Module 5:

Mass Transfer in Turbulant flows

Lecture 38:

Turbulent Flow: Time Averaging, Reynolds Stress and Eddy Viscosity

NPTEL , IIT Kharagpur, Prof. Saikat Chakraborty, Department of Chemical Engineering

Turbulence

Sub-topics:

(i) Time average equations

(ii) Eddy viscosity (conductivity, diffusivity)

(iii) Universal velocity profile

(iv) Analogies

(v) Statistical theories (e.g. homogeneous turbulence)

$$\underline{U} = \overline{\underline{U}} + \underline{U}'$$

Time averaging $$\overline{U}_z = \frac{1}{t_0}\int_t^{t+t_0} U_z dt$$

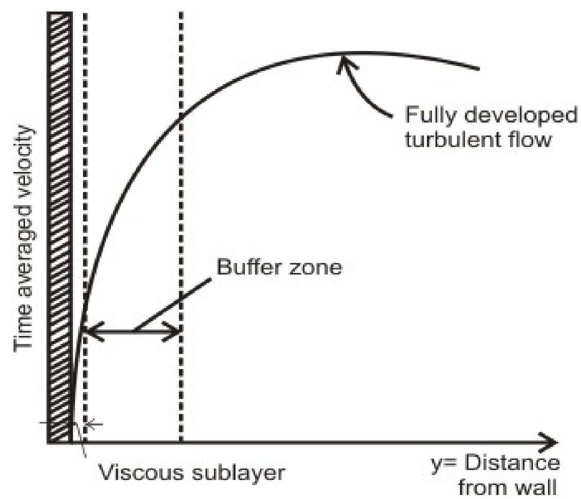
Rules:

$$\overline{A+B} = \overline{A} + \overline{B}$$

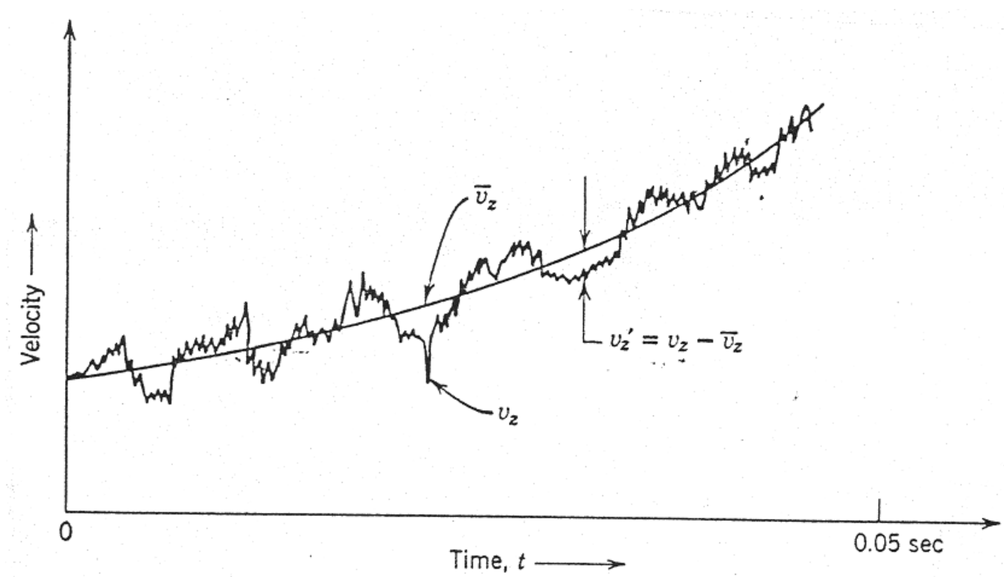
$$\overline{A'} = 0$$

$$\overline{\left(\frac{dA}{dX}\right)} = \frac{d\overline{A}}{dX}$$

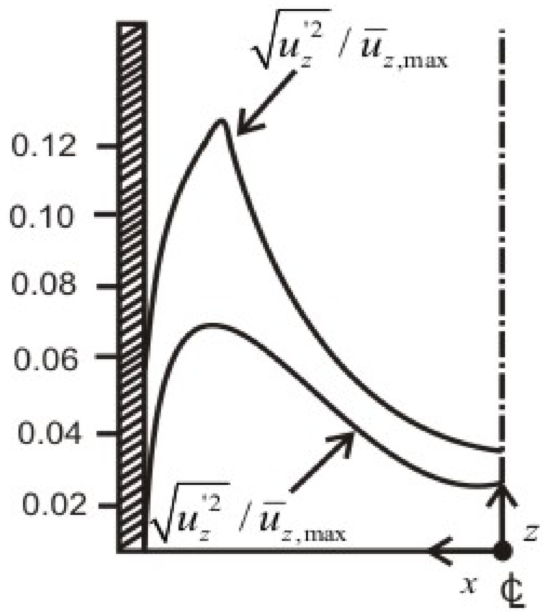
$$\overline{AB} = \overline{A}\,\overline{B} + \overline{A'B'}$$



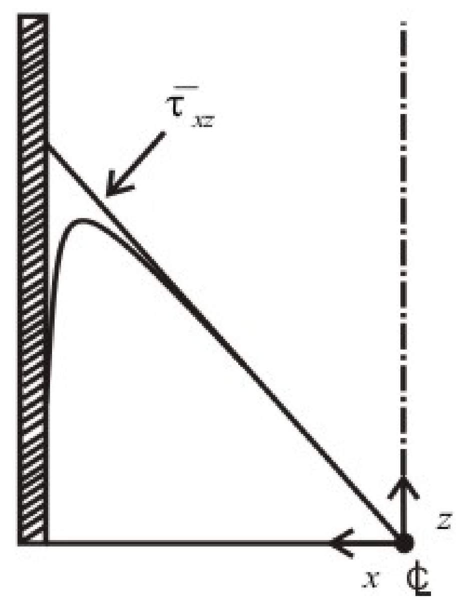
Velocity distribution for turbulent flow in tubes - regions near tube wall



Oscillation of velocity component about a mean value.



Turbulent fluctuations for flow in z-direction in a rectangular channel.



Reynolds stress for flow in a rectangular channel. The quantity $\bar{\tau}_{xz}$ is the sum of $\bar{\tau}_{xz}^{(l)}$ and $\bar{\tau}_{xz}^{(t)}$

Consider constant physical properties

<u>Equation of Continuity</u> $$\nabla.\underline{\bar{U}} = 0 \qquad (4.1a)$$

<u>Equation of Motion</u> $$\rho\frac{D\underline{\bar{U}}}{Dt} = -\nabla\bar{p} - [\nabla.\underline{\underline{\bar{\tau}}}^{(l)}] - [\nabla.\underline{\underline{\bar{\tau}}}^{(t)}] + \rho\underline{g} \qquad (4.1\ b)$$

$\underline{\underline{\bar{\tau}}}^{(t)}$ = Reynolds stresses ; turbulent momentum flux

$$\overline{\tau}_{xx}^{(t)} = \rho \overline{U_x^{'} U_x^{'}}, \quad \overline{\tau}_{xy}^{(t)} = \rho \overline{U_x^{'} U_y^{'}} \quad \text{etc.}$$

We need to know structure of turbulent flow to find $\underline{U}^{'}$ as a function of position. (Unsolved problem)

Semi-empirical expressions for Reynolds stresses

a) Boussinesq's Eddy Viscosity (By analogy to Newton's law of Viscosity)

$$\overline{\tau}_{yx}^{(t)} = -\mu^{(t)} \frac{d\overline{U}_x}{dy} \qquad (4.2)$$

Eddy viscosity (function of position)

b) Prandtl's Mixing length (analogy to kinetic theory of gases)

$$\overline{\tau}_{yx}^{(t)} = -\rho l^2 \left| \frac{d\overline{U}_x}{dy} \right| \frac{d\overline{U}_x}{dy} \qquad (4.3)$$

Mixing length *l* (function of positon): Roughly analogy to mean free path in gas kinetic theory

c) Von-Karman similarity hypothesis

$$\overline{\tau}_{yx}^{(t)} = -\rho k_2^2 \left| \frac{(d\overline{U}_x / dy)^3}{(d^2 U_x / dy^2)^2} \right| \frac{d\overline{U}_x}{dy} \qquad (4.4)$$

k_2 = "universal constant" (0.36-0.40)

d) Deissler's Empirical Formula for the Near Wall Region

$$\overline{\tau}_{yx}^{(t)} = -\rho\eta^2\overline{U}_x y(1-\exp(-\frac{\eta^2\overline{U}_x y}{v}))\frac{d\overline{U}_x}{dy} \qquad (4.5)$$

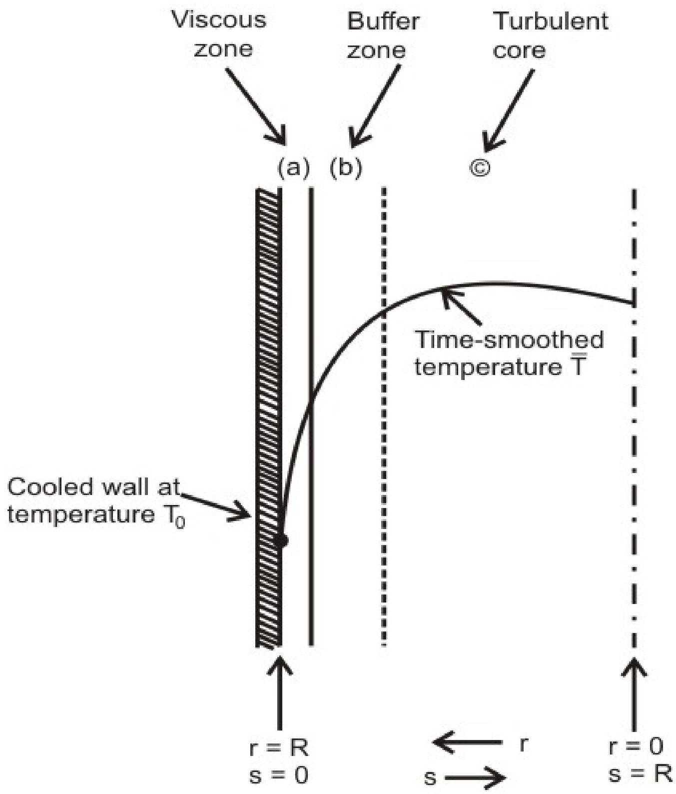
η = constant=0.124

y =distance from wall

Time-averaged Equation of Energy

$$\rho\widehat{C}_P\frac{D\overline{T}}{Dt} = -(\overline{V}.\underline{\overline{q}}^{(l)}) - (\overline{V}.\underline{\overline{q}}^{(t)}) + \mu\overline{\Phi}_u^{(l)} + \mu\overline{\Phi}_u^{(t)} \qquad (4.6)$$

$$with \quad \overline{q}_x^{(t)} = \rho\widehat{C}_P\overline{U_x'T'}, \; etc.$$



Semi-empirical expressions

a) $\bar{q}_y^{(t)} = -k^{(t)} \dfrac{d\bar{T}}{dy}$ (4.7)

b) $\bar{q}_y^{(t)} = -\rho \hat{C}_p l^2 \left| \dfrac{d\bar{U}_x}{dy} \right| \dfrac{d\bar{T}}{dy}$ (4.8)

l : same as in eq. (4.3) which implies $\nu^{(t)} = \alpha^{(t)}$

c) $$\overline{q}_y^{(t)} = -\rho \widehat{C}_p k_2^2 \left| \frac{(d\overline{U}_x / dy)^3}{(d^2 U_x / dy^2)^2} \right| \frac{d\overline{T}}{dy} \quad (4.9)$$

Where k_2 is same as in eqn. (4.4)

d) $$\overline{q}_y^{(t)} = -\rho \widehat{C}_p \eta^2 \overline{U}_x y (1 - \exp(-\frac{\eta^2 \overline{U}_x y}{v})) \frac{d\overline{T}}{dy} \quad (4.10)$$

Where η is same as in (4.5), assumes $v^{(t)} = \alpha^{(t)}$

Time-averaged equation of continuity for component A

$$\frac{D\overline{C}_A}{Dt} = -(\nabla . \underline{\overline{j}}_A^{(l)}) - (\nabla . \underline{\overline{j}}_A^{(t)}) - \begin{cases} k_1^{'''} \overline{C}_A \\ k_2^{'''} \left(\overline{C}_A + \overline{C'^2_A} \right) \end{cases} \quad (4.11)$$

With:

$$\underline{\overline{j}}^{(l)} = -D_{AB} \nabla \overline{C}_A$$

$$\overline{j}_i^{(t)} = \overline{U_i' C_A'}$$

Expression for $\overline{j}_{AY}^{(t)}$ is analogous to eqs. (4.7) – (4.10)