Module 3:

“Convective” heat and mass transfer

Lecture 27:

Heat Transfer to Boundary Layers

(Continued)

Continuity equation becomes

$$\frac{\partial}{\partial x}\left(Rv_x\right)+\frac{\partial}{\partial y}\left(Rv_y\right)=0$$

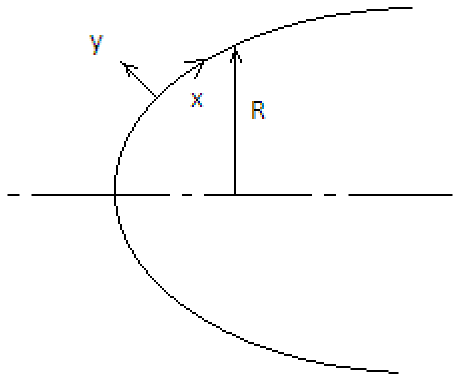

Fig. 1

$$v_x = B\left(x\right)y$$

$$v_y = -\frac{1/2y^2\left(RB\right)'}{R}$$

Convective-Diffusion equation for this case is given by

$$yB\frac{d\theta}{dx}-\frac{1}{2}\frac{y^2}{R}\frac{d\left(RB\right)}{dx}\frac{\partial\theta}{\partial y}=D\frac{\partial^2\theta}{\partial y^2}$$

Transformation is

$$\eta = \frac{y\left(RB\right)^{1/2}}{\left[9D\int_0^x R\left(RB\right)^{1/2}\,dx\right]^{1/3}} \tag{1}$$

and the solution is again

$$\theta = \frac{1}{\Gamma\left(\frac{4}{3}\right)}\int_0^{\eta} e^{-x^3}\,dx$$

$$N_y\Big|_{y=0} = -\frac{D\left(c_\infty - c_w\right)}{\Gamma\left(\frac{4}{3}\right)}\frac{\sqrt{B}}{\left[9D\int_0^x R\left(RB\right)^{1/2}dx\right]^{1/3}} \tag{2}$$

For small Pr (liquid metals) $\delta_T >> \delta$ and

$v_x = v_\infty\left(x\right) =$ free stream velocity

For 2-D (plane) flow use transformation

$$\eta = \frac{y v_\infty\left(x\right)}{\left[4\alpha\int_0^x v_\infty\left(x\right)dx\right]^{1/2}} \tag{3}$$

Temperature profile is

$$\theta = \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-x^2}\,dx \tag{4}$$

$$q_y\Big|_{y=0} = -\frac{k\left(T_\infty - T_w\right)}{\sqrt{\pi}} \frac{v_\infty\left(x\right)}{\left[\alpha\int_0^x v_\infty\left(x\right)dx\right]^{1/2}} \qquad (5)$$

<u>Mises Transformation (Laminar flow)</u>

Consider the equation:

$$\underbrace{v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y}}_{\text{Convective}} = \underbrace{\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)}_{\text{Conductive}} \qquad (6)$$

If y is replaced by ψ as the independent variables, this term will drop out because no flow occurs streamlines (i.e., lines of ψ = cont.)

First, let's write the equation by which a transformation from variables x and y to variables ξ and η is accomplished.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial y}$$

Now introduce x and ψ for ξ and η, and make use of equation defining the stream function ψ to obtain

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x} - v_y \frac{\partial}{\partial \psi}$$

$$\frac{\partial}{\partial y} = v_x \frac{\partial}{\partial \psi}$$

$$\left(\text{Note: } v_x = \frac{\partial \psi}{\partial y} \text{ and } v_y = -\frac{\partial \psi}{\partial x} \right)$$

$$\frac{\partial T}{\partial x} = \left.\frac{\partial T}{\partial x}\right|_{\psi} - v_y \frac{\partial T}{\partial \psi}$$

$$\frac{\partial T}{\partial y} = v_x \frac{\partial T}{\partial \psi}; \quad \frac{\partial^2 T}{\partial y^2} = v_x \frac{\partial}{\partial \psi}\left(v_x \frac{\partial T}{\partial \psi} \right)$$

Introducing these into eqn. (2) given in lecture 24, yields

$$\left(\frac{\partial T}{\partial x} \right)_{\psi} = \alpha \frac{\partial}{\partial \psi}\left(v_x \frac{\partial T}{\partial \psi} \right)$$

where α is a constant

$$\left(\frac{\partial T}{\partial x} \right)_{\psi} = \frac{\partial}{\partial \psi}\left(\left(\alpha v_x\right) \frac{\partial T}{\partial \psi} \right) \qquad (7)$$

This is like the heat conduction equation with variable thermal conductivity. It is easier to solve compared to convection equation (2) given in lecture 24.

B.C.: We can select ψ = 0 at the surface (y = 0)

Then

$$T = T_w \quad \text{at} \quad \psi = 0$$
$$T = T_\infty \quad \text{as} \quad \psi \to \infty$$

"Initial condition" : T = finite at x = 0

If we are interested in solutions of eqn. (7) near the surface (e.g. for $Pr \to \infty$), we can retain only the leading terms of a series expansion of $v_x(\psi, x)$.