Module 3:

“Convective” heat and mass transfer

Lecture 25:

Heat Transfer to Boundary Layers

(Continued)

For large Pr, eqn. (10) from lecture 24 implies that $\delta_T << \delta$. Hence a linear velocity profile can be assumed inside the boundary layer or

$$\frac{v_x}{v_\infty} = f'(\eta) = c\eta \quad \Rightarrow \quad f = c\frac{\eta^2}{2}$$

(using $f=0$ at $\eta=0$),

where 'c' is a constant

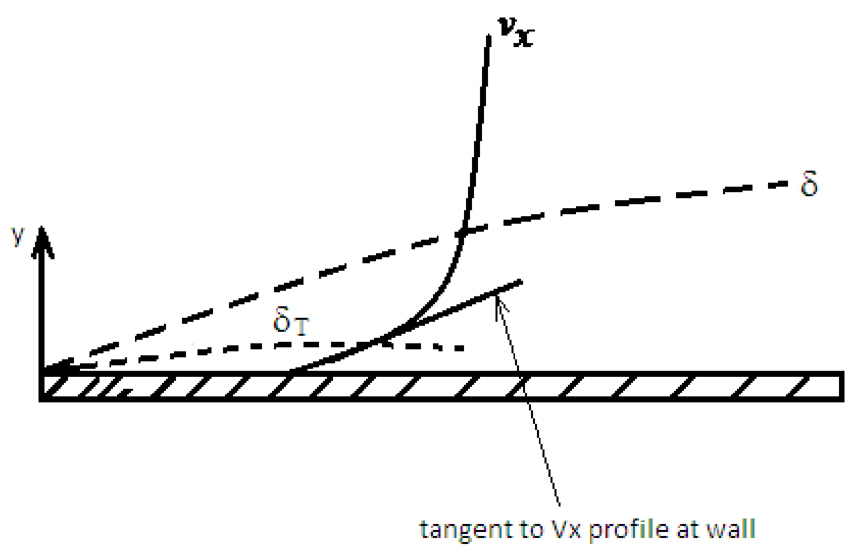

Fig 1 Boundary layer and thermal boundary layer on flat plate

Then

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = \frac{1}{\int_0^\infty \left[\exp\left(-\Pr C\frac{\eta^3}{12}\right)\right]d\eta} = C'\sqrt[3]{\Pr} \qquad (1)$$

Where C' is the constant.

All the other extreme of small Pr, $\delta_T >> \delta$ and for most thermal B.L. $v_x \approx v_\infty \Rightarrow \; f'(\eta) \cong 1 \;\; \Rightarrow \;\; f = \eta$.

Then

$$\left.\frac{d\theta}{d\eta}\right|_{\eta=0} = \frac{1}{\sqrt{\pi}}\sqrt{\mathrm{Pr}}$$

Hence

$$Nu_x = 0.564\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/2} \qquad (2)$$

The above equation is valid for $0.005 < Pr < 0.05$. Comparing to the "exact" solution (eqn. (6) of lecture 24) one finds that the coefficient 0.564 is a little too high. This is because the approximation $v_x \approx v_\infty$ is good **far from the surface** while we are trying to evaluate Nu_x **at the surface**.

Average heat transfer coefficient

$$q = h_w A\left(T_w - T_\infty\right) \;\text{ with }\; h_w = \frac{1}{L}\int_0^L h_x dx$$

For $\dfrac{h_x x}{k} = c\left(\dfrac{\nu}{\alpha}\right)^{1/3}\left(\dfrac{v_x x}{\nu}\right)^{1/2} \quad \Rightarrow \quad h_w = 2h_{x=L}$

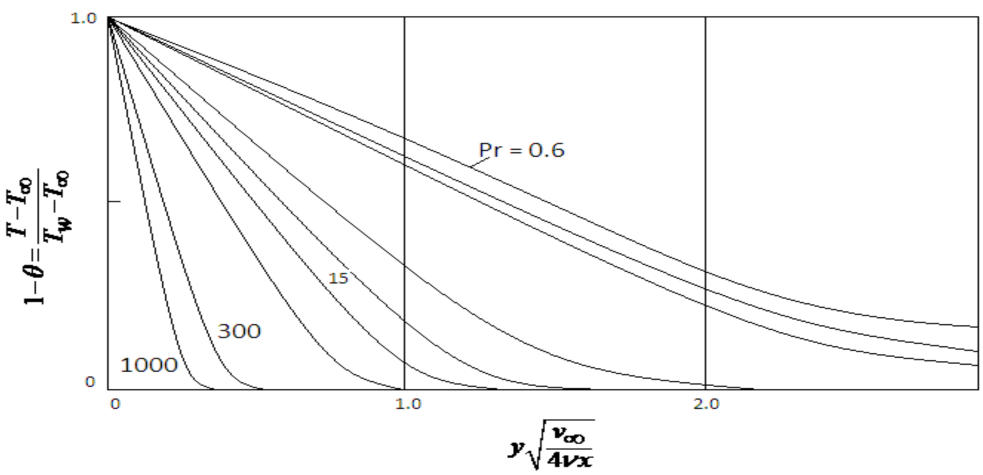

Fig. 2

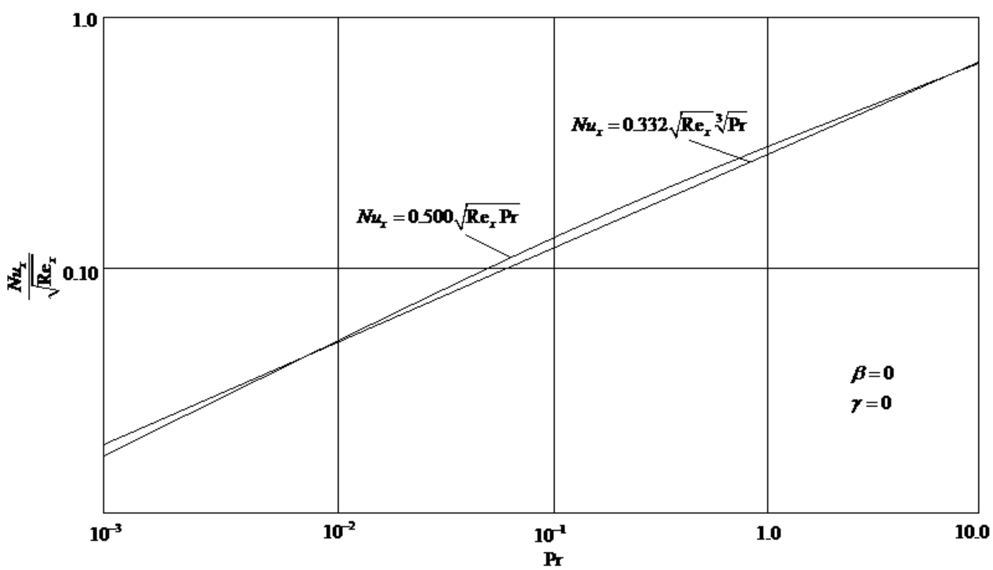

Fig. 3

This analysis can be generalised to **<u>wedge flows</u>** (shown below in Fig. 4).

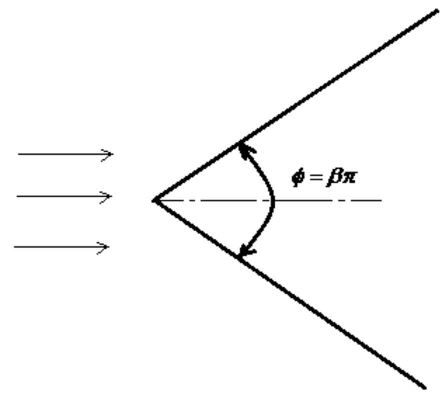

Fig. 4

$v_\infty = Cx^m$

Where C is a constant

In this case similarity variable becomes $\eta = \sqrt{m+1}\; y\sqrt{\dfrac{v_\infty}{\nu \bullet x}}$;

$$f(\eta) = \sqrt{m+1}\;\frac{\psi}{\sqrt{\nu v_\infty x}}$$

The case we analysed can be obtained by setting β = 0 (m = 0), thus $v_\infty = \text{constant}$.

Also similarity exists for cases in which

$$T_w - T_\infty = C_t x^r \qquad (3)$$

Where C_t is a constant

Then the energy balance becomes

$$\frac{d^2\theta}{d\eta^2}+\frac{1}{2}\Pr f\frac{d\theta}{d\eta}-\left(1-\frac{\beta}{2}\right)\Pr\gamma f'\theta=0 \qquad (4)$$

With $\begin{matrix}\theta=0 & \text{at} & \eta=0\\ \theta=1 & \text{at} & \eta\to\infty\end{matrix}$

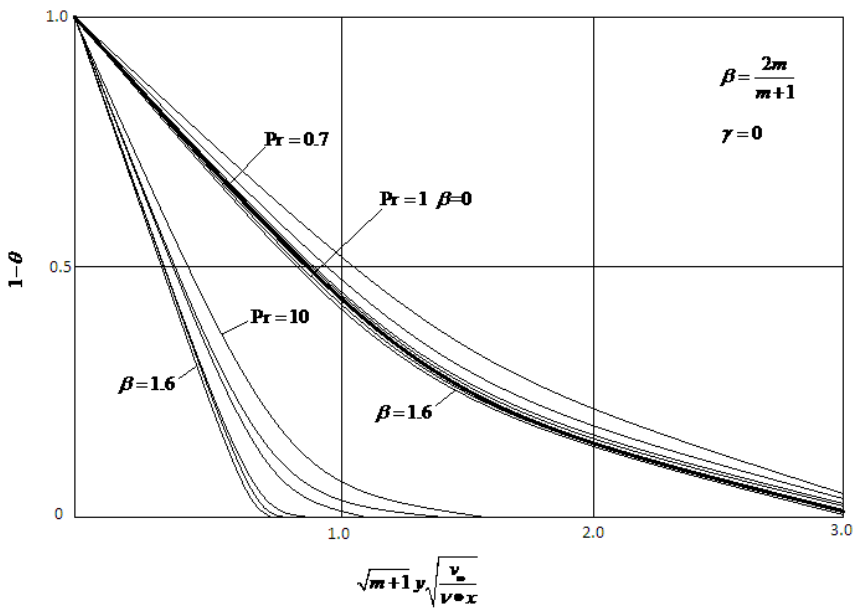

Fig. 5 Varying β does not have a strong effect on θ

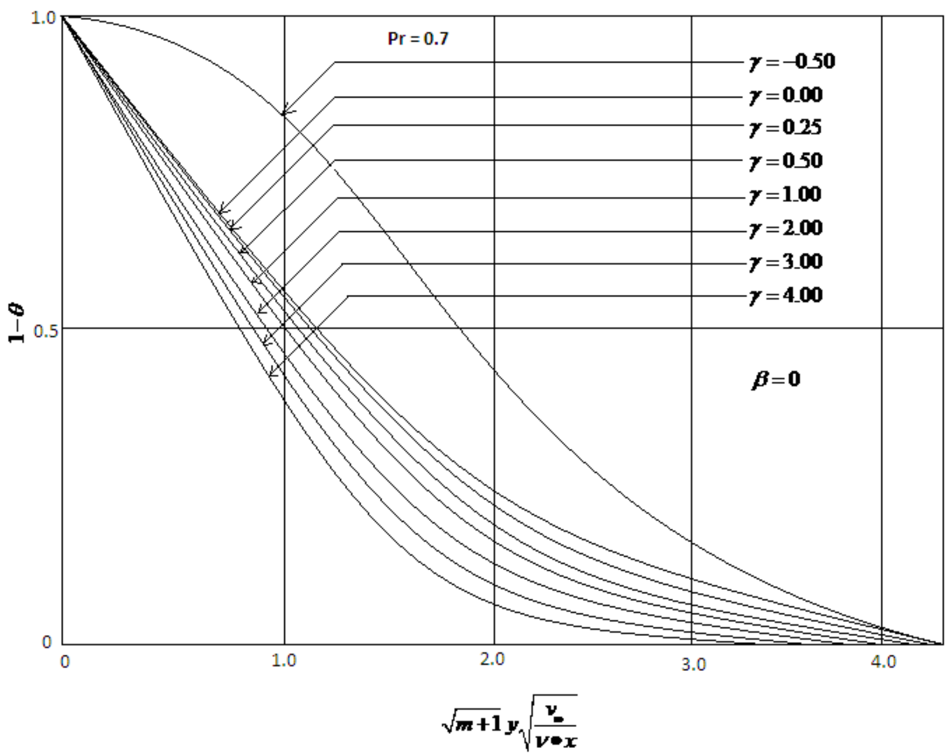

Fig. 6

For $\gamma = -0.5$ the surface flux vanishes. For $\gamma < -0.5$, heat flows into the wall even when the wall temperature is higher than the free stream temperature.