Module 3:

“Convective” heat and mass transfer

Lecture: 22

Mass Transfer to a Rotating Disk

We have seen in the previous lecture, $v_z = \sqrt{\nu\Omega} H\left(z\sqrt{\frac{\Omega}{\nu}}\right)$

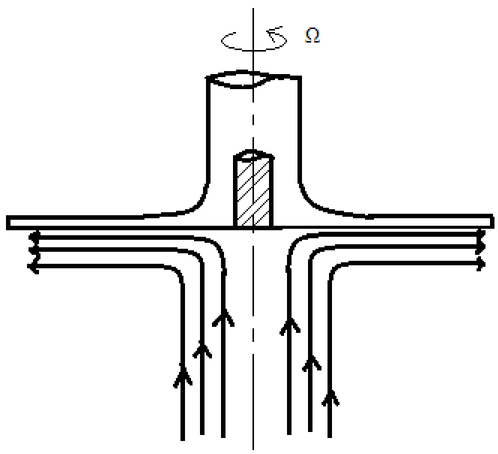

Fig.1. Rotating disk

Since $v_z \neq v_z(r)$ and "fresh" reactant is bought to the disk by v_z, there is no reason for the concentration to depend on anything besides z. Assuming dilute solutions and constant diffusivity, the convective transport equation becomes (Note also that we assume $R_i = 0$)

$$v_z \frac{dC_i}{dz} = D_i \frac{d^2 C_i}{dz^2} \qquad (1)$$

$$\text{B.Cs}: C_i = C_0 \text{ at } z = 0$$
$$C_i = C_\infty \text{ at } z \to \infty$$

Eqn. (1) is an ODE. The flux is $-D\left.\frac{dC_i}{dz}\right|_{z=0}$ independent of "r".

Uniform accessibility of the reactant to the surface

Eqn. (1) is first order w.r.t. $\frac{dC_i}{dz}$. Integration yields

$$\ln\frac{dC_i}{dz} = \frac{1}{D_i}\int_0^z v_z dz + \ln K$$
$$\frac{dC_i}{dz} = K\exp\left(\frac{1}{D_i}\int_0^z v_z dz\right) \qquad (2)$$

Second integration yields

$$C_i = C_0 + K\int_0^z \exp\left(\int_0^z \frac{v_z}{D_i}dz\right)dz \qquad (3)$$

Constant K is found using the B.C.

$$\frac{C_\infty - C_0}{K} = \int_0^z \exp\left(\int_0^z \frac{v_z}{D_i}dz\right)dz$$
$$= \sqrt{\frac{\nu}{\Omega}}\int_0^\infty \exp\left[Sc\int_0^\eta H(J)\,dJ\right]d\eta \qquad (4)$$

Here, $Sc = \frac{\nu}{D_i}$

Finally, setting $\theta = \frac{C_i - C_0}{C_\infty - C_0}$

$$\theta = \frac{\int_0^z \exp\left[\int_0^\eta H(J)\,dJ\right] d\eta}{\int_0^\infty \exp\left[\int_0^\eta H(J)\,dJ\right] d\eta} \qquad (5)$$

Note: θ is a function of J or z.

The normal component of surface flux can be given by $N_{in} = -D_i \left.\frac{dC_i}{dz}\right|_{z=0} = -D_i K$

(6)

In dimensionless form

$$\underbrace{\frac{1}{Sc}\theta'(0)}_{\text{Dimensionless MassTransfer Rate}} = \frac{1}{Sc\int_0^\infty \exp\left[Sc\int_0^\eta H(J)\,dJ\right] d\eta} \qquad (7)$$

$$= \frac{N_{in}}{(C_0 - C_\infty)\sqrt{\nu\Omega}}$$

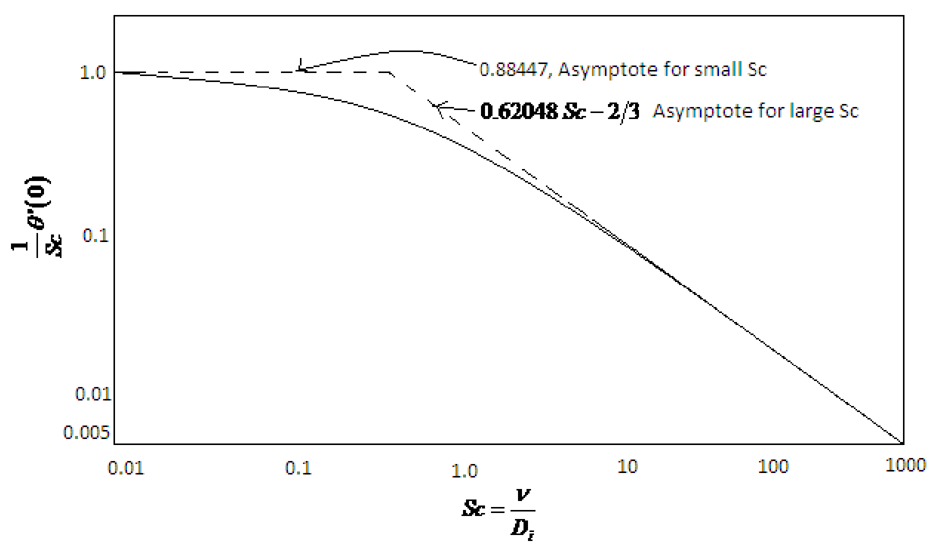

Fig.2. Dimensionless mass-transfer rates for a rotating disk.

The rotating disk is used to find the diffusion coefficient in liquids by an electrochemical method. In this method the current flowing to the disk is measured. If species '*i*' undergoes an electrochemical reaction,

$$\sum_i S_i M_i^{zi} \rightarrow ne^-$$

In this 'S_i' is the stoichiometric coefficient, 'M_i' is any chemical species, 'z_i' is the charge and 'n' is the number of electron exchanged.

The normal component of the flux is

$$N_{in} = -\underbrace{\frac{S_i}{nF}}_{\text{Faraday}\uparrow\text{ constant}} \underbrace{i_n}_{\text{Measured current}} \qquad (8)$$

Here n = 4 for O_2 electrode.

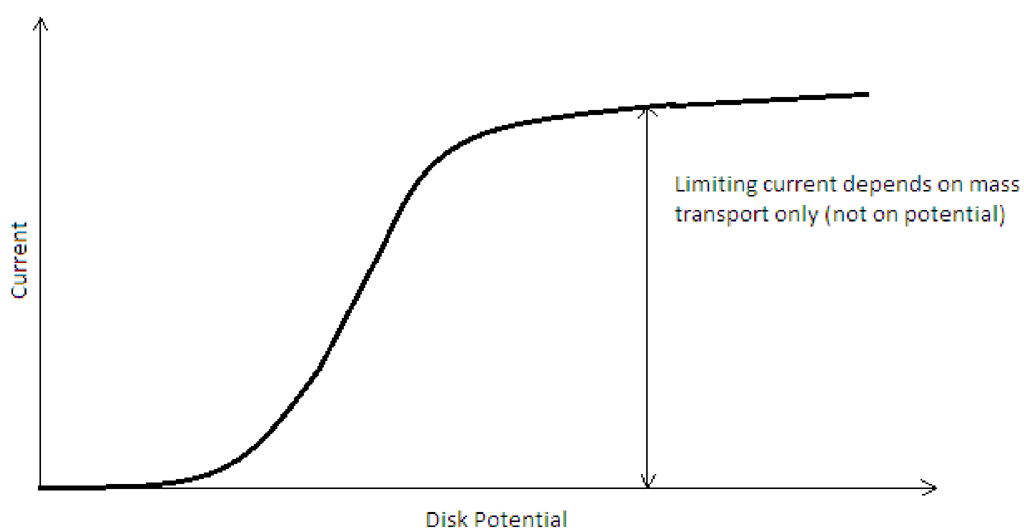

Fig.3. Current versus Disk potential

At the limiting current the surface concentration of reactant (e.g.O_2) goes to zero ($C_o \rightarrow 0$).

Procedure

1. Measure limiting current and find N_{in} from eqn. (8)

2. Calculate $\underbrace{\frac{1}{Sc}\theta'(0)}_{\text{Dimensionless MassTransfer Rate}} = \frac{N_{in}}{\left(C_0 - C_\infty\right)\sqrt{\nu\Omega}}$

3. Use fig. to find $Sc = \frac{\nu}{D_i}$ and calculate D_i.

Besides electrochemical systems, this procedure can be used for any system in which N_{in} can be determined.

$Sc = \text{Schmidt number} = \frac{\nu}{D_i}$ shows the relative importance of momentum transport to mass transport. When Sc >>1, the mass transfer boundary layer is much thinner than the hydrodynamic boundary layer. For liquids we have Sc ≈ 10^3. Therefore for Sc>>1, one can use only the first term of eqn. (12) (given in lecture 21) for H into eqn. (5) to obtain an equation for the Sc>>1 asymptote.

$$\frac{1}{Sc}\theta'(0) = 0.62048\,Sc^{-2/3} \qquad (9)$$

At the other extreme of Sc<<1 the diffusion layer extends a large distance from the disk and eqn. (13) (given in lecture 21) for H should be used. In that case,

$$\frac{1}{Sc}\theta'(0) = 0.88447\exp(-1.611Sc)\left[1+1.961(Sc)^2+\theta\left(Sc^3\right)\right] \tag{10}$$

As $Sc \to 0 \quad \Rightarrow \quad \theta'(0) = 0.88447Sc$

or, $N_{in,Max} = 0.88447(C_\infty - C_0)\sqrt{\nu\Omega}$ (11)

$Sc \to 0$ implies very large diffusivities and hence eqn. (11) gives the maximum flux to the disk. This is determined completely by the rate of convection of material from infinity, i.e. it is equal to $v_{z,\infty}(C_\infty - C_0)$ (see also Fig. 2 in Lecture 21 for plot of H).

Rotating disk electrode has well-defined hydrodynamics and mass transfer characteristics. It is used widely for determining transport properties (e.g. D) and reaction kinetics.