Module 2 :

"Diffusive" heat and mass transfer

Lecture 19:

Simultaneous Heat and Mass Transfer

Fog formation

Transport-reaction model of fog formation

Definitions

1. Condensable vapour:
2. Non-condensable vapour:
3. Fog:

$$\frac{\partial \rho_1}{\partial t} = -\nabla \bullet \underline{N_1} - r_3 \qquad (19.1)$$

where ρ is the mass density

$$\frac{\partial \rho_2}{\partial t} = -\nabla \bullet \underline{N_2} \qquad (19.2)$$

$$\frac{\partial \rho_3}{\partial t} = -\nabla \bullet \underline{N_3} + r_3 \qquad (19.3)$$

where $\mathbf{r_3}$ is the rate of fog formation

Simplified problem: "Stagnant" medium

Gas contains condensable vapors of liquid.

Initially vapor concentration is uniform at ω_∞.

Gas and liquid temperatures are uniform too

(initially) at T_∞ and T_I, respectively.

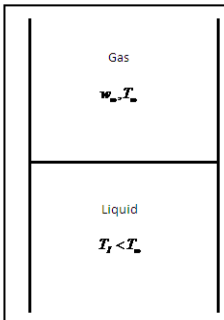

Fig. 1 gas –liquid interaction

Assume Lewis number $Le = \frac{\alpha}{D} = 1$ $\left(Le = 0.85 \text{ for air-water}\right)$.

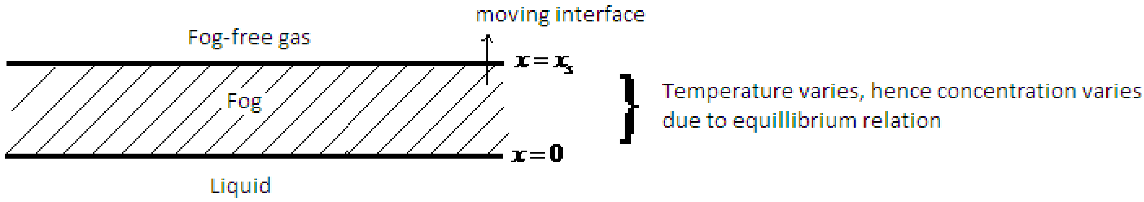

Fig. 2 fog formation and liquid- fog free interface

For inner region $x<x_s$ (i.e. fogged region)

$$\frac{\partial w_i}{\partial t} = D\frac{\partial^2 w_i}{\partial x^2} - \frac{r_3}{\rho} \qquad (19.4)$$

where w_i is the mass fraction of condensable vapor

Equilibrium relation: $w_i = w_i\left(T\right)$ (for air-water system from psychometric chart)

$$\frac{\partial T_i}{\partial t} = D\frac{\partial^2 T_i}{\partial x^2} + \frac{r_3 \lambda}{\rho C_p} \qquad (19.5)$$

where λ is the latent heat

For the outer region (fog-free) $x \geq x_s$

$$\frac{\partial w_0}{\partial t} = D\frac{\partial^2 w_0}{\partial x^2} \qquad (19.6)$$

$$\frac{\partial T_0}{\partial t} = D\frac{\partial^2 T_0}{\partial x^2} \qquad (19.7)$$

Initial and boundary conditions are

$$\begin{aligned}
&\text{I.C.} \quad t = 0 \Rightarrow w_0 = w_\infty,\ T_0 = T_\infty && (18.8a)\\
&\text{B.Cs.} \quad x = 0 \Rightarrow w_i = w_I,\ T_i = T_I && (b)\\
&\qquad x \to \infty \Rightarrow w_0 = w_\infty,\ T_0 = T_\infty && (c)\\
&\qquad x = x_s \Rightarrow w_i = w_0,\ T_i = T_0 && (d)\\
&\qquad \frac{\partial w_i}{\partial x} = \frac{\partial w_0}{\partial x},\ \frac{\partial T_i}{\partial x} = \frac{\partial T_0}{\partial x} && (e)
\end{aligned}$$

Eqn. (19.4) to (19.7) assumes "dilute" solution of vapor in gas.

For Le=1 the equation for mass transfer can be combined with that of heat transfer using enthalpy H as a new variable

$$dH = C_p dT + \lambda dw \qquad (19.9)$$

In fact the enthalpy equation can be given as

$$\frac{\partial H}{\partial t} = D\frac{\partial^2 H}{\partial x^2} \qquad (19.10)$$

$$\begin{aligned}
&t = 0 \quad \Rightarrow H = H_\infty\\
&x = 0 \quad \Rightarrow H = H_I\\
&x \to \infty \Rightarrow H = H_\infty
\end{aligned}$$

is satisfied in both inner and outer regions. For the derivation of eqn. (19.10) we assume both C_p and λ as constants quantities.

The solution of eqn. (19.10) can be given as

$$\frac{H-H_I}{H_\infty-H_I}=erf\left(\frac{x}{\sqrt{4Dt}}\right) \tag{19.11}$$

This gives a linear relationship between T and w.

Combining eqn. (19.11), (19.9) and equilibrium relationship, $w_i=w_i\left(T\right)$, we get for inner region (fog region)

$$\frac{T-T_I}{T_\infty-T_I}+\frac{\lambda\left(w\left(T\right)-w_I\right)}{C_p\left(T_\infty-T_I\right)}=\left[1+\frac{\lambda\left(w_\infty-w_I\right)}{C_p\left(T_\infty-T_I\right)}\right]erf\left(\frac{x}{\sqrt{4Dt}}\right) \tag{19.12}$$

Here subscript *i* has been dropped. Note that, w =w (T), so knowing T we can find out w from equilibrium relationship, $w_i=w_i\left(T\right)$.

For the outer region, we first postulate that at the fog boundary where x = xs, T = T*, w = w* i.e. constant values at all times. Thus even though the boundary is moving, eqns. (19.6) - (19.8) yield

$$\frac{T-T^*}{T_\infty-T^*}=\frac{w-w^*}{w_\infty-w^*} \tag{19.13}$$

(since dimensionless solutions have to be of the same form for temperature and concentration because the equations and conditions are same)

Combining eqn. (19.13) with (19.11), which is valid for the outer region as well,

$$\frac{T_\infty - T}{T_\infty - T_1} = \frac{1 + \dfrac{\lambda}{C_p}\dfrac{\left(w_\infty - w_I\right)}{\left(T_\infty - T_I\right)}}{1 + \dfrac{\lambda}{C_p}\dfrac{\left(w_\infty - w^*\right)}{\left(T_\infty - T^*\right)}}\left[1 - erf\left(\frac{x}{\sqrt{4Dt}}\right)\right] \tag{19.14}$$

where T_1 is liquid temperature at x = 0 (known value)

Knowing T from eqn. (19.14), w can be obtained using eqn. (19.13).

Using B.C. (19.8d) and (19.8e) along with eqn. (19.12) and eqn. (19.14), we get

$$\frac{w_\infty - w^*}{T_\infty - T^*} = w'\left(T^*\right) \tag{19.15}$$

Equations (19.13) and (19.15) fix w^* and T^*.

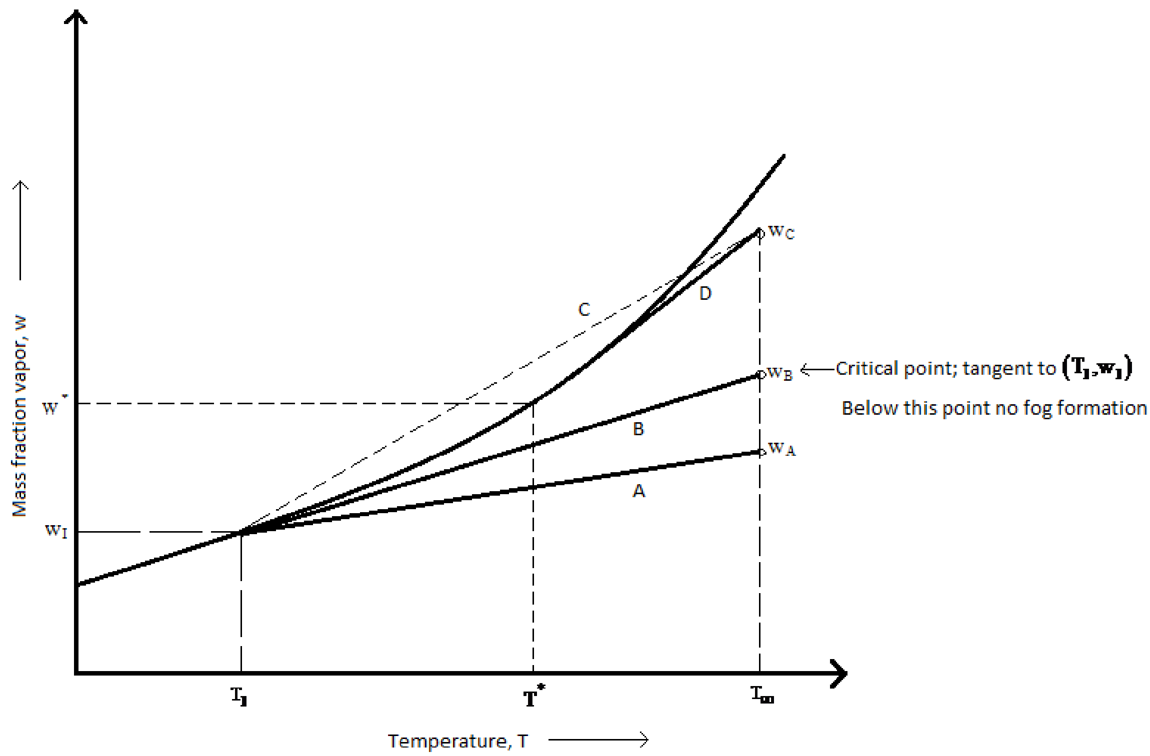

Fig. 3 Concentration-temperature relations $T_I < T_\infty$

Letting T=T* in eqn. (19.14) one obtains

$$erf\left(\eta_s\right)=\left(\frac{T^*-T_s}{T_\infty-T_I}\right)\frac{\left[1+\dfrac{\lambda\left(w^*-w_I\right)}{C_p\left(T^*-T_I\right)}\right]}{\left[1+\dfrac{\lambda\left(w_\infty-w_I\right)}{C_p\left(T_\infty-T_I\right)}\right]} \qquad (19.16)$$

With $x_s=\eta_s\sqrt{4Dt}$, a point where fog-free front is located

Equation (19.16) gives the value of η_s ; and hence x_s can be found out. Here we note that xs is proportional to $\sqrt{t}$.

If w∞ is reduced holding T_∞ constant (see fig. 3) T^* and w^* decreases and η_s also decreases so the fog region gets thinner. When w_∞ reaches w_B, $T^* \rightarrow T_I$; and $w^* \rightarrow w_I$ and thus $\eta_s \rightarrow 0$, which indicates that no fog is formed.

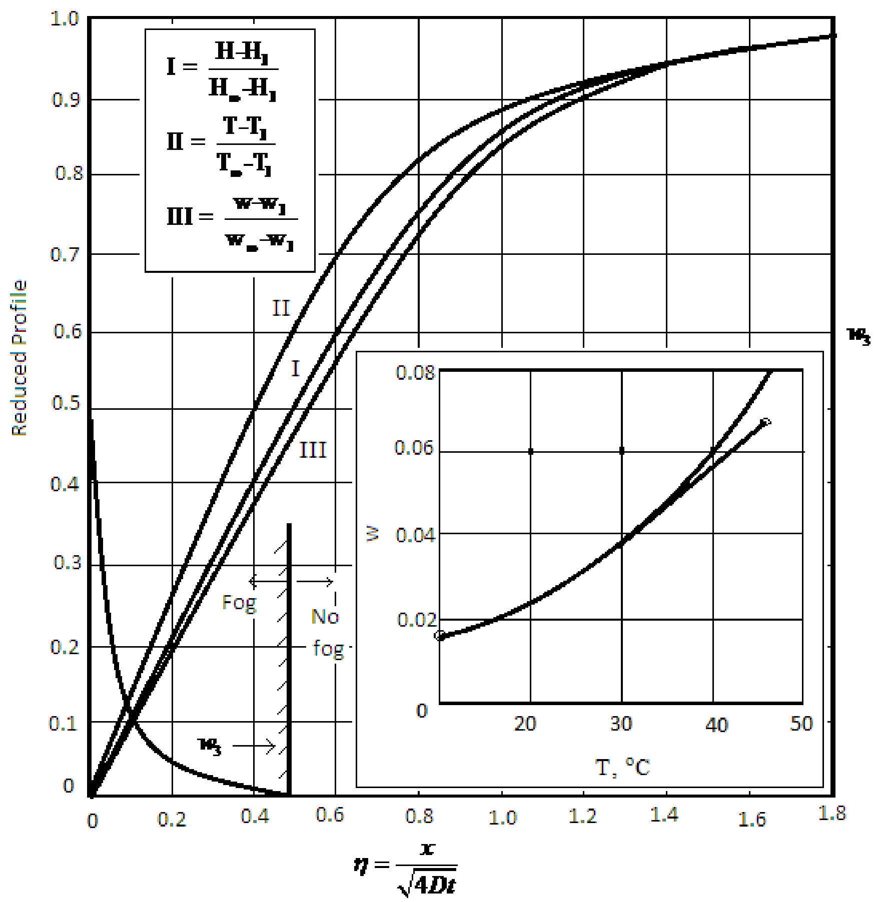

Fig.4 .Concentration and temperature profiles with fog formation in a semi-infinite air-water system