Module 2 :

"Diffusive" heat and mass transfer

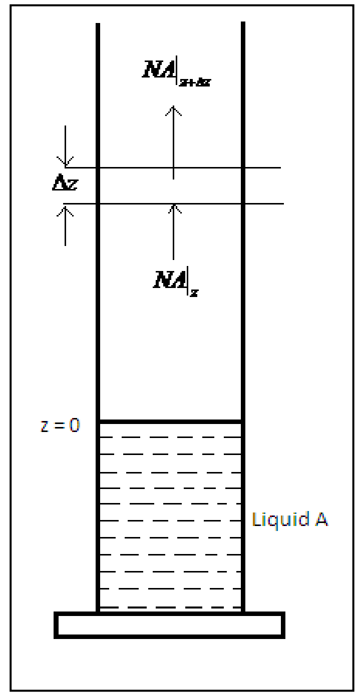
Lecture 20:

Unsteady-state Evaporation

Evaporation is a process by which liquid water passes directly to the vapor phase.

Methods of estimating evaporation rates:

- Energy balance method
- Aerodynamic method
- Combined method



Model derivation of Evaporation

Assumptions

1. P,T = constant
2. Ideal gases
3. B is insoluble in A
4. Liquid maintained at z = 0 at all times (i.e. no moving boundary)

Note: No dilute solution assumption has been made. Dilute solution assumption results in neglecting convective terms in mass balance.

Equation of mass continuity of component A and B

$$\frac{\partial C_A}{\partial t} = -\frac{\partial N_{Az}}{\partial z} \qquad (\text{No reaction}) \qquad (19.1)$$

$$\frac{\partial C_B}{\partial t} = -\frac{\partial N_{Bz}}{\partial z} \qquad (19.2)$$

Addition of eqn. (19.1) and (19.2) yields

$$\frac{\partial C}{\partial t} = -\frac{\partial}{\partial z}\left(N_{Az} + N_{Bz}\right) \qquad (19.3)$$

But $C=\dfrac{P}{RT}$ and $P_1T=\text{constant}\Rightarrow C=\text{constant}$

Thus, $\dfrac{\partial C}{\partial t}=c$

Hence,

$$N_{Az}+N_{Bz}=f\left(t\right)\qquad \left(\text{i.e. not a function of z}\right) \tag{19.4}$$

Now $N_A=x_A\left(N_A+N_B\right)-cD_{AB}\nabla x_A$ (19.5)

Applying at $z=0\Rightarrow N_{A0}=x_{A0}\left(N_{A0}+N_{B0}\right)-cD_{AB}\left.\dfrac{\partial x_A}{\partial z}\right|_{z=0}$

But component B does not dissolve in the liquid therefore $N_{B0}=0$

Thus $N_{A0}=-\dfrac{cD_{AB}}{1-x_{A0}}\left.\dfrac{\partial x_A}{\partial z}\right|_{z=0}$

Eqn. (19.4) implies that

$$N_{Az}+N_{Bz}=N_{A0}+N_{B0}=N_{A0}$$

Then,

$$N_{Az}+N_{Bz}=-\frac{cD_{AB}}{1-x_{A0}}\left.\frac{\partial x_A}{\partial z}\right|_{z=0} \tag{19.6}$$

Note: $N_{Az}+N_{Bz}=cv_z^{*}$

Inserting (19.6) into (19.5) we get,

$$N_{Az} = -cD_{AB}\frac{\partial x_A}{\partial z} - x_A \frac{cD_{AB}}{1-x_{A0}}\frac{\partial x_A}{\partial z}\bigg|_{z=0} \tag{19.7}$$

Inserting eqn. (19.7) into (19.1) we get,

$$\frac{\partial x_A}{\partial t} = D_{AB}\frac{\partial^2 x_A}{\partial x^2} + \left(\frac{D_{AB}}{1-x_{A0}}\frac{\partial x_A}{\partial z}\bigg|_{z=0}\right)\frac{\partial x_A}{\partial z} \tag{19.8}$$

With the initial and boundary

I.C. $\quad t = 0 \quad \Rightarrow x_A = 0$

conditions, B.C.1 $\quad z = 0 \quad \Rightarrow x_A = x_{A0} \quad \left(\text{saturation concentration, } \frac{P_v A}{P}\right)$

BC.2 $\quad z \to \infty \quad \Rightarrow x_A = 0$

Define $\quad X = \frac{x_A}{x_{A0}},\ Z = \frac{z}{\sqrt{4D_{AB}t}}$

Anticipate $\quad X = f(Z)$ only

Then eqn. (19.8) can be written as

$$X'' + 2(Z - \phi)X' = 0 \tag{19.9}$$

With $\quad \phi(x_{A0}) = -\frac{1}{2}\frac{x_{A0}}{1-x_{A0}}\frac{\partial X_A}{\partial Z}\bigg|_{Z=0} \qquad (19.10)$

$= \text{constant depending on } x_{A0}$

Eqn. (19.9) must be solved subject to

B.C.1 $\quad z = 0 \quad \Rightarrow X = 1$

BC.2 $\quad z \to \infty \quad \Rightarrow X = 0$

The solution to (19.9) satisfying B.C.1 and B.C.2 is

$$X = \frac{1 - erf\left(Z - \phi\right)}{1 + erf\,\phi} \tag{19.11}$$

Take $\left.\frac{\partial X_A}{\partial Z}\right|_{Z=0}$ and substitute into eqn. (19.10) to get

$$X_{A0} = \frac{1}{1 + \left[\sqrt{\pi}\left(1 + erf\,\phi\right)\phi \exp \phi^2\right]^{-1}} \tag{19.12}$$

X_{A0}	ϕ	ψ
0	0	1.000
0.25	0.1562	1.108
0.50	0.3578	1.268
0.75	0.6618	1.564
1.0	∞	∞

For given x_0, calculate using (19.12), and then use eqn. (19.11) to obtain the concentration profiles

The evaporation rate of A is

$$\frac{dV_A}{dt} = \frac{N_{A0}S}{c} = S\phi\sqrt{\frac{D_{AB}}{t}}$$

The amount of vapor produced between t =0 and t = t is

$$V_A = S\phi\sqrt{4D_{AB}t} \tag{19.13}$$

Note: compare eqn. (19.8) with the "general" form of the mass balance equation

$$\frac{\partial C_A}{\partial t} + C_A\left(\nabla \bullet U^*\right) + \underline{U}^* \bullet \nabla C_A = D_{AB}\nabla^2 C_A + R_A$$

The second term on right hand side is zero in this case since $N_{Az} + N_{Bz}$ = not a function of z = cv_z^* i.e. $\nabla . U^* = 0$

We see that the term $\left.\frac{D_{AB}}{1-X_{A0}}\frac{\partial X_A}{\partial Z}\right|_{Z=0}$ plays the role of convective molar average velocity.

Question: What would be the result if we had neglected this convective term?

Then eqn. (19.8) would become $\frac{\partial X_A}{\partial t} = D_{AB}\frac{\partial^2 X_A}{\partial X^2}$ with same I.C. and B.Cs as before.

The solution to this case was given in the section on Semi-Infinite slab with constant wall T.

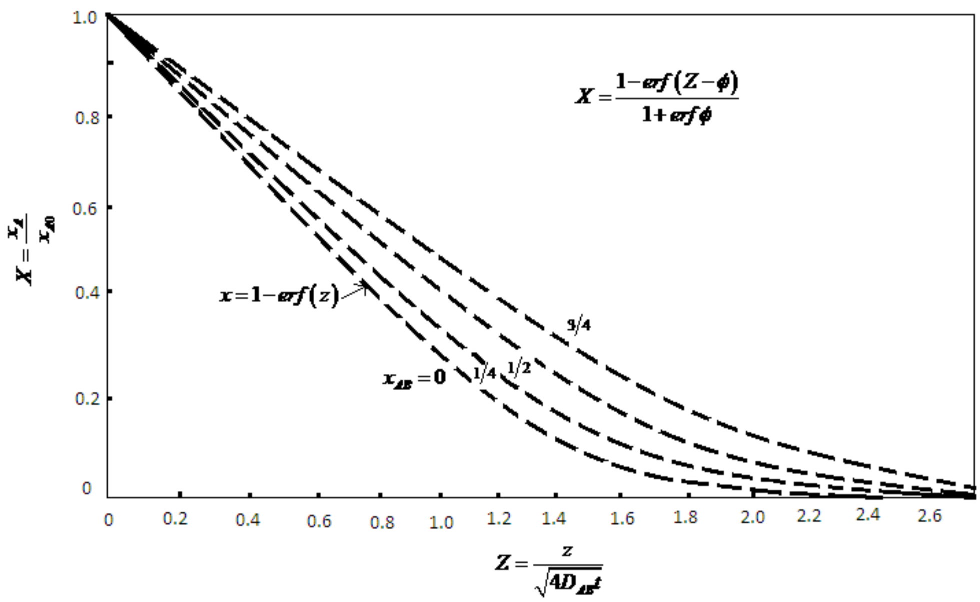
The corresponding volume of 'A' vaporized would be (when convective terms are neglected)

$$V_{A,appx.} = SX_{A0}\sqrt{\frac{4D_{AB}t}{\pi}}$$

Eqn. (19.13) can also be written as

$$V_A = \underbrace{SX_{A0}\sqrt{\frac{4D_{AB}t}{\pi}}}_{V_{A,apprx}} \underbrace{\left(\phi\frac{\sqrt{\pi}}{X_{A0}}\right)}_{\psi = \text{a correction factor}} \qquad (19.14)$$

The correction factor ψ is given in the table below eqn. (19.12).



Note: eq. (19.14) may be solved for DAB to give

$$D_{AB} = \frac{\pi}{t}\left(\frac{V_A}{2Sx_{A0}\psi}\right)^2 \qquad (19.15)$$

This equation can be used to get diffusivities of volatile liquids.