Module 3:

“Convective” heat and mass transfer

Lecture 29:

Convective Transport in channels

Flow in pipes: cylindrical axis-symmetric (no θ) geometry

The assumptions are: constant properties, Newtonian fluid, no viscous dissipation

Four categories

I. Velocity and Temperature profiles both developing. Need to keep inertia terms in N-S equations.

II. V and T profiles both fully developed: solve for V and insert to find T profile. Analytical solution possible depending on B.C.

III. Fully developed V, developing T

IV. Fully developed T, developing V (plug flow)

(Fully developed means only profile shape is invariant with z, but absolute value of dependant variable may change with z)

Two limiting boundary conditions at the pipe wall

T = const. or $\frac{\partial T}{\partial r} = \text{const}\left(\text{constant heat flux}\right)$

Flow-average or Cup-Mixing temperature $\frac{\langle v_z \; T\rangle}{\langle v_z\rangle} = \frac{\int_0^{2\pi}\int_0^R v_z(r)T(r)r \; dr \; d\theta}{\int_0^{2\pi}\int_0^R v_z(r)r \; dr \; d\theta} = T_M$

This is the temperature one would measure if the tube were chopped off at z and if the fluid issuing forth were collected in a container and thoroughly mixed.

Case (I). This case will not be examined.

Case (II). $\underbrace{\frac{T_0 - T}{T_0 - T_M}}_{\left(\substack{\text{not a function of z;} \\ \text{as ratio is not a function of z}}\right)} = f(r)\,;\ \frac{\partial}{\partial z}\left(\frac{T_0 - T}{T_0 - T_M}\right) = 0$

$$\frac{\partial T}{\partial z} = \frac{\partial T_0}{\partial Z} - \frac{T_0 - T}{T_0 - T_M}\frac{dT_0}{dz} + \frac{T_0 - T}{T_0 - T_M}\frac{dT_M}{dz} \qquad (1)$$

Case (**II-1**): Constant wall heat flux $\mathbf{q_0}$**;**

Examples

1. Electrical heating
2. Heating by radiation
3. Special kind of heat exchangers

Define heat transfer coefficient by $q_0 = h\left(T_0 - T_M\right) = \text{Const.}$

Indeed h = Const. (since fully developed temperature profile, hence no change of slope of temperature profile at the wall)

Since $\left(T_0 - T_M\right) = \text{Const.}$

$$\frac{dT_0}{dz} = \frac{\partial T_M}{\partial Z} \quad \Rightarrow \quad \frac{\partial T}{\partial z} = \frac{dT_0}{dz} = \frac{dT_M}{dz}$$

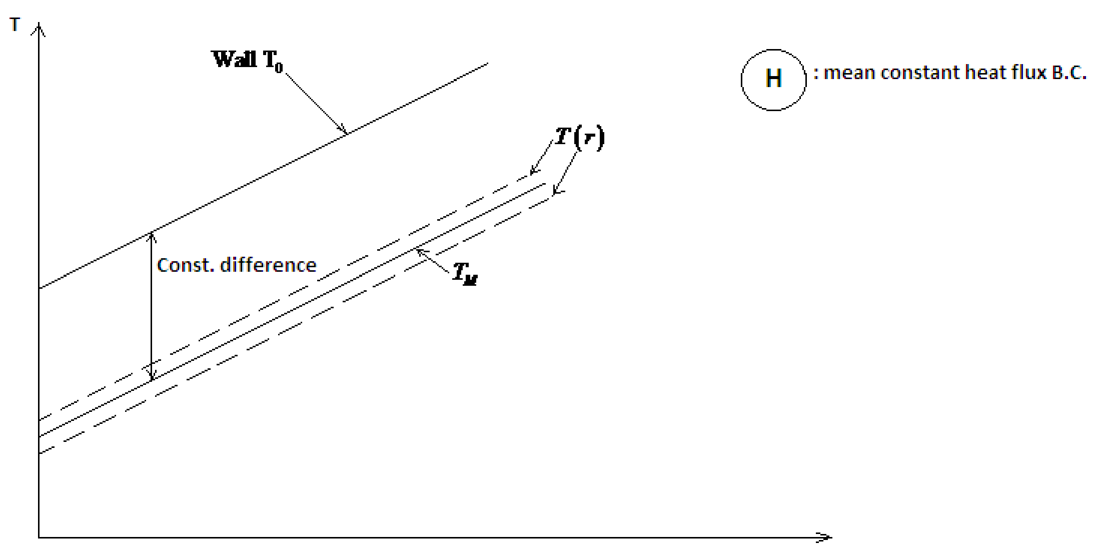
Another way of arriving at $\frac{dT_M}{dz} = \text{Const.}$ Use overall energy balance

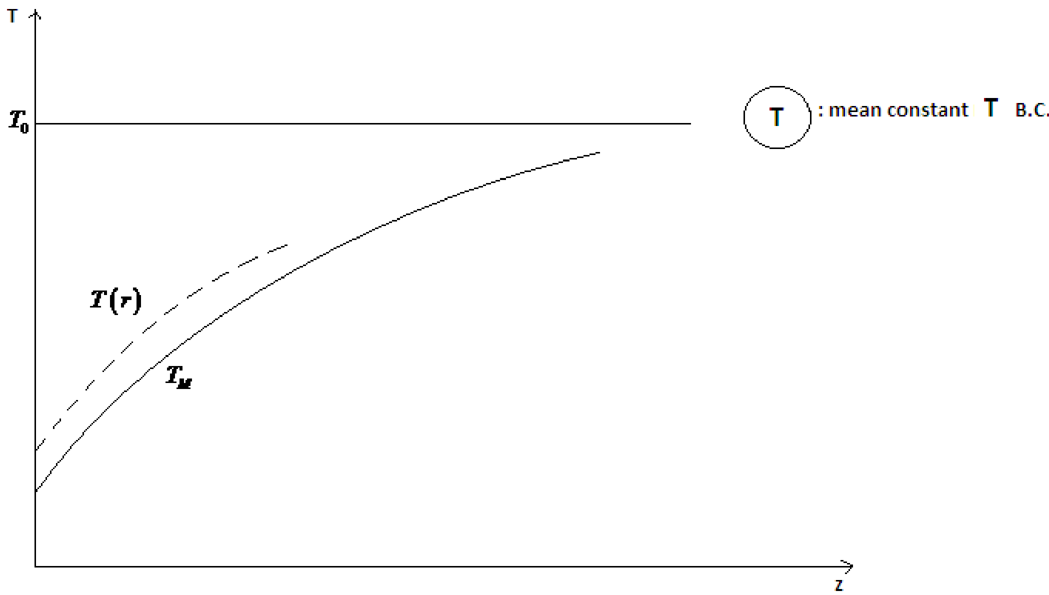
$$C_p A\rho \langle v_z \rangle T_M \big|_{z=z} - C_p A\rho \langle v_z \rangle T_M \big|_{z=z+\Delta z} + q_0 (2\pi R)\Delta z = 0$$

$$q_0 = \frac{\rho \langle v_z \rangle C_p R}{2} \frac{dT_M}{dz} \qquad (2)$$

$$\frac{dT_M}{dz} = \text{Const.}$$

Expected T profiles for fully developed V and T





In both cases $\theta = \dfrac{T_0 - T}{T_0 - T_M} = f(r)$ only and does not depend on z.

Governing PDE (neglect axial diffusion)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{v_z}{\alpha}\frac{\partial T}{\partial z} = \frac{v_z}{\alpha}\frac{\partial T_M}{\partial z}$$

with $v_z = 2\langle v_z \rangle \left(1 - \dfrac{r^2}{R^2}\right)$

Direct integration yields

$$T = T_0 - \frac{2\langle v_z \rangle}{\alpha}\left(\frac{dT_M}{dz}\right)\left(\frac{3}{16}R^2 + \frac{r^4}{16R^2} - \frac{r^2}{4}\right)$$

$T_M = T_0 - \frac{11}{96}\frac{2\langle v_z\rangle}{\alpha}\left(\frac{dT_M}{dz}\right)R^2$ and $q_0 = h\left(T_0 - T_M\right)$

$q_0 = h\frac{11}{96}\frac{2\langle v_z\rangle}{\alpha}\left(\frac{dT_M}{dz}\right)R^2$. But we also have eqn. (2)

Thus $Nu = \frac{h(2R)}{K} = 4.364$ for $\{(H)$ and fully developed T and V profiles$\}$ **Case (II-2): Constant**

T_0 at the pipe wall

Then from eqn. (1)

$\frac{\partial T}{\partial z} = \frac{T_0 - T}{T_0 - T_M}\frac{dT_M}{dz}$ and PDE becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2\langle v_z\rangle}{\alpha}\left(1 - \frac{r^2}{R^2}\right)\frac{T_0 - T}{T_0 - T_M}\left(\frac{dT_M}{dz}\right)$$

Solve numerically by successive substitution.

The asymptotic $Nu = 3.658$

<u>Summary of the Results</u>

	Asymptotic *Nu* (Fully developed T)
Parabolic v_z, (H)	4.36
Parabolic v_z, (T)	3.66

Plug flow, (H)	8.0
Plug flow, (T)	5.86

How long does it take for fully developed (Laminar flow)?

Velocity profile: $\frac{z}{d} > 0.05\,\mathrm{Re}$

Where d is the tube diameter and z is the entrance length

Re is based on the tube diameter

Temperature profile: $\frac{z}{d} > 0.05\,\mathrm{Re}\,\mathrm{Pr}$

Concentration profile: $\frac{z}{d} > 0.05\,\mathrm{Re}\,\mathrm{Sc}$

Example: assume water Re = 100, Pr = 7, Sc = 1000

Then $\frac{z}{d} > 5$ for velocity

$\frac{z}{d} > 35$ for temperature

$\frac{z}{d} > 5000$ for concentration

Skelland correlation for fully developed V, but developing T for constant wall temperature

$$Nu = 3.66 + \frac{0.0668 \frac{d}{z} \mathrm{Re}\,\mathrm{Pr}}{1 + 0.04 \left[\frac{d}{z} \mathrm{Re}\,\mathrm{Pr} \right]^{2/3}} \tag{3}$$

For $\frac{z}{d} > 0.05\,\mathrm{Re}\,\mathrm{Pr}$; *Nu* approaches 3.66

For mass transfer replace *Nu* by *Sh* and *Pr* by *Sc*

Lewis number is given by $Le = \frac{Sc}{\mathrm{Pr}}$. If $Le \sim 1$, temperature and concentration boundary layer are of the same thickness.