Module 1 :

The equation of "continuity"

Lecture 4:

Fourier's Law of Heat Conduction

Fourier's Law of Heat Conduction

According to Fourier's law, the rate of heat flow, $\underline{q}$, through a homogeneous solid is directly proportional to the area A, of the section at the right angles to the direction of the heat flow, and to the temperature difference ∇T along the path of heat flow. Mathematically, it can be written as

$$\boxed{\underline{q} = -k\,\nabla T}$$
(2.17)

Fourier's law of heat conduction is an empirical relationship based on observation. Equation (2.16) holds for **isotropic media**. k is the thermal conductivity. The figure shown below illustrates the Fourier law of heat conduction.
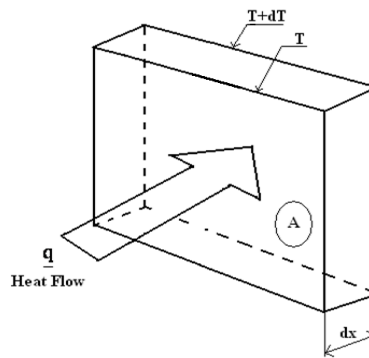

Fig.2.3 Heat flow through a homogeneous (isotropic) solid

Heat Conduction

Heat transfer by conduction involves transfer of energy within a bulk material without any motion of the material as a whole. Conduction takes place when a temperature gradient exist in a

solid (or stationary fluid) medium. Energy is transferred from the more energetic to the less energetic molecules when neighboring molecules collide.

Thermal conductivity

Thermal conductivity (k) is the intrinsic property of a material which relates its ability to conduct heat. Thermal conductivity is defined as the quantity of heat (Q) transmitted through a unit thickness (x) in a direction normal to a surface of unit area (A) due to a unit temperature gradient (ΔT) under steady state conditions and when the heat transfer is dependant only on temperature gradient. So mathematically it can be written as

$$k = \frac{\underline{q} \bullet dx}{dT} \qquad \text{(W/m K)} \qquad (2.18)$$

where $\underline{q} =$ Rate of heat flow

For non-isotropic media, $\underline{q} = -\underline{\underline{k}} \bullet \nabla T$, where $\underline{\underline{k}}$ is the thermal conductivity tensor.

Viscous Dissipation Function

A function used to take into account the effect of the forces of viscous friction on the motion of a mechanical system. The dissipation function describes the rate of decrease of mechanical energy of the system. It is used, commonly, to allow for the transition of energy of ordered motion to the energy of disordered motion (ultimately to thermal energy). For **Newtonian fluids**, the term: $\left(- \underline{\underline{\tau}} : \nabla \underline{U}\right) = \mu\, \Phi_U$, where Φ_U is the **viscous dissipation function**.

In rectangular coordinates we can write it as

$$\Phi_U = 2\left[\left(\frac{\partial U_x}{\partial x}\right)^2 + \left(\frac{\partial U_y}{\partial y}\right)^2 + \left(\frac{\partial U_z}{\partial z}\right)^2\right] + \left(\frac{\partial U_y}{\partial x} + \frac{\partial U_x}{\partial y}\right)^2 + \left(\frac{\partial U_z}{\partial y} + \frac{\partial U_y}{\partial z}\right)^2 + \left(\frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}\right)^2$$

(2.19)

Under most circumstances the term $\mu\,\Phi_U$ can be neglected. It may be important in high speed boundary layer flow, where velocity gradient are large.

Special Forms of the energy equation

General assumptions

   a) Neglect viscous dissipation

   b) Assume constant thermal conductivity k

Ideal gas

An ideal gas is defined as one in which all collisions of atoms or molecules are perfectly elastic and in which there are no intermolecular attractive forces. In such a gas, all the internal energy is in the form of kinetic energy and any change in the internal energy is accompanied by a change in temperature.

An ideal gas can be characterized by three state variables: absolute pressure (P), volume (V) and absolute temperature (T). The relationship between them may be deduce from the kinetic theory of gases and is called the **Ideal Gas Law**, which is given as

$$PV = nRT \qquad (2.20)$$

where n is the number of moles of a gas and R is the **Universal Gas Constant** which is equal to 8.314 J/mol K.

I. Fluid at constant volume

For ideal gas, $\left(\dfrac{\partial P}{\partial T}\right)_{\hat{V}} = \dfrac{P}{T}$ $\qquad (2.21)$

Inserting equation (1.22) into equation (2.16) and neglecting viscous effects, then for ideal gas we get

$$\rho \hat{C}_v \frac{DT}{Dt} = k\,\nabla^2 T - P(\nabla \bullet \underline{U}) + \dot{Q} \qquad (2.22)$$

II. Fluid at Constant Pressure

At constant pressure, $dP = 0$ and maintaining all other conditions same as the previous case (I) we get

$$\rho \hat{C}_p \frac{DT}{Dt} = k \nabla^2 T + \dot{Q} \qquad (2.23)$$

III. Fluid with density independent of temperature

According to Continuity equation, we can write $(\nabla \bullet \underline{U}) = 0$, and maintaining all other conditions same as the previous case (II) we get

$$\rho \hat{C}_p \frac{DT}{Dt} = k \nabla^2 T + \dot{Q} \qquad (2.24)$$

IV. Solids

The density may be considered constant and $(\nabla \bullet \underline{U}) = 0$. Then (1.27) yields

$$\rho \hat{C}_p \frac{\partial T}{\partial t} = k \nabla^2 T + \dot{Q} \qquad (2.25)$$

Typical Boundary Conditions on the Energy equation

For heat transfer problems, there are five types of boundary condition at the solid surfaces that are pertinent.

I. **Initial Condition: At t=0** $\rightarrow T(x, y, z, 0) = T_0(x, y, z)$

Constant Surface Heat Flux: Here dependant variable (T) and its normal derivative are specified. k is the material **thermal conductivity**.
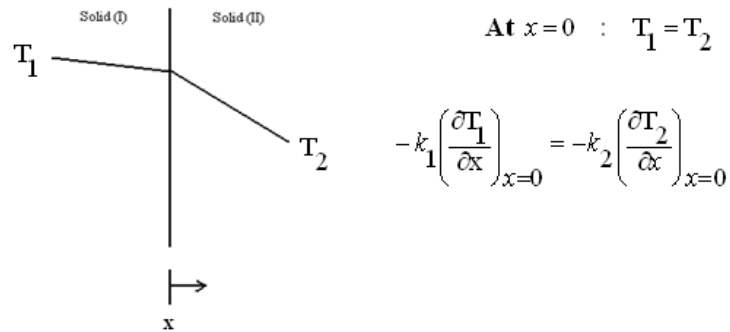
Fig.2.4 heat transfer through composite wall

It is also known as **Neumann Condition**.

II. Here, Convection coefficient (h) is prescribed at the surface. In this B.C., dependant variable (T) and its normal derivative are connected by an algebraic equation. **This type of B.C. is known as Robin Mixed B.C**. This is also an example of flux type B.C. This type of B.Cs. are encountered in solid-fluid problems
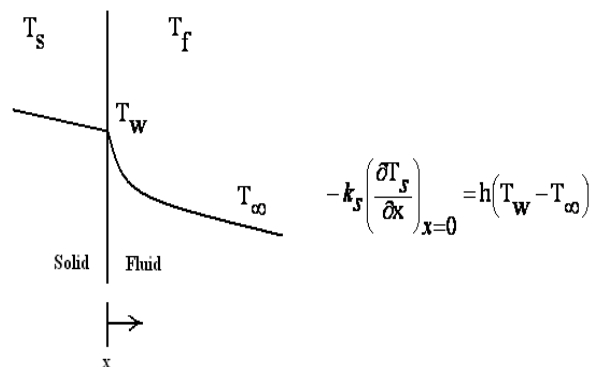

Fig.2.5 heat transfer at the fluid-solid interface

III. **Given temperature (may be a function of time) or given flux at a surface. It can be given as**

$$\frac{\partial T}{\partial t} = 0 \quad \text{(for steady state condition)}$$

IV. Here normal derivative of the dependant variable is specified at the plane surface or at the wall.

insulation

Solid

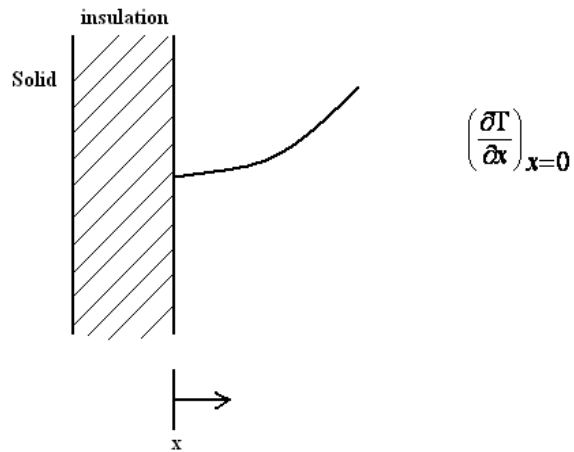$$\left(\frac{\partial T}{\partial x}\right)_{x=0}$$

x

Fig.2.6 heat transfer through insulated walls

V. In this, normal derivative of the dependant variable is defined for cylindrical or spherical geometry.
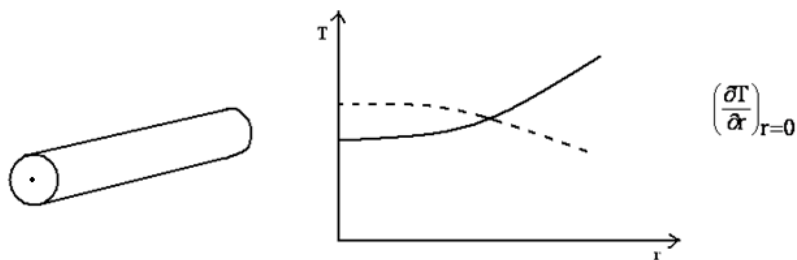
T

$$\left(\frac{\partial T}{\partial x}\right)_{r=0}$$

r

Fig.2.7 heat transfer through walls of cylinder or sphere