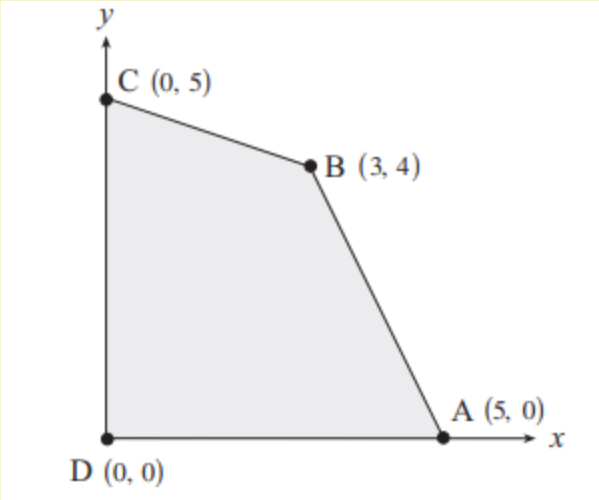
Subject to $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$

$\boldsymbol{x} \geq 0, \quad \boldsymbol{b} \geq 0$

A: m n matrix x: n 1 vector

b: m 1 vector c: n 1 vector

The feasible solution space for a linear programming problem is a polygon. The optimal solution will be at one of the corner points. Thus, an *enumeration approach* to solve the LPP will be to substitute the coordinates of each corner point into the objective function and determine which corner point is optimal. But this will be an inefficient for large scale problems.



Linear Programming Problem: Simplex Method

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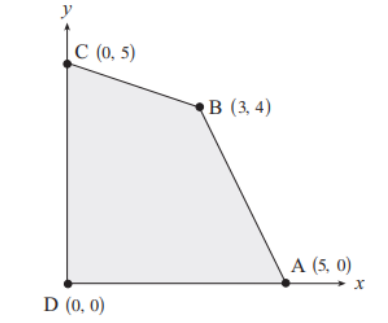
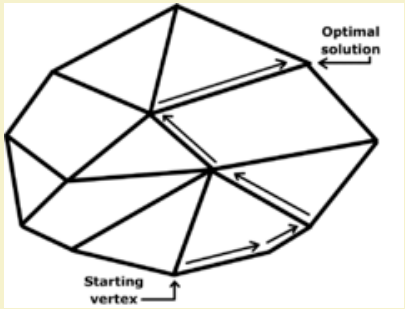
The simplex method is an iterative procedure. Beginning at a vertex of the feasible region, each iteration brings us to another vertex of the feasible region with an *improved* value of the objective function. The iteration ends when the optimal solution is reached. The algorithm tries to find the optimal solution by visiting minimum number of corner points.





The Simplex Method: Canonical Form

Maximize: $Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$

Subject to
$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$
$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$
$$b_1 \geq 0, b_2 \geq 0, \ldots, b_m \geq 0$$

In general, $m < n$ which leads to infinite number of feasible solutions. Hence selection of best feasible solution which maximizes Z is not an easy problem.

To generate the solutions, use first m variables (x_1, …, x_m) to reduce the system to canonical or row echelon form by Gauss-Jordon elimination.

The Simplex Method: Canonical Form

By performing n pivotal operations for any m variables (say, x_1, x_2, …, x_m) called pivotal variables the system of equations can be reduced to canonical form as follows:

$$1x_1 + 0x_2 + \cdots\cdots + 0x_m + \bar{a}_{1,m+1}x_{m+1} + \cdots + \bar{a}_{1s}x_s + \cdots\cdots + \bar{a}_{1n}x_n = \bar{b}_1$$

$$0x_1 + 1x_2 + \cdots\cdots + 0x_m + \bar{a}_{2,m+1}x_{m+1} + \cdots + \bar{a}_{2s}x_s + \cdots\cdots + \bar{a}_{2n}x_n = \bar{b}_2$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots$$

$$0x_1 + \cdots + 1x_r + \cdots + 0x_m + \bar{a}_{r,m+1}x_{m+1} + \cdots + \bar{a}_{rs}x_s + \cdots\cdots + \bar{a}_{rn}x_n = \bar{b}_r$$

$$\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots$$

$$0x_1 + 0x_2 + \cdots\cdots + 1x_m + \bar{a}_{m,m+1}x_{m+1} + \cdots + \bar{a}_{ms}x_s + \cdots\cdots + \bar{a}_{mn}x_n = \bar{b}_m$$

A pivot operation is sequence of elementary row operations that reduce the coefficients of a specified variable to unity in one of the equation and zero elsewhere.

The Simplex Method: Canonical Form

$$
\begin{aligned}
&1x_1 + 0x_2 + \cdots\cdots + 0x_m + \bar{a}_{1,m+1}x_{m+1} + \cdots + \bar{a}_{1s}x_s + \cdots\cdots + \bar{a}_{1n}x_n = \bar{b}_1 \\
&0x_1 + 1x_2 + \cdots\cdots + 0x_m + \bar{a}_{2,m+1}x_{m+1} + \cdots + \bar{a}_{2s}x_s + \cdots\cdots + \bar{a}_{2n}x_n = \bar{b}_2 \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
&0x_1 + \cdots + 1x_r + \cdots + 0x_m + \bar{a}_{r,m+1}x_{m+1} + \cdots + \bar{a}_{rs}x_s + \cdots\cdots + \bar{a}_{rn}x_n = \bar{b}_r \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
&0x_1 + 0x_2 + \cdots\cdots + 1x_m + \bar{a}_{m,m+1}x_{m+1} + \cdots + \bar{a}_{ms}x_s + \cdots\cdots + \bar{a}_{mn}x_n = \bar{b}_m
\end{aligned}
$$

In the canonical form, $x_1, \ldots, x_m$ are termed the basic variables or dependent variables.

$x_{m+1}, \ldots, x_n$ are called nonbasic variables or the independent variables (or non-pivotal variables)

Basic Variables, Basic Feasible Solution

The solution obtained from a canonical form by setting the nonbasic variable or independent variable to zero is called a basic solution.

$$x_i = \bar{b}_i \quad \text{for} \quad i = 1, \cdots, m$$
$$x_i = 0 \quad \text{for} \quad i = (m+1), \cdots, n$$

This solution is known as *basic solution*.

Variables, $x_1, x_2, \ldots, x_m$, are also known as ***basic variables***.
Variables, $x_{m+1}, \ldots, x_n$, are known as ***non-basic variables***.

A basic feasible solution (BFS) is a basic solution in which the values of basic or dependent variables are non-negative. That is, for the above basic solution, $\bar{b}_i \geq 0$

Number of Basic Feasible Solutions

Let us consider that we have a m linearly independent equations in n variables. Any set of m variables can be selected as basic variables out of the possible n variables to obtain a canonical system and a basic solution. Thus the maximum number of basic solutions is

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

Every basic feasible solution is also a basic solution. Hence, the maximum number of basic feasible solution is also

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

Note that by evaluating each basic feasible solution, one can find the optimal solution for the LPP. The simplex method is an efficient iterative procedure that finds the optimal solution by evaluating a small number of the total basic feasible solutions.

The Simplex Method: General Steps

1. Express the Linear Programming Problem in standard form.

2. Start with an initial basic feasible solution in canonical form.

3. Improve the solution if possible by finding another basic feasible solution with a better objective function value. At this step, the simplex method eliminates from consideration all those basic feasible solutions which are inferior compared to the present basic feasible solution.

4. Continue to find improved basic feasible solution. When a particular basic feasible solution is found, and cannot be improved by finding new basic feasible solution, the optimality is reached.

The Simplex Method: How to Obtain Improved BFS?

Let us consider we have an initial basic feasible solution (BFS) in canonical form as follows:

Basic: $x_i = \bar{b}_i \geq 0$ for $i = 1, \cdots, m$

Nonbasic: $x_i = 0$ for $j = m+1, \cdots, n$

The set of basic variables is called a basis, x_B. Let the objective function coefficients of the basic variables be denoted as c_B.

$$\mathbf{x}_B = (x_1, \cdots, x_m), \qquad \mathbf{c}_B = (c_1, \cdots, c_m)$$

Since the nonbasic variables are zero, the value of the objective function Z corresponding to initial BFS is given by:

$$Z = c_B x_B = c_1 \bar{b}_1 + \cdots\cdots + c_m \bar{b}_m = \sum_{i=1}^{m} c_i \bar{b}_i$$

The Simplex Method: How to Obtain Improved BFS?

Given the initial BFS, the simplex method

(a) First examines whether the present BFS is optimal or not

(b) If it is not optimal, the simplex method finds an adjacent basic feasible solution with better (or at least equal) value of objective function (Z)

Definition: An adjacent basic solution differs from the present basic solution in exactly one basic variable.

In order to obtain an adjacent basic feasible solution, the simplex method makes one of the basic variables a nonbasic variable and converts a nonbasic variable to a basic variable in its place.

Question: How to select appropriate basic and nonbasic variables that gives maximum improvement to the objective function?

The Simplex Method: How to Select an Adjacent BFS?

A basic feasible solution has non-negative value. The nonbasic variables are always zero. Thus when we convert a nonbasic variable to a basic variable we increase its value from zero to some positive quantity.

We have to choose that nonbasic variable that causes maximum improvement in the objective function. To determine this, we can increase the value of nonbasic variable by one unit (0 to 1) and check the change in the objective function value. Note that values of other nonbasic variables will remain zero.

The Simplex Method: How to Select Adjacent BFS?

Let us select the nonbasic variable x_s and increase its value from 0 to 1. Values of all other nonbasic variables remain zero. The original canonical form can be rewritten as:

$$
\begin{aligned}
&1x_1 + 0x_2 + \cdots\cdots + 0x_m + \bar{a}_{1,m+1}x_{m+1} + \cdots + \bar{a}_{1s}x_s + \cdots\cdots + \bar{a}_{1n}x_n = \bar{b}_1 \\
&0x_1 + 1x_2 + \cdots\cdots + 0x_m + \bar{a}_{2,m+1}x_{m+1} + \cdots + \bar{a}_{2s}x_s + \cdots\cdots + \bar{a}_{2n}x_n = \bar{b}_2 \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
&0x_1 + \cdots + 1x_r + \cdots + 0x_m + \bar{a}_{r,m+1}x_{m+1} + \cdots + \bar{a}_{rs}x_s + \cdots\cdots + \bar{a}_{rn}x_n = \bar{b}_r \\
&\vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \\
&0x_1 + 0x_2 + \cdots\cdots + 1x_m + \bar{a}_{m,m+1}x_{m+1} + \cdots + \bar{a}_{ms}x_s + \cdots\cdots + \bar{a}_{mn}x_n = \bar{b}_m
\end{aligned}
$$

$$
\begin{aligned}
x_1 + \quad \cdots \quad + \bar{a}_{1s}x_s &= \bar{b}_1 \\
x_2 + \cdots + \bar{a}_{2s}x_s &= \bar{b}_2 \\
x_r + \cdots + \bar{a}_{rs}x_s &= \bar{b}_r \\
&\vdots \\
x_m + \bar{a}_{ms}x_s &= \bar{b}_m
\end{aligned}
$$

The Simplex Method: How to Select Adjacent BFS?

When we increase x_S from 0 to 1, the new solution are obtained as:

$$
\begin{aligned}
x_1 + \quad \cdots \quad + \bar{a}_{1s} x_s &= \bar{b}_1 \\
x_2 + \cdots + \bar{a}_{2s} x_s &= \bar{b}_2 \\
x_r + \cdots + \bar{a}_{rs} x_s &= \bar{b}_r \\
&\vdots \\
x_m + \bar{a}_{ms} x_s &= \bar{b}_m
\end{aligned}
$$

$$
\begin{aligned}
x_i &= \bar{b}_i - \bar{a}_{is} \qquad \text{for } i = 1, \ldots\ldots, m \\
x_s &= 1 \\
x_j &= 0 \qquad \text{for } j = m+1, \ldots\ldots, n \text{ and } j \neq s
\end{aligned}
$$

The new value of the objective function becomes: $Z_{new} = \sum_{i=1}^{m} c_i \left(\bar{b}_i - \bar{a}_{is} \right) + c_s$

Note here x_s = 1 and c_s = cost coefficient of x_s

Inner Product Rule: Which Nonbasic Variable Becomes Basic Variable?

The net change in value of Z per unit change in x_S (known as relative-profit of nonbasic variable x_S),

$$\bar{c}_S = Z_{new} - Z = \left(\sum_{i=1}^{m} c_i \left(\bar{b}_i - \bar{a}_{is} \right) + c_s \right) - \sum_{i=1}^{m} c_i \bar{b}_i = c_s - \sum_{i=1}^{m} c_i \bar{a}_{is}$$

Inner Product Rule

If the relative profit $\bar{c}_s > 0$ then the objective function Z can be improved by making x_s a basic variable. For a maximization problem, we should choose that nonbasic variable which has maximum positive relative profit value. Note, for basic variables $\bar{c}_j = 0$

The relative profit coefficient of a nonbasic variable x_j is given by $\bar{c}_j = c_j - c_B \bar{P}_j$

Here c_B corresponds to the profit coefficients of the basic variables and $\bar{P}_j$ corresponds to the j-th column in the canonical system of the basis under consideration.

The Simplex Method: Condition of Optimality

Inner Product Rule:

For a maximization problem, we should choose that nonbasic variable which has maximum relative profit.

Condition of Optimality:

In a maximization problem, a BFS is optimal if the relative profits of all the nonbasic variables are negative or zero. That is: $\overline{c}_j \leq 0$

When the relative profits of all nonbasic variables are less or equal to zero, then every adjacent BFS has an objective function value lower than the present solution. Thus the current solution is a local maximum. Since a LPP is a convex programming problem, the local maximum becomes global maximum.

Which Basic Variable Becomes Nonbasic?

Let us consider that $\bar{c}_S = \max \bar{c}_j > 0$ and the nonbasic variable x_S has been chosen to enter as basic variable. Then, the values of the basic variables change as:

$$x_i = \bar{b}_i - \bar{a}_{is} x_s \qquad \text{for } i = 1, \ldots, m$$

If $\bar{a}_{is} < 0$, then x_i increases as x_s is increased

If $\bar{a}_{is} = 0$, then x_i does not change as x_s is increased

If $\bar{a}_{is} > 0$, then x_i decreases as x_s is increased and may turn negative (infeasible)

Thus, the maximum increase in x_S is given by the following Minimum Ratio Rule:

$$\max x_s = \min_{\bar{a}_{is} > 0} \left[\frac{\bar{b}_i}{\bar{a}_{is}} \right], \quad \forall i$$

If this happens at $i = r$, x_s is increased to $\dfrac{\bar{b}_r}{\bar{a}_{rs}} = \min_{\bar{a}_{is} > 0} \left[\dfrac{\bar{b}_i}{\bar{a}_{is}} \right], \ \forall i$

Thus x_s is increased to $\bar{b}_r / \bar{a}_{rs}$, the basic variable x_r becomes zero and is replaced by x_s.

Inner Product Rule and Minimum Ratio Rule

Maximization Problems

Inner Product Rule:

$$\bar{c}_S = Z_{new} - Z = \left(\sum_{i=1}^{m} c_i \left(\bar{b}_i - \bar{a}_{is} \right) + c_s \right) - \sum_{i=1}^{m} c_i \bar{b}_i = c_s - \sum_{i=1}^{m} c_i \bar{a}_{is}$$

Nonbasic variable which has maximum positive relative profit value becomes basic variable.

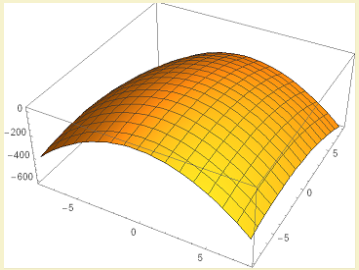
Minimum Ratio Rule:

$$\max x_s = \min_{\bar{a}_{is} > 0} \left[\frac{\bar{b}_i}{\bar{a}_{is}} \right], \quad \forall i$$

Basic variable which has minimum ratio becomes nonbasic variable.

The simplex method then checks if the current BFS is optimal by calculating the relative-profit coefficients for all nonbasic variables and the cycle is repeated until optimality conditions are reached.

Optimization in Chemical Engineering



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Week 9:

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- **Today's Topic:**

- Simplex method (Cont'd)

Simplex Method: Summary of Steps: Maximization Problem

1. Express problem in standard form.
2. Start with an initial BFS.
3. Use Inner Product Rule to find the relative-profit coefficients.
4. If all the relative profit coefficients are nonpositive, then the current BFS is optimal. Otherwise select the nonbasic variable with most positive relative-profit coefficient to enter as basic variable.
5. Apply Minimum Ratio Rule to determine the basic variable that will become nonbasic variable.
6. Checks if the current BFS is optimal by calculating the relative-profit coefficients for all nonbasic variables and repeat the cycle until optimality conditions are reached.

Linear Programming Problem: Simplex Method

Example:

Maximize $Z = 6x_1 + 8x_2$

Subject to $5x_1 + 10x_2 \leq 60$

$x_1 + x_2 \leq 10$

$x_1, x_2 \geq 0$

Express the LPP in standard form by introducing two slack variables S_1 and S_2.

$$\text{Maximize } Z = 6x_1 + 8x_2 + 0 \cdot S_1 + 0 \cdot S_2$$

$$\text{Subject to: } 5x_1 + 10x_2 + S_1 = 60$$

$$x_1 + x_2 + S_2 = 10$$

$$x_1, x_2, S_1, S_2 \geq 0$$

The Simplex Method: Tableau 1

Maximize $Z = 6x_1 + 8x_2 + 0 \cdot S_1 + 0 \cdot S_2$

Subject to: $5x_1 + 10x_2 + S_1 = 60$

$x_1 + x_2 + S_2 = 10$

$x_1, x_2, S_1, S_2 \geq 0$

Most positive $\bar{c}_j$ enters basis

Row with minimum $\frac{\bar{b}_i}{\bar{a}_{is}}$ leaves basis

$$\overline{C_j} = C_j - \sum_{i=1}^{2} (C_B)_i (a_{ij})$$

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
0	S_1	5	10	1	0	60	$\frac{60}{10} = 6$
0	S_2	1	1	0	1	10	$\frac{10}{1} = 10$
$\bar{C}_j$		6	8	0	0	$Z = 0$	

S_1 Leaves basis

x_2 enters basis

The Simplex Method: Tableau 1 (Cont'd)

Perform elementary row operations on Pivot Row (Row 1).

1. Multiply 1st Row by 1/10
2. Perform: 2nd Row - (1st Row) (1/10)

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
0	S_1	5	10	1	0	60	$\frac{60}{10}=6$
0	S_2	1	1	0	1	10	$\frac{10}{1}=10$
$\bar{C}_j$		6	8	0	0	$Z=0$	

The Simplex Method: Tableau 2

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
8	x_2	$\frac{1}{2}$	1	$\frac{1}{10}$	0	6	$\frac{6}{1/2}=12$
0	S_2	$\frac{1}{2}$	0	$-\frac{1}{10}$	1	4	$\frac{4}{1/2}=8$
$\bar{C}_j$		2	0	$-\frac{4}{5}$	0	$Z=48$	

S_2 Leaves basis

x_1 enters basis

$$\overline{C_j} = C_j - \sum (C_B)_i (a_{ij})$$

$$j=1 \Rightarrow \overline{C_j} = 6 - \left(8\times\frac{1}{2} + 0\times\frac{1}{2}\right) = 2$$

$$j=3 \Rightarrow \overline{C_j} = 0 - \left(8\times\frac{1}{10} + 0\left(\frac{-1}{10}\right)\right) = -\frac{8}{10} = -\frac{4}{5}$$

$$Z = 6x_1 + 8x_2 + 0\cdot S_1 + 0\cdot S_2$$
$$= 8\times 6 = 48$$

The Simplex Method: Tableau 2 (Cont'd)

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
8	x_2	$\frac{1}{2}$	1	$\frac{1}{10}$	0	6	$\frac{6}{1/2}=12$
0	S_2	$\frac{1}{2}$	0	$-\frac{1}{10}$	1	4	$\frac{4}{1/2}=8$
$\bar{C}_j$		2	0	$-\frac{4}{5}$	0	$Z=48$	

Perform elementary row operations on Pivot Row (Row 2).

1. Multiply 2nd Row by 2
2. Perform: 1st Row + 2nd Row (-1)

The Simplex Method: Tableau 3

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
8	x_2	0	1	$\frac{1}{5}$	-1	2	
6	x_1	1	0	$-\frac{1}{5}$	2	8	
$\bar{C}_j$		0	0	$-\frac{2}{5}$	-4	$Z = 64$	

All $\overline{C_j} \leq 0$ optimality reached

$$j = 1 \Rightarrow \overline{C_j} = 6 - (8 \times 0 + 6 \times 1) = 6 - 6 = 0$$

$$j = 2 \Rightarrow \overline{C_j} = 8 - (8 \times 1 + 6 \times 0) = 8 - 8 = 0$$

$$j = 3 \Rightarrow \overline{C_j} = 0 - \left(\frac{8}{5} - \frac{6}{5}\right) = \frac{-2}{5}$$

$$j = 4 \Rightarrow \overline{C_j} = 0 - (-8 + 12) = -4$$

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
0	S_1	5	10	1	0	60	$\frac{60}{10}=6$
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$\bar{C}_j$		6	8	0	0	$Z=0$	

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
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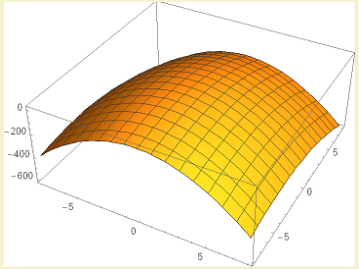
0	S_2	$\frac{1}{2}$	0	$-\frac{1}{10}$	1	4	$\frac{4}{1/2}=8$
$\bar{C}_j$		2	0	$-\frac{4}{5}$	0	$Z=48$	

Summary

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		6	8	0	0		
		x_1	x_2	S_1	S_2		
8	x_2	0	1	$\frac{1}{5}$	-1	2	
6	x_1	1	0	$-\frac{1}{5}$	2	8	
$\bar{C}_j$		0	0	$-\frac{2}{5}$	-4	$Z=64$	

Optimization in Chemical Engineering



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2. Start with an initial BFS in canonical form and set up the initial Tableau.
3. Use the Inner Product Rule to find the relative-profit coefficients $\overline{c}_j$
4. If all the $\overline{c}_j \leq 0$, then the current BFS is optimal. Otherwise select the nonbasic variable with most positive $\overline{c}_j$ value to enter the basis as basic variable.
5. Apply the Minimum Ratio Rule to determine the basic variable that will leave the basis and become nonbasic variable.
6. Perform the pivot operation to get new Tableau and new BFS.
7. Check if the current BFS is optimal by calculating the relative-profit coefficients for all nonbasic variables and go to Step-4. Repeat the cycle until optimality conditions are reached.

Simplex Method: Another Example

A chemical manufacturing company uses 3 different raw materials A, B, and C to produce 3 different products D, E, and F. What production schedule will maximize the daily profit?

Raw Material	kg required per kg of product			Maximum availability per day, kg
	D	E	F	
A	1	1	4	100
B	1	2	1	150
C	3	2	1	320
Profit per kg	**2**	**3**	**1**	

Linear Programming Problem: Formulation

Let the daily production schedule be: x_1 = kg of D
x_2 = kg of E
x_3 = kg of F

Constraint on A: $x_1 + x_2 + 4x_3 \leq 100$

Constraint on B: $x_1 + 2x_2 + x_3 \leq 150$

Constraint on C: $3x_1 + 2x_2 + x_3 \leq 320$

Profit per day:

$Z = 2x_1 + 3x_2 + x_3$

Raw Material	kg required per kg of product			Maximum availability per day, kg
	D	E	F	
A	1	1	4	100
B	1	2	1	150
C	3	2	1	320
Profit per kg	2	3	1	

Complete formulation:

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to:

$x_1 + x_2 + 4x_3 \leq 100$

$x_1 + 2x_2 + x_3 \leq 150$

$3x_1 + 2x_2 + x_3 \leq 320$

$x_1, x_2, x_3 \geq 0$

Simplex Method: LPP in Standard Form

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to:

$x_1 + x_2 + 4x_3 \leq 100$

$x_1 + 2x_2 + x_3 \leq 150$

$3x_1 + 2x_2 + x_3 \leq 320$

$x_1,\ x_2,\ x_3 \geq 0$

Express the LPP in standard form by introducing two slack variables S_1, S_2, and S_3.

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to:

$x_1 + x_2 + 4x_3 + S_1 = 100$

$x_1 + 2x_2 + x_3 + S_2 = 150$

$3x_1 + 2x_2 + x_3 + S_3 = 320$

$x_1,\ x_2,\ x_3, S_1, S_2, S_3 \geq 0$

The Simplex Method: Tableau 1

C_B	Basis		C_j					Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		2	3	1	0	0	0		
		x_1	x_2	x_3	S_1	S_2	S_3		
0	S_1	1	1	4	1	0	0	100	100
0	S_2	1	2	1	0	1	0	150	$\frac{150}{2} = 75$
0	S_3	3	2	1	0	0	1	320	$\frac{320}{2} = 160$
$\bar{C}_j$		2	3	1	0	0	0	$Z = 0$	

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to: $x_1 + x_2 + 4x_3 + S_1 = 100$

$x_1 + 2x_2 + x_3 + S_2 = 150$

$3x_1 + 2x_2 + x_3 + S_3 = 320$ $\quad x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Most positive $\bar{c}_j$ enters basis $\quad \overline{C_j} = C_j - \sum_{i=1}^{3} (C_B)_i (a_{ij})$

Row with minimum $\frac{\bar{b}_i}{\bar{a}_{is}}$ leaves basis

The Simplex Method: Tableau 1(Cont'd)

C_B	Basis		C_j					Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		2	3	1	0	0	0		
		x_1	x_2	x_3	S_1	S_2	S_3		
0	S_1	1	1	4	1	0	0	100	100
3	x_2	1	2	1	0	1	0	150	$\frac{150}{2} = 75$
0	S_3	3	2	1	0	0	1	320	$\frac{320}{2} = 160$
$\bar{C}_j$			0		0		0	Z	

Perform elementary row operations on Pivot Row (Row 2).

1. Multiply 2nd Row by 1/2
2. Perform: $R_1 + (-1/2)R_2$
3. Perform: $R_3 + (-1)XR_2$

Nonbasic x_2 enters basis
Basic S_2 leaves basis

The Simplex Method: Tableau 2

C_B	Basis		C_j					Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		2	3	1	0	0	0		
		x_1	x_2	x_3	S_1	S_2	S_3		
0	S_1	1/2	0	7/2	1	-1/2	0	25	$\frac{25}{1/2} = 50$
3	x_2	1/2	1	1/2	0	1/2	0	75	$\frac{75}{1/2} = 150$
0	S_3	2	0	0	0	-1	1	170	$\frac{170}{2} = 85$
$\bar{C}_j$		1/2	0	-1/2	0	-3/2	0	$Z = 225$	

$$\overline{C_j} = C_j - \sum_{i=1}^{3} (C_B)_i (a_{ij})$$

The Simplex Method: Tableau 2

C_B	Basis		C_j					Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		2	3	1	0	0	0		
		x_1	x_2	x_3	S_1	S_2	S_3		
2	x_1	1/2	0	7/2	1	-1/2	0	25	$\frac{25}{1/2} = 50$
3	x_2	1/2	1	1/2	0	1/2	0	75	$\frac{75}{1/2} = 150$
0	S_3	2	0	0	0	-1	1	170	$\frac{170}{2} = 85$
$\bar{C}_j$		0	0				0	Z	

$$\bar{C}_j = C_j - \sum_{i=1}^{3} (C_B)_i (a_{ij})$$

Perform elementary row operations on Pivot Row (Row 1).

1. Multiply 1st Row by 2
2. Perform: $R_2 - R_1$
3. Perform: $R_3 - 4R_2$

Nonbasic x_1 enters basis
Basic S_1 leaves basis

The Simplex Method: Tableau 3

C_B	Basis		C_j					Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		2	3	1	0	0	0		
		x_1	x_2	x_3	S_1	S_2	S_3		
2	x_1	1	0	7	2	-1	0	50	
3	x_2	0	1	-3	-1	1	0	50	
0	S_3	0	0	-14	-4	1	1	70	
$\bar{C}_j$		0	0	-4	-1	-1	0	$Z = 250$	

$$\overline{C_j} = C_j - \sum_{i=1}^{3} (C_B)_i (a_{ij})$$

All $\bar{C}_j \le 0$

Optimality condition is reached

$x_1 = 50, x_2 = 50, x_3 = 0$

$\text{Profit} = 2x_1 + 3x_2 + x_3$

$= 2(50) + 3(50) = 250$

The optimal daily production schedule is: x_1 = 50 kg of D

x_2 = 50 kg of E

x_3 = 0 kg of F

The Simplex Method: Geometric Interpretation

The simplex method is based on the same geometrical considerations as the graphical method. This can be seen by looking at a problem that can be readily solved by both methods.

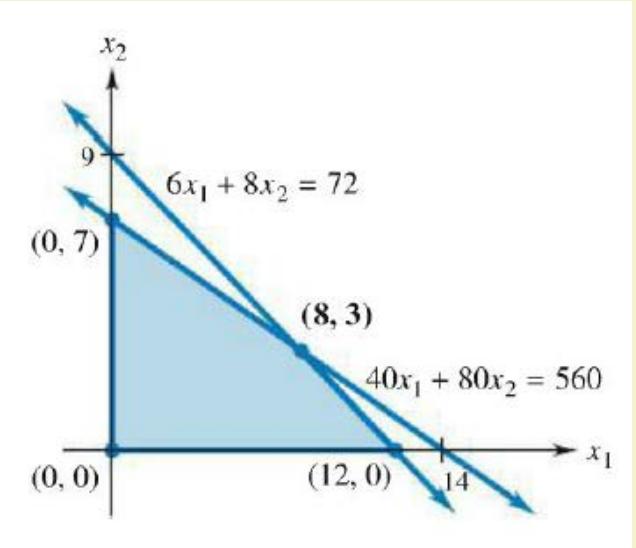
Graphical Solution

$$\text{Maximize } Z = 8x_1 + 12x_2$$

$$\text{Subject to: } 40x_1 + 80x_2 \le 560$$

$$6x_1 + 8x_2 \le 72$$

$$x_1, x_2 \ge 0$$



The Simplex Method: Geometric Interpretation

The simplex method provides a means of moving from one basic feasible solution to another until we reach the optimal value of the objective function.

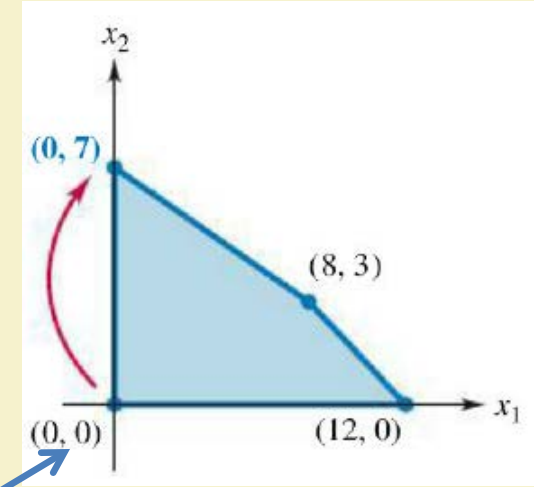
Solution by Simplex Method

Maximize $Z = 8x_1 + 12x_2$

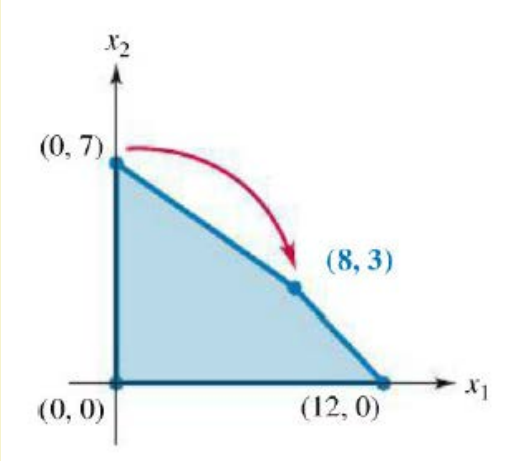
Subject to: $40x_1 + 80x_2 \leq 560$

$6x_1 + 8x_2 \leq 72$

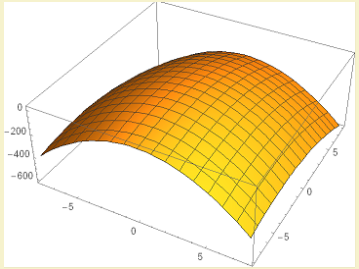
$x_1, x_2 \geq 0$



Starting BFS



Optimization in Chemical Engineering



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CERTIFICATION COURSES

OPTIMIZATION IN CHEMICAL ENGINEERING

Linear Programming: The Simplex Method

(WEEK – 9: LECTURE – 44)

Dr. Debasis Sarkar

Department of Chemical Engineering

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Linear Programming: The Simplex Method

Week 9:

- Simplex method
- Use of artificial variables
- Two phase method

- **Today's Topic:**

- Simplex method: Special cases

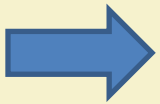
Simplex Method: Minimization Problems

There are two approaches to solve minimization problems:

Approach-1:

Convert the minimization problem to an equivalent maximization problem by multiplying the objective function by -1.

$$\text{Minimize } f = x_1 + 2x_2 + x_3$$
$$\text{Subject to: } 2x_1 + x_2 + x_3 \le 50$$
$$x_1 + 4x_2 + x_3 \le 70$$
$$x_1 + 2x_2 + 3x_3 \le 100$$
$$x_1,\ x_2,\ x_3 \ge 0$$



$$\text{Maximize } (-f) = -x_1 - 2x_2 - x_3$$
$$\text{Subject to: } 2x_1 + x_2 + x_3 \le 50$$
$$x_1 + 4x_2 + x_3 \le 70$$
$$x_1 + 2x_2 + 3x_3 \le 100$$
$$x_1,\ x_2,\ x_3 \ge 0$$

Simplex Method: Minimization Problems

Approach-2:

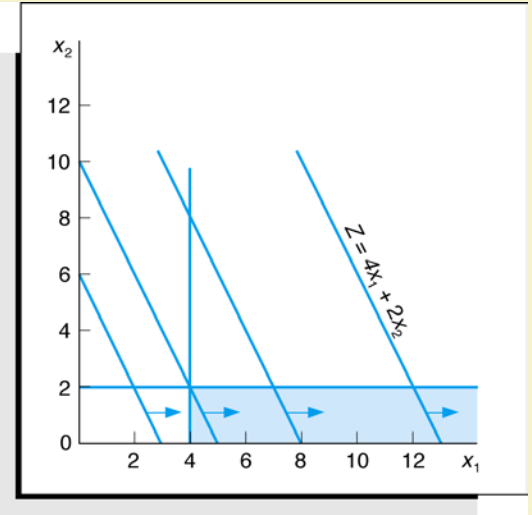
The coefficients in the $\overline{c}_j$ row give the net change in the value of Z per unit increase in the nonbasic variable. A negative coefficient in the $\overline{c}_j$ indicates that the corresponding nonbasic variable will decrease the value of the objective function.

Thus, the nonbasic variable with most negative $\overline{c}_j$ enters the basis. Note that the Minimum Ration Rule remain unchanged.

Optimality criteria: All $\overline{c}_j \geq 0$

Simplex Method: Unbounded Optimum

A Linear Programming Problem has unbounded optimum when its optimal value is not finite. For maximization problem, the optimum tends to $+\infty$ and for minimization problem it tends to $-\infty$.



Maximize $Z = 4x_1 + 2x_2$

subject to: $x_1 \geq 4$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

Simplex Method: Unbounded Optimum

In Simplex Method, unbounded optimum of LPP is indicated when the Minimum ratio Rule fails to identify the basic variable that will leave the basis.

Minimum Ratio Rule: $$\max x_s = \min_{\bar{a}_{is}>0} \left[\frac{\bar{b}_i}{\bar{a}_{is}} \right], \quad \forall i$$

Suppose we have identified the nonbasic variable with most positive reduced-profit coefficient to enter the basis. Now we use the Minimum Ratio rule to decide which basic variable will leave basis. At this stage, the Minimum Ratio rule may fail if none of the constraint coefficient of the nonbasic variable selected to enter the basis is positive.

Simplex Method: Unbounded Optimum

Consider the following Simplex Tableau for a maximization problem.

C_B	Basis	C_j				Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		2	3	0	0		
		x_1	x_2	S_1	S_2		
0	S_1	-2	0	1	0	6	??
3	x_2	-3	1	0	1	4	??
$\bar{C}_j$		11	0	0	-3	$Z = 12$	

$$\min_{\bar{a}_{is} > 0} \left[\frac{\bar{b}_i}{\bar{a}_{is}} \right]$$

$$x_i = \bar{b}_i - \bar{a}_{is} x_s \qquad \text{for } i = 1, \ldots\ldots, m$$

As x_1 increases, both the basic variables x_2 and S_1 can be increased indefinitely without being infeasible. Thus, we have an unbounded optimum to the LPP.

Simplex Method: Degeneracy and Cycling

Definition: A BFS in which one or more of the basic variables are zero is called a degenerate basic feasible solution.

A BFS in which all the basic variables are positive is called nondegenerate.

Consider that there is tie between two rows while applying the Minimum Ratio rule for a Simplex Tableau. This tie can be broken arbitrarily by choosing any of the basic variables to leave basis. When a tie happens, at least one basic variable will be zero in the next iteration and we will get a degenerate solution.

Simplex Method: Degeneracy and Cycling

What is the problem with degenerate solution?

When we have degenerate BFS, it is possible that the minimum ratio will be zero. This implies that when a basis change is performed there will not be any improvement in the value of the objective function.

Thus Simplex method can go through a series of iterations without making any improvement in the objective function. This is known as cycling.

Note that cycling may not happen often. Also, a solution may be temporarily degenerate.

Simplex Method: Degeneracy and Cycling: Bland's Rule

Maximize $Z = 10x_1 + 3x_2$

Subject to: $x_1 + x_2 \leq 4$

$5x_1 + 2x_2 \leq 11$

$x_2 \leq 4$

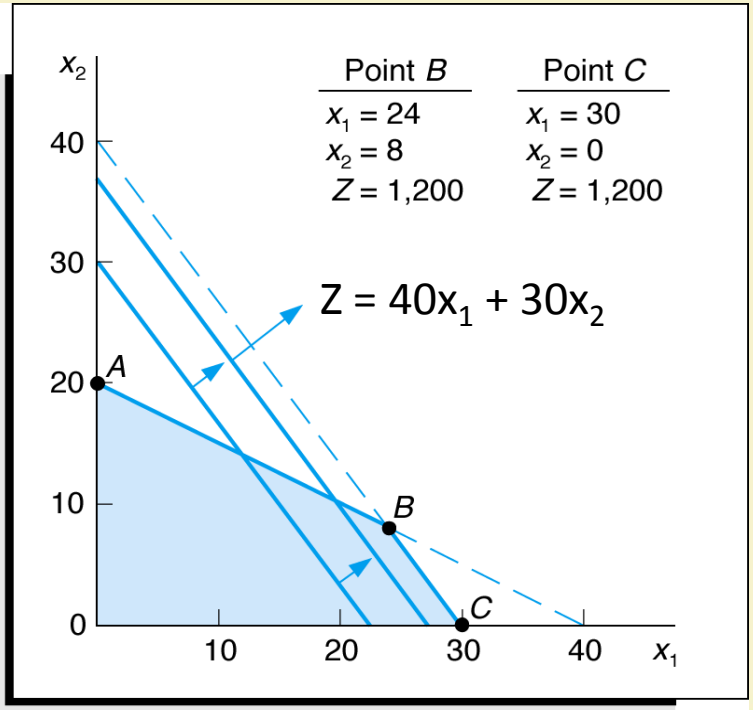
$x_1, x_2 \geq 0$

This is a simple example where cycling is possible.

Bland's Rule to Prevent Cycling (Smallest subscript pivoting rule):

1. Among nonbasic variables that have a positive relative-profit coefficient, choose the one with least index.
2. Among rows that satisfy the Minimum Ratio rule, choose the one with least index (in case of ties in the ratio test).

Simplex Method: Alternate Optimum



LP will have alternate optimal solution when the objective function is parallel to a constraint line.

Maximize $Z = 40x_1 + 50x_2$

subject to: $x_1 + 2x_2 \leq 40$

$4x_1 + 3x_2 \leq 120$

$x_1, x_2 \geq 0$

Every point on line BC is an optimal solution.

Simplex Method: Alternate Optimum

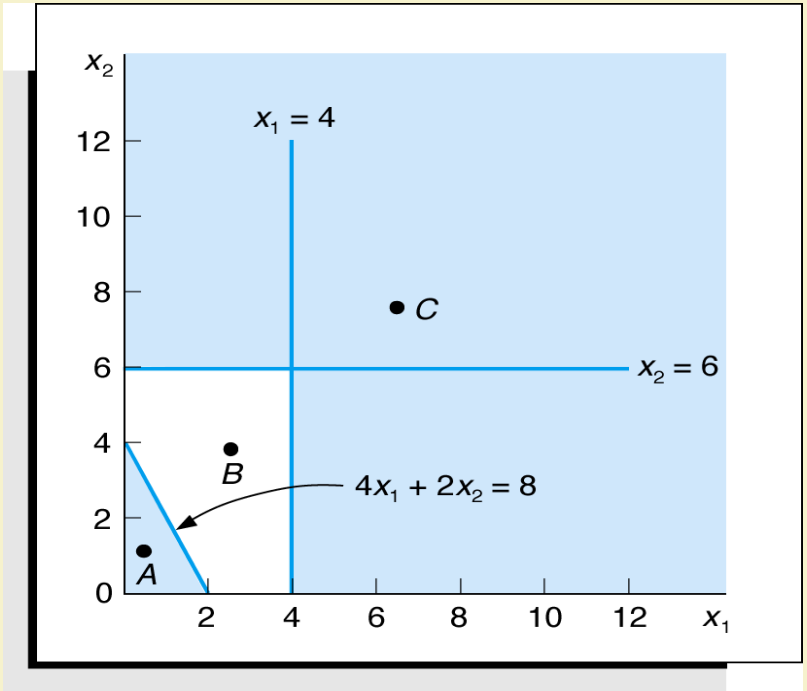
An alternate optimal solution is indicated when there exists a nonbasic variable whose relative profit coefficient is zero in the optimal Tableau. When all relative-profit coefficients for nonbasic variables are strictly Less-Than-Zero (< 0), we have unique solution.

C_B	Basis	C_j					Solution b_j (constants)	$\frac{\bar{b}_i}{\bar{a}_{is}}$ ratio
		3	2	0	0	0		
		x_1	x_2	S_1	S_2	S_3		
0	S_1	0	0	1	-1/5	8/5	60	
0	x_2	0	1	0	1/5	-3/5	10	
	x_1	1	0		1/5	2/5		
$\bar{C}_j$		0	0	0	-1	0	$Z = 14$	

Final Tableau.

Note that S_5 can also be made a basic variable and Z will still be 14. Thus we have alternate optimal solution.

Simplex Method: Infeasible Solution



Every possible solution violates at least one constraint. Consider points A, B, and C.

Maximize $Z = 5x_1 + 3x_2$

subject to: $4x_1 + 2x_2 \leq 8$

$x_1 \geq 4$

$x_2 \geq 6$

$x_1, x_2 \geq 0$

Simplex Method: Infeasible Solution

A Linear Programming Problem with inconsistent constraints have no feasible solution.

This situation can never occur if all the constraints are Less-Or-Equality (≤) type and the Right Hand Side constants are nonnegative. In such cases, introduction of slack variables always provides a BFS.

For other types of constraints, we will use artificial variables to get initial BFS. But to get a solution, the LPP must have a feasible space.

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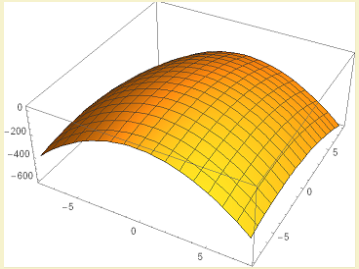

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OPTIMIZATION IN CHEMICAL ENGINEERING

Linear Programming: The Simplex Method

(WEEK – 9: LECTURE – 45)

Dr. Debasis Sarkar

Department of Chemical Engineering

Indian Institute of Technology Kharagpur

Linear Programming: The Simplex Method

Week 9:

- Simplex method
- Use of artificial variables
- Two phase method

- **Today's Topic:**

- Use of artificial variables
- Two phase method
- MATLAB for LPP

Simplex Method: Summary of Computational Steps: Maximization Problem

1. Express problem in standard form.
2. Start with an initial BFS in canonical form and set up the initial Tableau.
3. Use the Inner Product Rule to find the relative-profit coefficients $\overline{c}_j$
4. If all the $\overline{c}_j \leq 0$, then the current BFS is optimal. Otherwise select the nonbasic variable with most positive $\overline{c}_j$ value to enter the basis as basic variable.
5. Apply the Minimum Ratio Rule to determine the basic variable that will leave the basis and become nonbasic variable.
6. Perform the pivot operation to get new Tableau and new BFS.
7. Check if the current BFS is optimal by calculating the relative-profit coefficients for all nonbasic variables and go to Step-4. Repeat the cycle until optimality conditions are reached.

Initial BFS Not Available: Use of Artificial Variable

If initial BFS is not readily available, the Simplex Tableau can not be formed.

When initial BFS is not available follow these steps:

1. Convert the LPP in its standard form (constraints are equations, nonnegative variables, nonnegative RHS).
2. Examine each constraint for existence of a basic variable.
3. If a basic variable is not available, a new variable is added to act as basic variable.
4. Finally, all constraints will have a basic variable and the system will be in canonical form. A BFS can now easily be obtained.

The added variables are called artificial variables. They have no meaning to the original problem and they are used only to get the canonical form. As a part of Simplex procedure, these artificial variables will be forced to zero in later steps.

Linear Programming Problem: Artificial Variables

$$\text{Maximize } Z = 3x_1 - x_2 - x_3$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 \le 11$$

$$-4x_1 + x_2 + 2x_3 \ge 3$$

$$2x_1 \quad - x_3 = -1$$

$$x_1,\ x_2,\ x_3 \ge 0$$

Given LPP

$$\text{Maximize } Z = 3x_1 - x_2 - x_3$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 + S_1 = 11$$

$$-4x_1 + x_2 + 2x_3 - S_2 = 3$$

$$-2x_1 \quad + x_3 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2 \ge 0$$

LPP in standard form

S_1 = slack variable

S_2 = surplus variable

There are no basic variables for constraints 2 and 3. Note that S_2 can not be a basic variable. Because if we consider x_1, x_2, x_3 as non-basic variables, S_2 will be -3 < 0.

Linear Programming Problem: Artificial Variables

$$\text{Maximize } Z = 3x_1 - x_2 - x_3$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 + S_1 = 11$$

$$-4x_1 + x_2 + 2x_3 - S_2 = 3$$

$$-2x_1 + x_3 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2 \geq 0$$

LPP in standard form

S_1 = slack variable

S_2 = surplus variable

$$\text{Maximize } Z = 3x_1 - x_2 - x_3$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 + S_1 = 11$$

$$-4x_1 + x_2 + 2x_3 - S_2 + R_1 = 3$$

$$-2x_1 + x_3 + R_2 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2, R_1, R_2 \geq 0$$

"Artificial LPP" in canonical form. Note we have added two Artificial variables: R_1 and R_2

BFS for the "artificial LPP": $x_1 = x_2 = x_3 = S_2 = 0$, $S_1 = 11$, $R_1 = 3$, $R_2 = 1$

This is not BFS for original LPP as R_1, R_2 are not equal to zero. How to make R_1 and R_2 zero?

Approach-1: Big M Method: Artificial Variables

$$\text{Maximize } Z = 3x_1 - x_2 - x_3 - MR_1 - MR_2$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 + S_1 = 11$$

$$-4x_1 + x_2 + 2x_3 - S_2 + R_1 = 3$$

$$-2x_1 + x_3 + R_2 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2, R_1, R_2 \geq 0$$

"Artificial LPP" in canonical form

To force the artificial variables R_1 and R_2 to become zero, we penalize them in the objective function.

Here M is taken as a large number (say 100 or above). Thus to maximize the objective function R_1 and R_2 should be zero.

Approach-2: Two-Phase Simplex Method

We have assumed that a feasible basis is always given. But in practice, it may not be always easy to spot a feasible basis.

Two-phase method is an approach to handle artificial variables whenever they are added. Here LPP is solved in two phases as follows.

Phase 1: We solve an auxiliary LPP to either get a feasible basis or conclude that the given LPP is infeasible.

Phase 2: We solve the original LPP starting from the feasible basis found in Phase 1.

Two-Phase Simplex Method: Example

Phase – 1: Objective is to get a BFS to the given LPP.

Given LPP:

$$\text{Maximize } Z = 3x_1 - x_2 - x_3$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 + S_1 = 11$$

$$-4x_1 + x_2 + 2x_3 - S_2 = 3$$

$$-2x_1 + x_3 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2 \geq 0$$

Auxiliary LPP in Standard Form:

$$\text{Minimize } f = R_1 + R_2$$

$$\text{Subject to: } x_1 - 2x_2 + x_3 + S_1 = 11$$

$$-4x_1 + x_2 + 2x_3 - S_2 + R_1 = 3$$

$$-2x_1 + x_3 + R_2 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2, R_1, R_2 \geq 0$$

Two-Phase Simplex Method: Example

Phase – 1: Objective is to get a BFS for the given LPP.

Note the objective function *f*. If the minimum value of the artificial problem is zero, then both artificial variables are zero and we will have a BFS to the original problem.

If the minimum value of artificial problem is not zero, then at least one of the artificial variables is positive. This means that the original problem is infeasible and we stop.

Auxiliary LPP in Standard Form:

Minimize $f = R_1 + R_2$

Subject to: $x_1 - 2x_2 + x_3 + S_1 = 11$

$$-4x_1 + x_2 + 2x_3 - S_2 + R_1 = 3$$

$$-2x_1 + x_3 + R_2 = 1$$

$$x_1,\ x_2,\ x_3, S_1, S_2, R_1, R_2 \geq 0$$

Two-Phase Simplex Method: Example

Phase – 2: Objective is to solve the original LPP with the BFS obtained in Phase-1.

Maximize $Z = 3x_1 - x_2 - x_3$

Subject to: $x_1 - 2x_2 + x_3 + S_1 = 11$

$-4x_1 + x_2 + 2x_3 - S_2 + a_1 = 3$

$-2x_1 + x_3 + a_2 = 1$

$x_1, x_2, x_3, S_1, S_2, a_1, a_2 \geq 0$

The final Tableau of Phase-1 becomes the initial Tableau for Phase-2, but the objective function is now changed to the original form.

The Simplex method can now be followed to find the optimal solution.

Simplex Method: Handling Unrestricted Variables

In the standard form of LPP, all the variables must be non-negative. A variable which is free in sign may be replaced by difference of two non-negative variables.

Maximize $Z = 3x_1 - x_2 - x_3$

Subject to: $x_1 - x_2 + x_3 \le 9$

$4x_1 - x_2 + 2x_3 \ge 5$

$-2x_1 + x_3 = -2$

$x_1, x_2 \ge 0,\ x_3$ free in sign

Replace x_3 as: $x_3 = x_4 - x_5$

where, $x_4, x_5 \ge 0$

Maximize $Z = 3x_1 - x_2 - x_4 - x_5$

Subject to: $x_1 - x_2 + x_4 - x_5 + S_1 = 9$

$4x_1 - x_2 + 2x_4 - 2x_5 - S_2 = 5$

$2x_1 - x_4 + x_5 = 2$

$x_1, x_2, x_4, x_5 \ge 0$

MATLAB for Linear Programming problem

MATLAB solves a linear programming problem:

$$\underset{x}{\text{Minimize}} \quad f^T x$$

$$\text{Such that: } A \cdot x \leq b$$

$$A_{eq} = b_{eq}$$

$$LB \leq x \leq UB$$

f, *x*, *b*, b_{eq}, *lb*, and *ub* are vectors, and *A* and A_{eq} are matrices.

MATLAB for Linear Programming problem

```
x = linprog(f,A,b)
x = linprog(f,A,b,Aeq,beq)
x = linprog(f,A,b,Aeq,beq,lb,ub)
x = linprog(f,A,b,Aeq,beq,lb,ub,options)
```

x = linprog(f,A,b) solves min f'*x such that A*x ≤ b.

x = linprog(f,A,b,Aeq,beq) includes equality constraints Aeq*x = beq. Set A = [] and b = [] if no inequalities exist.

x = linprog(f,A,b,Aeq,beq,lb,ub) defines a set of lower and upper bounds on the design variables, x, so that the solution is always in the range lb ≤ x ≤ ub. Set Aeq = [] and beq = [] if no equalities exist.

MATLAB for Linear Programming problem

```
[x,fval] = linprog(___)
[x,fval,exitflag,output] = linprog(___)
[x,fval,exitflag,output,lambda] = linprog(___)
```

[x,fval] = linprog(___), for any input arguments, returns the value of the objective function fun at the solution x: fval = f'*x.

[x,fval,exitflag,output,lambda] = linprog(___) additionally returns a structure lambda whose fields contain the Lagrange multipliers at the solution x.

exitflag that describes the exit condition, and a structure output that contains information about the optimization process.

MATLAB for Linear Programming problem: Example

Maximize $Z = 150x_1 + 175x_2$

Subject to:

$7x_1 + 11x_2 \leq 77$

$10x_1 + 8x_2 \leq 80$

$x_1 \leq 9$

$x_2 \leq 6$

$x_1, x_2 \geq 0$

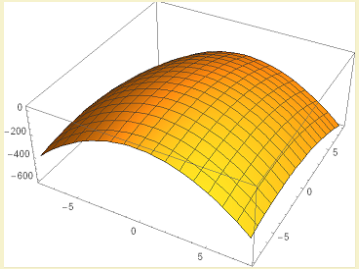
```
A = [7  11; 10  8];
b = [77  80];
f = [-150  -175];
lb = [0  0];
ub = [9  6];
x = linprog(f, A, b, [],[],lb,ub)
```

```
Optimal solution found.

x =
4.8889
3.8889
```

Try this: Same solution will be obtained: `x = linprog(f, A, b)`

Optimization in Chemical Engineering



Thank You