Module 3:

“Convective” heat and mass transfer

Lecture 32:

Dispersion (Continued)

(A) Taylor's Experiment for measuring Taylor Dispersion

Measurements of C_m vs. x at a certain time can be used to deduce D_{eff} and thus D.

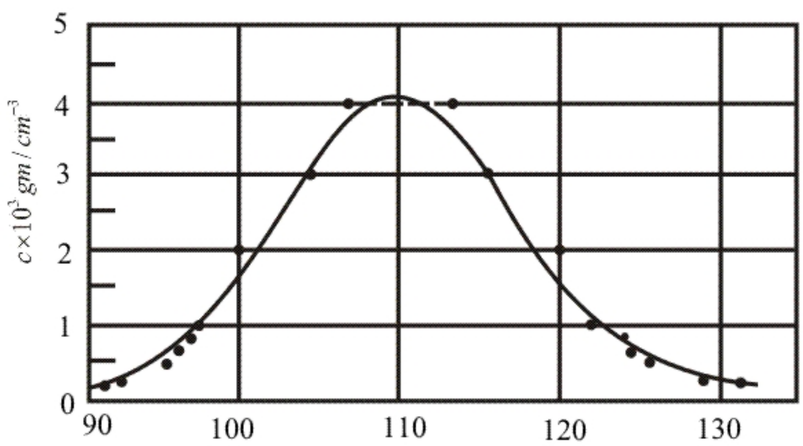


Figure showing concentration distribution of $KMnO_4$ in the stream at 11 minutes after injection of $KMnO_4$ (Taylor's experiment).

Under what conditioning is the above analysis valid?

1) Neglecting axial diffusion compared to convection implies that

$$D << D_{eff} \quad \text{Or, (using equation 3.5.8)}$$

$$\frac{RU}{D} = Pe >> 7 \qquad (3.5.11)$$

2) We can regard C as a slowly changing function of X (so that to a first approximation $\frac{\partial C}{\partial X} \approx \text{const.}$) when equation (3.5.6) gives

$$\frac{\partial C}{\partial X} = \frac{\partial C_m}{\partial X} + \frac{R^2 U}{4D}\frac{\partial^2 C_m}{\partial X^2}\left(-\frac{1}{3} + \eta^2 - \frac{1}{2}\eta^4\right) \approx \frac{\partial C_m}{\partial X}$$

Or, $\frac{\partial C_m}{\partial X} >> \frac{R^2U}{4D}\frac{\partial^2 C_m}{\partial X^2}$.

If L is the length over which C can change noticeably, this inequality can be written as

$$\frac{4LD}{R^2U} >> 1 \quad or, \quad \frac{4L}{R} >> \frac{UR}{D} \qquad (3.5.12)$$

Combination of (3.5.11) and (3.5.12) yields the condition

$$\frac{4L}{R} >> \frac{UR}{D} >> 7$$

Condition (3.5.12) actually implies "far downstream" where the concentration profile is "fully developed"

(B) Elliptic Regions

General PDE is given by

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} = F\left(x, y, u, u_{x,} u_y\right)$$

Elliptic if $AC - B^2 > 0$ (e.g., Laplace Equations)

Parabolic if $AC - B^2 = 0$ (e.g., unsteady – state heat conduction)

Hyperbolic if $AC - B^2 < 0$ (e.g., Wave equation)

Elliptic equations: Information can "travel" upstream by "diffusion". Conditions at the outflow boundary affect the solution.

Prabolic Equations: Information “travels” downstream only. Whatever happens downstream does not affect the upstream profile.

Example: Axial diffusion effects in Graetz problem

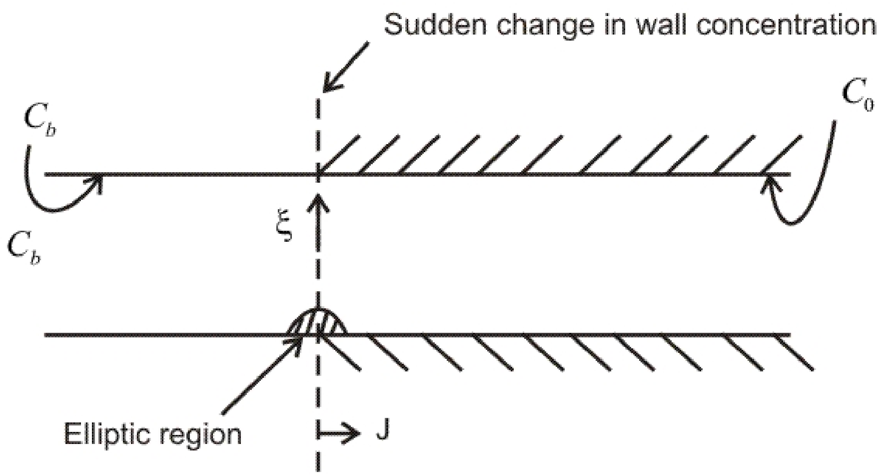
PDE : $U_Z \dfrac{\partial C}{\partial Z} = D\left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial C}{\partial r}\right) + \dfrac{\partial^2 C}{\partial Z^2}\right]$

$$\xi = \frac{r}{R}, \quad \theta = \frac{C_A - C_0}{C_b - C_0}, \quad j = \frac{ZD}{2\langle U_Z\rangle R^2} = \frac{1}{\mathrm{Re}\, Sc}\frac{Z}{R}$$

Dimensionless Equation

$$\underset{A}{\left(1-\xi^2\right)\frac{\partial \theta}{\partial j}} = \underset{B}{\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial \theta}{\partial \xi}\right)} + \underset{C}{\frac{1}{Pe^2}\frac{\partial^2 \theta}{\partial j^2}} \qquad (3.6.1)$$

$$Pe = \frac{2\langle U_Z\rangle R}{D}$$



Term C describes axial diffusion. Without this term equation (3.6.1) is parabolic. With this term equation (3.6.1) is elliptic.

In elliptic region, the full equation (3.6.1) (i.e., including term C) is expected to be important.

Near the wall, velocity changes linearly (Especially for mass Boundary Layer)

$\Rightarrow 1-\xi^2 = 2(1-\xi)$

Introduce "Stretched" Coordinates to observe *Pe* dependence:

$$\begin{matrix} Y = (1-\xi)Pe^n \\ X = jPe^m \end{matrix} \quad \Rightarrow \quad \begin{matrix} n = \frac{1}{2} \\ m = \frac{3}{2} \end{matrix} \quad \Rightarrow \quad \begin{matrix} Y = (1-\xi)Pe^{1/2} \\ X = \left(\frac{Z}{R}\right)Pe^{1/2} \end{matrix}$$

Substitute in eq. (3.6.1) and set terms with *Pe* to zero

(3.6.1) yields

$$2Y\frac{\partial\theta}{\partial X} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} \qquad (3.6.2)$$

Boundary Conditions:

$$\theta = 0 \quad at \quad \xi = 1, \quad j > 0$$

$$\theta \to 1 \quad as \quad j \to -\infty$$

$$\frac{\partial \theta}{\partial \xi} = 0 \qquad at \quad \xi = 0 \qquad (symmetry)$$

$$\theta \to 0 \quad as \quad j \to +\infty$$

$$\theta = 1 \quad at \quad \xi = 1 \ , \quad j < 0 \quad \left(or, \left. \frac{\partial \theta}{\partial \xi} \right|_{\xi = 1} = 0 \right) at \ j < 0$$

Size of elliptic region: $\left. \begin{aligned} (1-\xi) &= Y \, Pe^{-1/2} \\ 0(Y) &= 1 \end{aligned} \right\} \Rightarrow 1 - \xi = 0\left(Pe^{-1/2}\right)$

As *Pe* increases, elliptic region shrinks.
