Module 2 :

"Diffusive" heat and mass transfer

Lecture 7:

Heat Transfer in Extended surfaces (Fins)

"Diffusive" Heat and Mass Transport

Heat conduction in a cooling Fin

Fins: Heat flow mainly depends on three factors (1) area of the surface (2) temperature difference and (3) the convective heat transfer coefficient. Rate of heat transfer can be increased by enhancing any one of these factors. Out of these, the base surface area is limited because of the design of the object; temperature difference depends on process and having process limitations. The only choice appears to be the convection heat transfer coefficient and this can not be increased beyond a certain value. Thus the possible option is to increase the base surface area by the extended surfaces also known as fins.

Fins are thus used whenever the available surface area is found insufficient to transfer required quantity of heat with available temperature gradient and heat transfer coefficient. In the case of fins the direction of heat transfer by convection is perpendicular to the direction of conduction heat flow.

Some of the examples of the use of extended surfaces are in cylinder heads of air cooled engines and compressors and on electric motor bodies. In radiators and air conditioners, tubes with circumferential fins are normally used to increase the heat flow. Electronic chips cannot function without using fins to dissipate heat generated. Fins with different shapes are in use. These are (i) Plate fins of constant sectional area (ii) Plate fins of variable sectional area (iii) Annular of circumferential fins with constant thickness (iv) Annular fins of variable thickness (v)

Pin fins with constant sectional area and (vi) Pin fins with variable sectional area. Fig.(7.1) shows fins with different geometrical patterns.

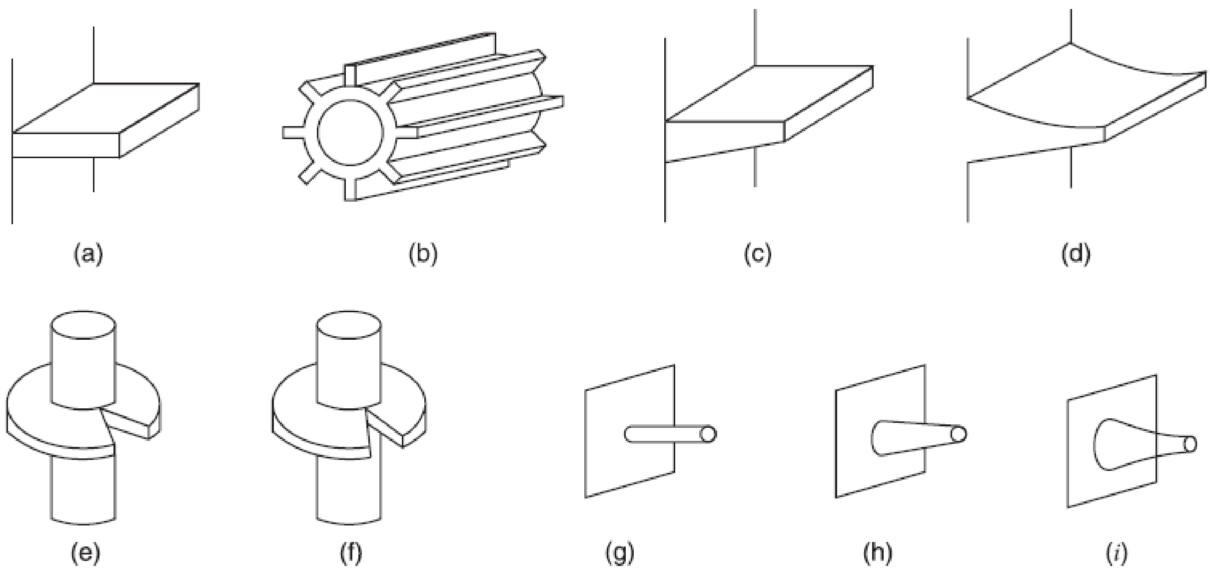

Fig. 7.1 **(a) Longitudinal Fin of Rectangular profile** **(b)Cylindrical tubes with Fins of Rectangular profile**

(c)Longitudinal Fin of Trapezoidal profile **(d)Longitudinal Fin of Parabolic profile**

(e)Cylindrical tube with Radial Fin of Rectangular profile **(f)Cylindrical tube with Radial Fin of truncated Conical profile**

(g)Cylindrical pin Fin **(h)Truncated Conical spine**

(i)Parabolic Spine

Mathematical Model for Heat Transfer in Fins

A simplified model for heat transfer through rectangular fin with constant surface area is shown in fig. (7.2). The surface of the slab from which heat is to be dissipated to the surrounding fluid, is extended by a fin on it. Heat is transferred from the surface to the fin from at its base by conduction. This heat is mainly convected to the surrounding fluid over the fin surface (it may radiate also). The assumptions made are,

Assumption

1. Temperature (T) varies along the length of the fin (Z-direction) only
2. Negligible heat loss from the edge and from the end
3. Constant thermal conductivity k and heat transfer coefficient h
4. Steady state
5. Heat transfer by radiation is neglected

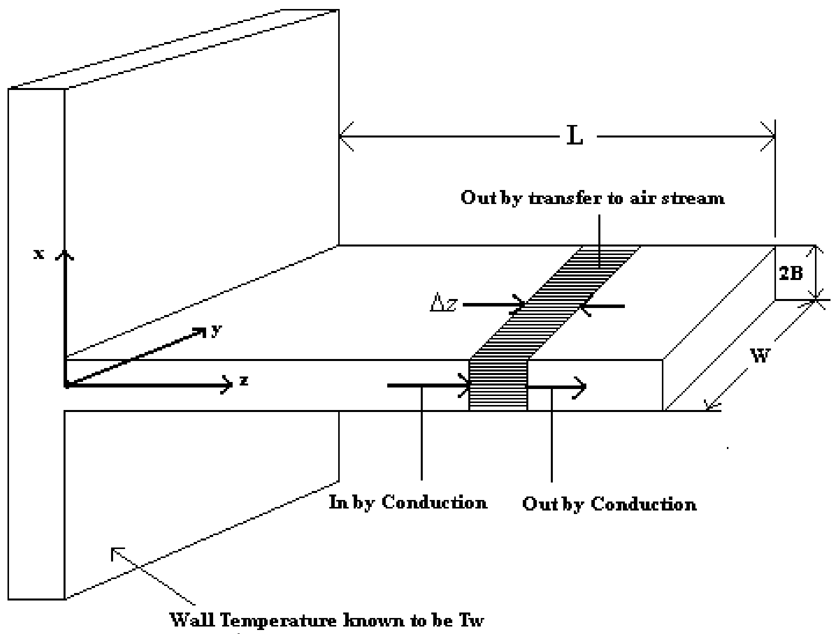

Fig. 7.2 Heat transfer through Fin of rectangular profile

The Conservation Equation can be derived using

1. Differential shell balance
2. Starting from General Conservation equation

1. Differential Shell Balance

The heat flux per unit area over a specified region is given as

Heat flux in at Z – Heat flux out at Z+ΔZ – Heat flux out due to convection=0

$$(q_z)_z 2BW - (q_z)_{z+\Delta z} 2BW - 2hW\Delta z(T - T_a) = 0 \tag{7.1}$$

where 2B is the fin thickness and W is the width of the fin.

$$-\frac{dq_z}{dz} = \frac{h}{B}(T - T_a) \tag{7.2}$$

But, Rate of heat transfer is given by

$$q_z = -k\frac{dT}{dz} \tag{7.3}$$

Inserting equation (7.3) into (7.2), we get

$$\frac{d^2T}{dz^2} = \frac{h}{kB}(T - T_a) \tag{7.4}$$

where h is the heat transfer coefficient and k is the thermal conductivity.

2. General Conservation Equation

The analysis can be done by considering the energy balance for small elemental volume of a length Δz as shown in fig. (7.2). For this case the applicable energy equation in terms of temperature T is

$$\rho \hat{C}_p \frac{DT}{Dt} = k\,\nabla^2 T + \dot{Q} \qquad (7.5)$$

where $\dot{Q}$ is the **rate of heat generation**

Note: equation (7.5) holds for solids with constant ρ and *k*

The heat balance under steady state condition is given by

Heat conducted in at Z – Heat conducted out at Z+ΔZ

– Heat convected over the surface of element= 0

According to assumption (3), i.e. constant k and also considering constant surface area (not influenced by temperature), hence for the steady state condition, equation (7.5) yields

$$k\nabla^2 \mathrm{T} + \dot{\mathrm{Q}} = 0 \qquad (7.6)$$

If we were to solve the 2-D fin problem (in x and z) then $\dot{Q} = 0$ and the heat transfer coefficient would enter through the boundary condition. But, by assuming 1-D transport, the heat loss due to h will enter as a volumetric sink, as $-\dot{Q}$, in the energy balance. [Note that $\dot{Q}$ is a source in equation (7.5)]. The heat removal rate will then be given by

$$\dot{Q} = -\frac{\mathrm{h}}{\mathrm{B}}\left(\mathrm{T} - \mathrm{T_a}\right) \qquad (7.7)$$

where $\frac{1}{\mathrm{B}}$ is the surface-to-volume ratio

$$\left(\frac{Surface}{Volume}=\frac{2\mathrm{W}\Delta \mathrm{z}}{2\mathrm{BW}\Delta \mathrm{z}}=\frac{1}{\mathrm{B}};\ \text{edges of the fin are neglected here}\right)$$

Put equation (7.7) in equation (7.6), so we get

$$\frac{\mathrm{d}^2\mathrm{T}}{\mathrm{dz}^2}-\frac{\mathrm{h}}{k\mathrm{B}}\left(\mathrm{T}-\mathrm{T_a}\right)=0 \qquad (7.8)$$

Boundary Conditions:

at $z = 0$ $\qquad T = T_w$

at $z = L$ $\qquad \frac{\mathrm{dT}}{\mathrm{dz}}=0$ (No heat loss from edge, see assumption (2))

Non-dimensional equation and B.Cs

Define $\theta=\frac{\mathrm{T}-\mathrm{T_a}}{\mathrm{T_w}-\mathrm{T_a}}$, $J=\frac{\mathrm{z}}{\mathrm{L}}$, $N=\sqrt{\frac{hL^2}{k\mathrm{B}}}$

After inserting non-dimensional quantities and algebraic manipulations, equation (7.8) becomes

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}J^2}=N^2\theta \qquad (7.9)$$

with the flollowing B.Cs.

$$\left.\begin{array}{l} \text{At } J=1 \Rightarrow (\theta)_{J=0}=1 \text{ and} \\ \text{At } J=0 \Rightarrow \left(\dfrac{\mathrm{d}\theta}{\mathrm{d}J}\right)_{J=1}=0 \end{array}\right\} \tag{7.10}$$

The general solution to eqn. (7.7) is

$$\theta = C_1 e^{NJ} + C_2 e^{-NJ} \tag{7.11}$$

Using B.C.s given in eqn. (7.10) in eqn. (7.11), and Utilizing $\text{Cosh}(\text{x}) = \dfrac{\left(e^x + e^{-x}\right)}{2}$, we get

$$\boxed{\theta = \frac{\cosh N(1-J)}{\cosh N}} \tag{7.12}$$

In this model the heat convected through tip of the fin is neglected. The error due o this can be minimized by increasing the length by ΔL equal to B (half the thickness) where 2B is the thickness of the fin. In case of circular fin ΔL is equal to D/4.