Module 5:

Mass transfer in turbulant flows

Lecture 39:

Transport in Turbulent Flow: Universal Velocity Profile

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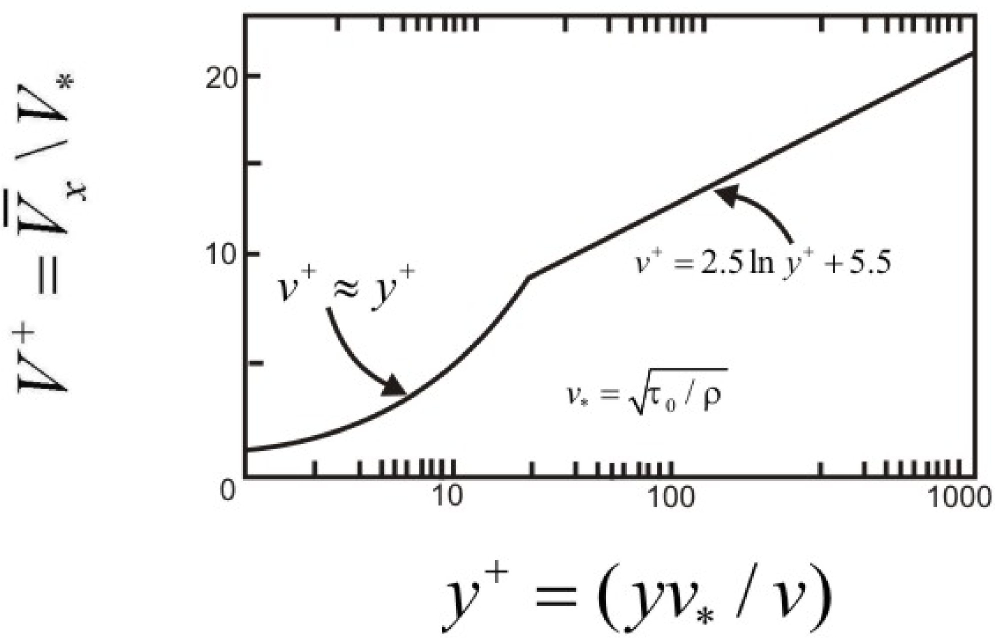
Universal Velocity Profile

For fully developed turbulent flow near a smooth wall: Good for pipe flow as well as boundary layers.

$$v^{+} = \frac{\bar{U}_x}{U_*}, y^{+} = \frac{yU_*}{v} \quad \text{with } u_* = \sqrt{\frac{\tau_0}{\rho}} \qquad (4.12)$$

Where y is the distance from wall and Shear stress τ_0 deduced from pressure drop measurements.



Near the wall => $u^{+} \approx y^{+} \qquad y^{+} < 5 \qquad (4.13)$

Far from wall => $u^+ \cong 2.5 \ln y^+ + 5.5 \qquad y^+ > 20 \qquad (4.14)$

In the logarithmic region

$$\bar{u}_x = 2.5\sqrt{\frac{\tau_0}{\rho}} \ln y + \left(2.5 \ln \frac{\sqrt{\tau_0/\rho}}{\nu} + 5.5\right)\sqrt{\frac{\tau_0}{\rho}} \qquad (4.15)$$

Viscosity enters only in the additive constant term

Eddy viscosity

$$\bar{\tau}_{xy}^{(t)} = -\mu^{(t)} \frac{\partial \bar{u}_x}{\partial y}$$

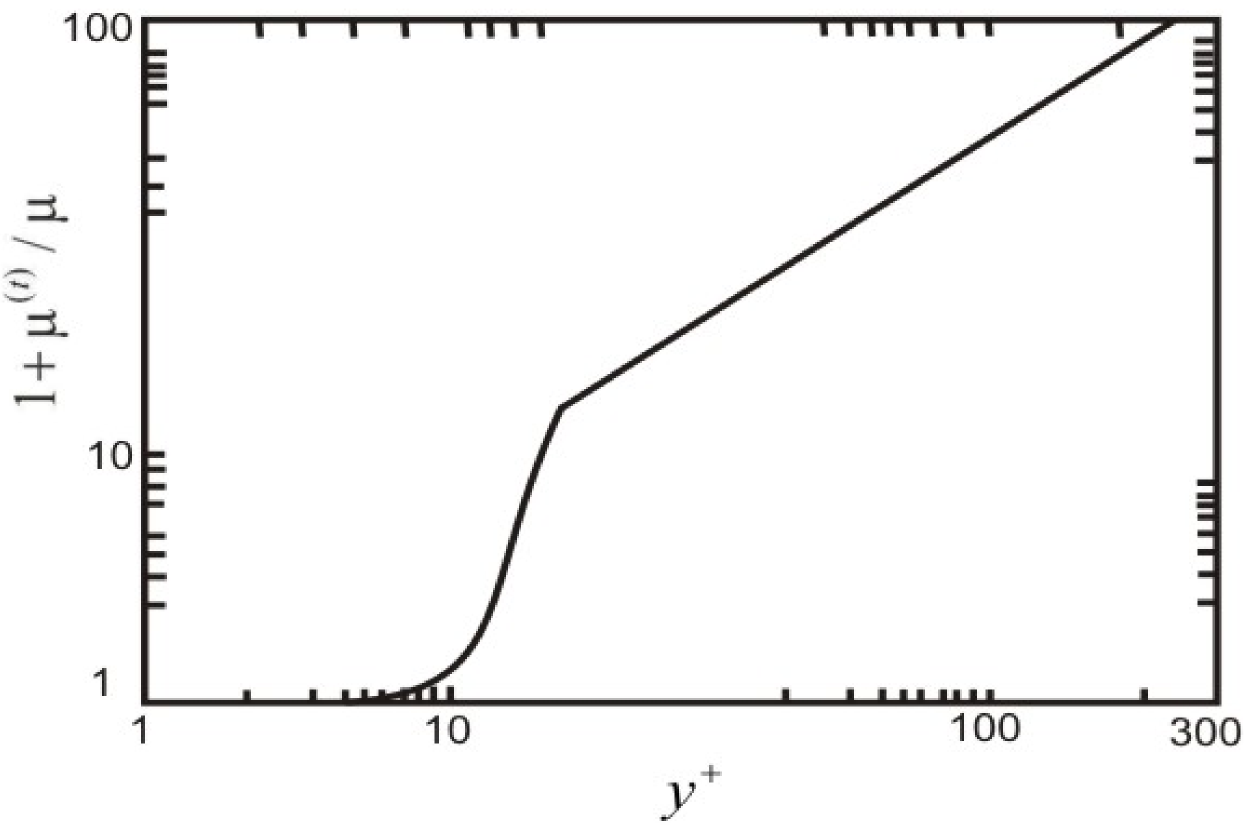
Assuming that $\bar{\tau}_{xy} \approx -\tau_0$ (i.e. not very far from wall)

Then,

$$\tau_0 = (\mu + \mu^{(t)}) \frac{\partial \bar{u}_x}{\partial y} = \frac{\mu + \mu^{(t)}}{\mu} \tau_0 \frac{du^+}{dy^+}$$

$$\Rightarrow \left(1 + \frac{\mu^{(t)}}{\mu}\right) \frac{du^+}{dy^+} = 1 \qquad (4.16)$$

$$\Rightarrow (1+\frac{\mu^{(t)}}{\mu}) \text{ is a universal function of } y^{+}.$$



Measured velocity profiles do not give information for $\mu^{(t)}$ near the wall since $\mu^{(t)} << \mu$ in that region.

Mass Transfer in Turbulent Flow

It is common to assume $D^{(t)} = \nu^{(t)} = \dfrac{\mu^{(t)}}{\rho}$ (4.17)

and $\overline{J}_{iy}^{(t)} =$ turbulent mass flux $= \overline{-c_i' u_y'} = -D^{(t)} \dfrac{\partial \overline{c_i}}{\partial y}$

Near the wall $(y^+ < 5) \Rightarrow \nu^{(t)} << \nu$

No info on $\nu^{(t)}$ can be gathered very near wall.

But if $Sc>>1$ (i.e. $\nu>>D_i$), it is necessary to know $D^{(t)}$ closer to the wall. Thus, even if $\nu^{(t)} = D^{(t)}$,

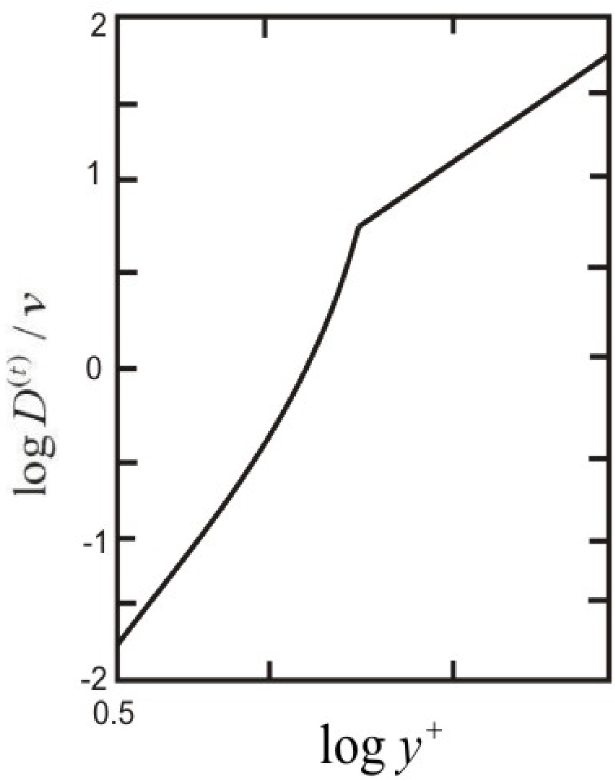
we can have at some distant $D^{(t)} >> D_i$, even where $\nu^{(t)} << \nu$, if $Sc >> 1$.

Thus much of our information about $D^{(t)}$ (and possibly $\nu^{(t)}$) near the wall comes from mass transfer experiments.

For example, in a pipe

$$St = \frac{D_i}{<\overline{u}_z> \Delta c_i} \frac{\partial \overline{c}_i}{\partial r}\Big|_{r=R} = \frac{Sh}{\mathrm{Re}\ Sc} \qquad (4.18)$$

$$For\ \ Sc >> 1, if\ \ D^{(t)} \sim y^n \Rightarrow St \sim Sc^{\frac{1-n}{n}}, \qquad \text{where n} = 2, 3, 4, \ldots$$


2
1
0
-1
-2
0.5
$\log D^{(t)} / \nu$
$\log y^+$