

MODULE I

ESSENTIAL PROCESS CONTROL BASICS

In this module, we cover essential aspects of process control theory, necessary for proper control system design. A hands-on approach to covering process dynamics, PID control algorithm, identification, tuning, advanced control structures and multivariable decentralized control is used, in contrast to the mathematically elegant but abstruse treatment in most controls texts. Only the most essential and relevant aspects are covered. In the interest of brevity, since this is a course on plantwide control and not control theory, we do not provide many detailed solved examples to back the theory and refer the reader to standard text-books for the same.

Chapter 1. Process Dynamics

Process dynamics refers to the time trajectory of a variable in response to a change in an input to the process. All of us have an inherent appreciation of process dynamics in the sense that the effect of a cause takes time to manifest itself. It thus takes 20 minutes for a pot of rice to cook over a flame, 5-10 minutes for the water in the geyser to heat up sufficiently, years and years of dedicated practice to become an adept musician (or a good engineer, for that matter!) and so on so forth. In each of these examples, a change in the causal variable (flame, electric heating or dedicated practice) results in a change over time in the effected variable (degree of “cookedness” of rice, geyser water temperature or a musician’s virtuosity). Process dynamics deals with the systematic characterization of the time response of the effected variable to a change in the causal variable. In process control parlance, the causal variable is referred to as an input variable and the effected variable is referred to as an output variable.

In order to fix ideas in the context of chemical processes, Figure 1.1 shows the schematic of a simple distillation column. An equimolar ABC feed is separated to recover nearly pure A as the distillate with the bottoms being a BC mixture with trace amounts of A. The fresh feed, reflux and reboil constitute the inputs to the column while the distillate and bottoms flow / composition and the tray composition / temperature profiles constitute the outputs.

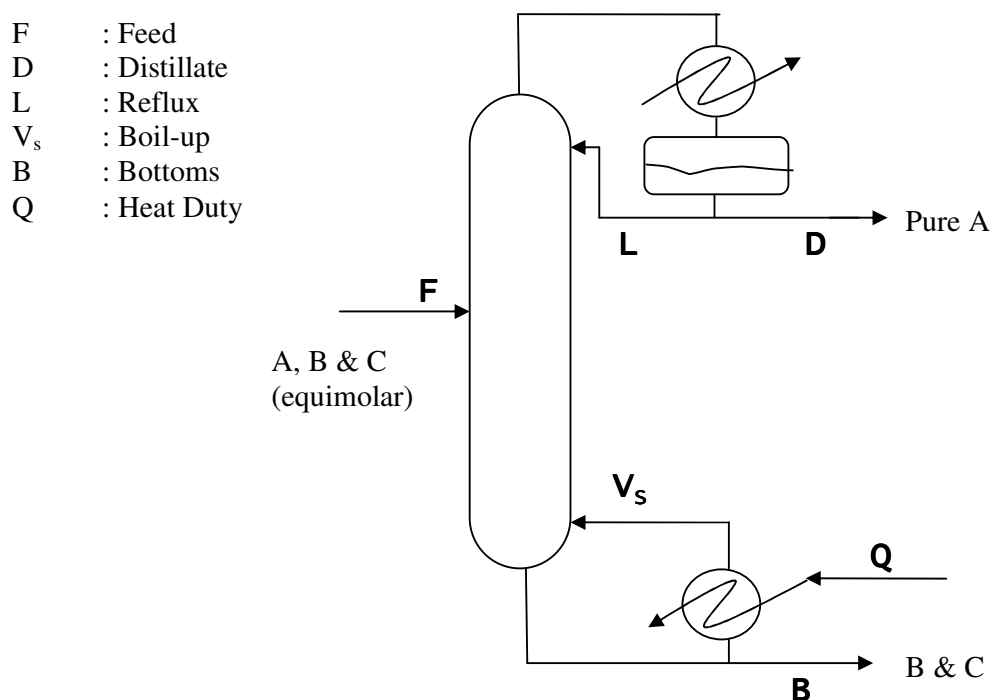


Figure 1.1. Schematic of a distillation column

1.1. Standard Input Changes

To systematically characterize the transient response of an output to a change in the input, the input change is usually standardized to a step change, a pulse change or an impulse change. These standard input changes are depicted in Figure 1.2. A step change in the input, the simplest input change pattern, is used in this work to characterize the process dynamics.

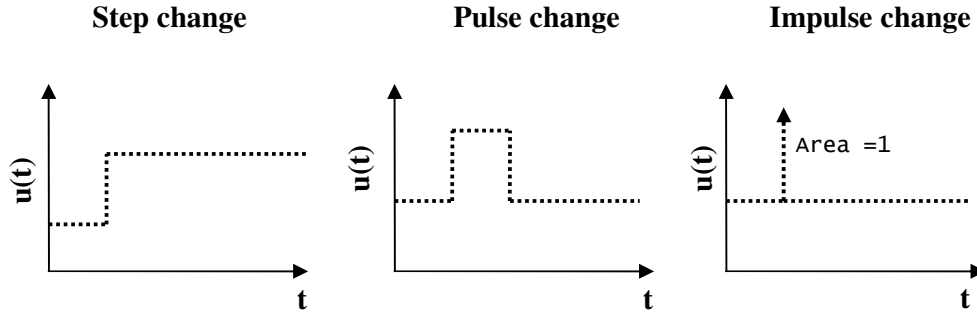


Figure 1.2. Standard input changes

1.2. Basic Response Types

The dynamics of every process are. Even so, the variety of transient responses can be characterized as an appropriate combination of one or more basic response types. These transient responses correspond to the solution of linear ordinary differential equations (ODEs). Linear ODEs can be compactly represented using Laplace transforms. For example consider a second order differential equation

$$\tau^2 \frac{d^2 y(t)}{dt^2} + 2\zeta\tau \frac{dy(t)}{dt} + y(t) = K_p u(t)$$

where $y(t)$ and $u(t)$ are the process output and input respectively. The Laplace transform representation in the s domain is obtained by replacing the n^{th} order derivative operator by s^n so that for the second order ODE above

$$\tau^2 s^2 y(s) + 2\zeta\tau s y(s) + y(s) = K_p u(s)$$

Rearranging, the input-output transfer function becomes

$$G_p = \frac{y(s)}{u(s)} = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

The ODEs and corresponding Laplace transform representation is noted in Table 1.1.

1.2.1. First Order Lag

The first order lag is the simplest transient response where the output immediately responds to a step change in the input (see Figure 1.3(a)). The ratio of the change in the output to the change in the input is referred to as the process gain, K_p . The time it takes for the output to reach 63.2% of its final value corresponds to the first order time constant τ_p . The output reaches ~95% of its final value in 3 time constants.

Table 1.1. Various differential equations and their Laplace transform

Terminology	Differential equation	Laplace Transform	$\frac{y(s)}{u(s)}$
Gain	$y(t) = K.u(t)$	K	
Derivative	$y(t) = \frac{du(t)}{dt}$	s	
Integrator	$y(t) = \int_0^t u(t).dt$	$\frac{1}{s}$	
First order lag	$\tau \frac{dy(t)}{dt} + y(t) = u(t)$	$\frac{1}{\tau s + 1}$	
First-order lead	$\tau \frac{du(t)}{dt} + u(t) = y(t)$	$\tau s + 1$	
Second Order Lag			
Underdamped $\zeta < 1$	$\tau^2 \frac{d^2 y(t)}{dt^2} + 2\zeta\tau \frac{dy(t)}{dt} + y(t) = K_p u(t)$	$\frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$	
Critically damped $\zeta = 1$	$\tau^2 \frac{d^2 y(t)}{dt^2} + 2\tau \frac{dy(t)}{dt} + y(t) = K_p u(t)$	$\frac{K_p}{(\tau s + 1)^2}$	
Overdamped $\zeta > 1$	$\tau_1\tau_2 \frac{d^2 y(t)}{dt^2} + \zeta(\tau_1 + \tau_2) \frac{dy(t)}{dt} + y(t) = K_p u(t)$	$\frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$	
Deadtime	$y(t) = u(t - \theta)$	$e^{-\theta s}$	
Lead-lag	$\tau_2 \frac{dy(t)}{dt} + y(t) = \tau_1 \frac{du(t)}{dt} + u(t)$	$\frac{\tau_1 s + 1}{\tau_2 s + 1}$	

1.2.2. Higher Order Lags

If the output from a first order lag is input to another first order lag, the latter's output behaves as a second order lag with respect to the input to the first lag. The overall transient response is S shaped with the output not responding immediately to a change in the input. When the time constant of the two lags are different, the response is called an over-damped second order response. The response for the special case where the two time constants are equal is called the critically damped second order response. Higher order systems result as more first order lags are connected in series with the transient response becoming increasingly sluggish.

1.2.3. Second Order Response

Sometimes, a step change in the input causes the output to oscillate before settling at the final steady state. The simplest such response corresponds to a second order underdamped system. The damping coefficient, ζ , can be used to characterize all second order responses – overdamped ($\zeta > 1$), critically damped ($\zeta = 1$) and underdamped ($\zeta < 1$). The second order response is shown in Figure 1.3(b).

To gain an appreciation of the impact of damping coefficient on the transient response, Table 1.2 reports the ratio of the second overshoot to the first overshoot for

different values of ζ . A quarter decay ratio is observed for a damping coefficient of 0.218. Sustained oscillations (decay ratio = 1) are observed for a damping coefficient of 0. As ζ increases to 1, the overshoot in the output disappears.

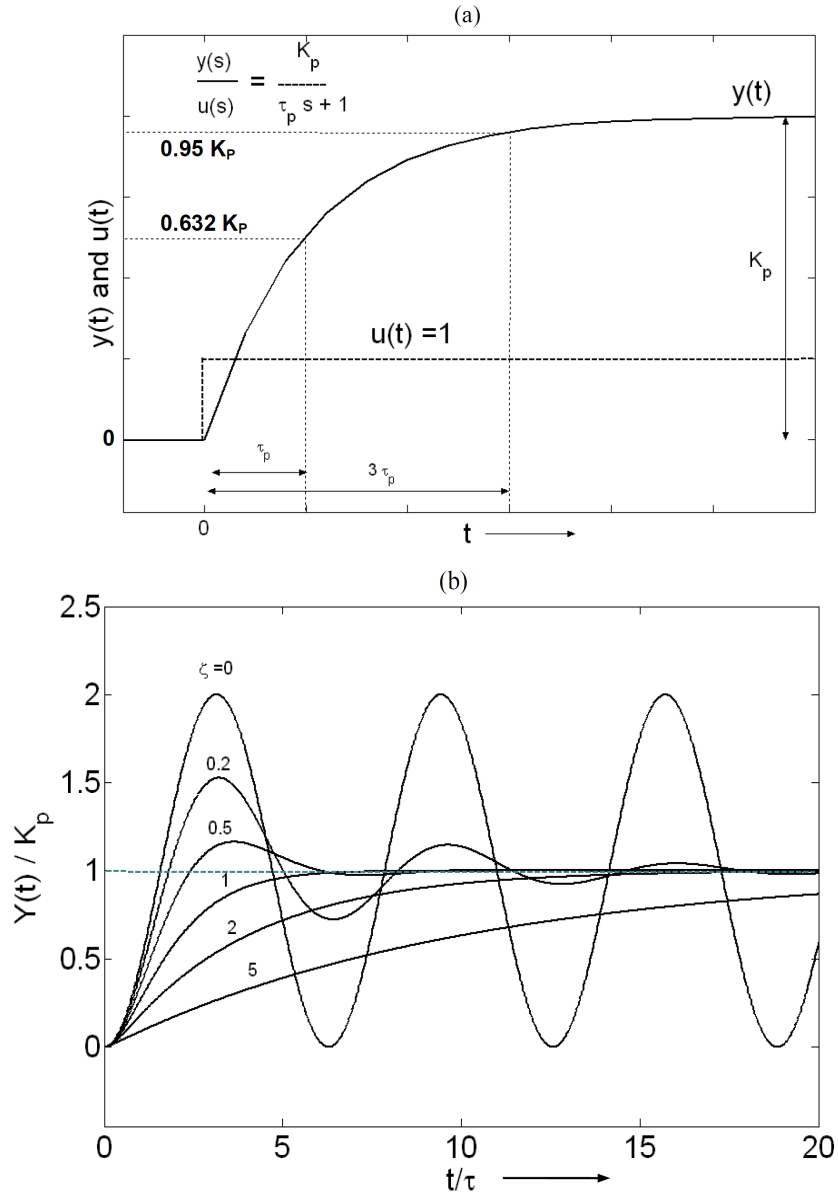


Figure 1.3. Output response for unit step change to (a) First order & (b) Second order process.

Table 1.2. Decay ratio for various different damping coefficients

Damping Coefficient, ζ	0	0.05	0.1	0.2	0.218	0.4	0.6	1
Decay ratio	1.000	0.730	0.532	0.277	0.250	0.064	0.009	0.000

1.2.4. Other Common Response Types

Other types of responses include the pure integrator, the pure dead-time, and the inverse response. The transient response to a unit step change can be seen in Figure 1.4 and are self explanatory.

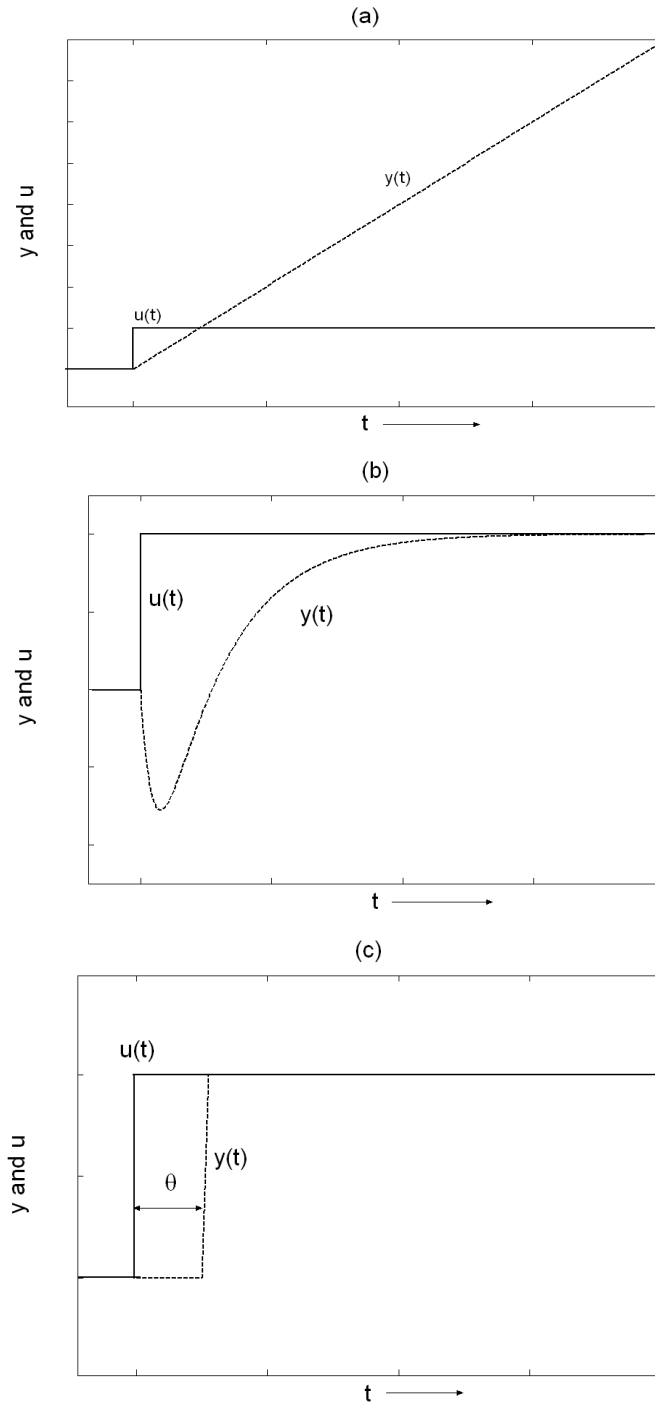


Figure 1.4. The output response for a unit step change for (a) pure integrator, (b) inverse response and (c) pure dead time process.

The most common example of a pure integrator is the response of the tank level to change in the inlet / outlet feed rate. Unless the inlet and outlet flows are perfectly equal, the tank level is either rising or falling in direct proportion to the mismatch in the flows. The level in a tank is thus non-self regulating with respect to the connected flows. A controller must be used to stabilize all such non-self regulating process variables. Dead time is very common in chemical processing systems and is due to transportation delay. A very common example of the inverse response is the response of the liquid level in a boiler to a change in the heating duty. As the heating duty is increased, the vapour volume entrapped in the liquid increases causing the liquid interface level to rise initially. Over longer duration, the level of course reduces since more liquid is being vaporized. As will be seen later, dead time and inverse response can create control difficulties.

1.2.5. Unstable Systems

Some systems may be inherently unstable. Unstable transient responses are shown in Figure 1.5. The unstable response may be non-oscillatory or oscillatory as in the Figure. Reactor temperature runaway is an example of an unstable process. A control system must be used to stabilize an inherently unstable system.

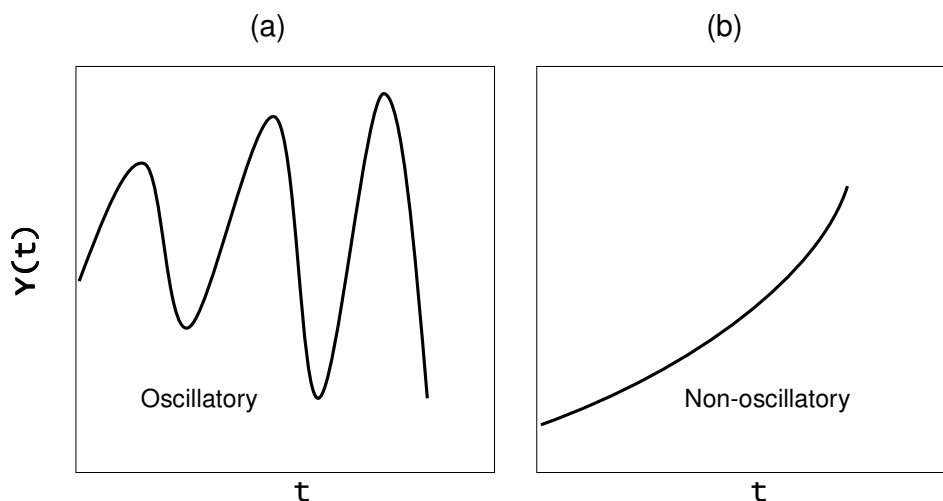
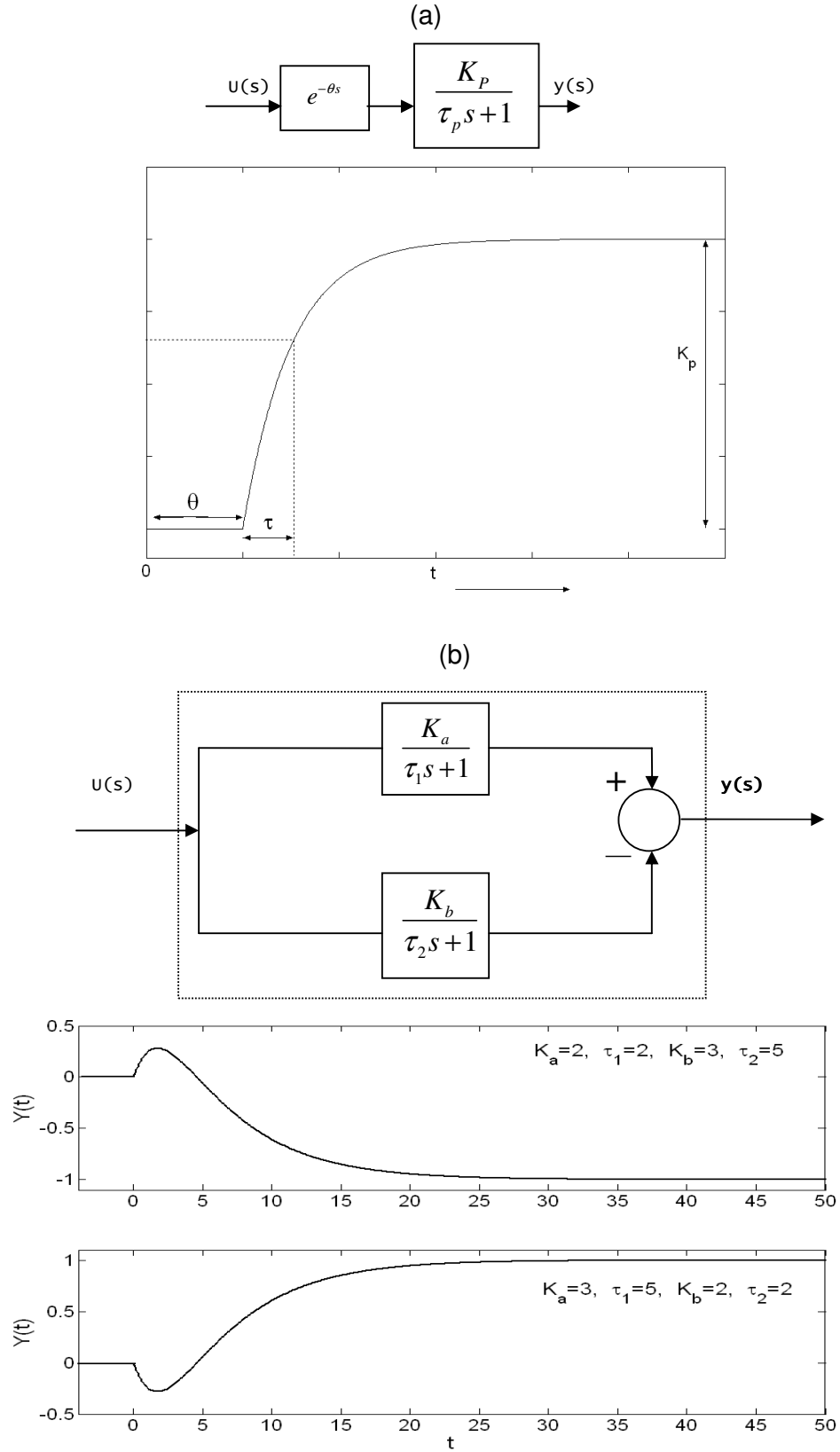


Figure 1.5. The output response for unstable process. (a) Oscillatory and (b) non-oscillatory

1.3. Combination of Basic Responses

Any transient response can be reasonably represented as a combination of the above basic response types. One such combination is the first order lag plus dead time that has been found to represent the transient response of many chemical processing systems very well. The response is illustrated in Figure 1.6(a). Another example of such a combination is the inverse response which can be represented by the parallel combination of two first order lags. One of the lags has a small gain and a small time constant (ie a fast response) while the other lag has a gain of larger magnitude and opposite sign with a much larger time constant (i.e. a slow response in the opposite direction). Figure 1.6(b) illustrates this concept.



Chapter 2. Feedback Control

The safe and stable operation of a process requires that key variables be maintained at or close to their design values in the face of disturbances entering the process. For example, it may be necessary to hold a process stream flow rate nearly constant even as the upstream / downstream pressure fluctuates. Similarly the temperature at the inlet to a packed bed reactor must be maintained at its design value to prevent reactor run-away and also ensure the desired conversion to products(s) for varying flow rates of the process stream. Maintaining a process variable at or near a certain value requires a manipulation handle that can be appropriately adjusted. For example, the valve opening can be adjusted to maintain the flow rate through the pipe. Similarly the heating duty of the furnace can be used to heat the process to maintain the reactor inlet stream temperature. This leads to the idea of feedback control where the deviation in the variable to be maintained at / near its design value is used to make appropriate adjustments in the manipulation handle. The variable to be maintained at its design value is referred to as the controlled variable and the adjustment handle is called the manipulated variable. The algorithm / procedure used to quantitatively translate the deviation in the controlled variable to the adjustment in the manipulated variable is known as the control algorithm.

2.1. The Feedback Loop and its Components

A feedback control loop is schematically illustrated in Figure 2.1. Its primary components are the sensor, transducer, transmitter, controller, I/P converter and the final control element. The sensor is the sensing element used to measure the controlled variable (and other important process variables that may not be controlled). Flow, temperature and pressure sensors are routinely used in the process industry. Composition analyzers are used less frequently to measure only key compositions such as the product purity. Most sensors translate a change in the state of the variable to be measured into an equivalent mechanical signal such as the stretching / bending of a Bourdon tube. The mechanical signal needs to be converted into an electrical signal for onward transmission to the control room (or stand-alone controller). This is accomplished by the transducer. For standardization across different manufacturers, the range of the input and output signal from a controller is 4-20 mA. The range corresponds to the sensor / final control element span. The transmitter converts the electrical signal from the transducer to the 4-20 mA range. The transmitter signal is input to the controller. The desired value for the controlled variable, referred to as the set-point, is also input to the controller. The controller output signal is again between 4-20 mA. In the process industry, this electrical signal is converted to an equivalent 3-15 psig pneumatic pressure signal using an I/P converter. The pressure signal (or rather change in the pressure signal) is used to move the final control element to bring about a change in the manipulated variable. In the process industry, almost all final control elements are control valves that adjust the flow rate of a material stream.

The controller subtracts the current value of the controlled variable from its set-point to obtain the error signal as

$$e_t = y^{SP} - y_t$$

where y is the controlled variable. The subscript t refers to the current time. The error signal is input to the control algorithm to determine the change in the manipulated variable (control input) to be implemented. This is schematically illustrated in Figure 2.2. The most popular control algorithm, namely the PID algorithm is discussed next.

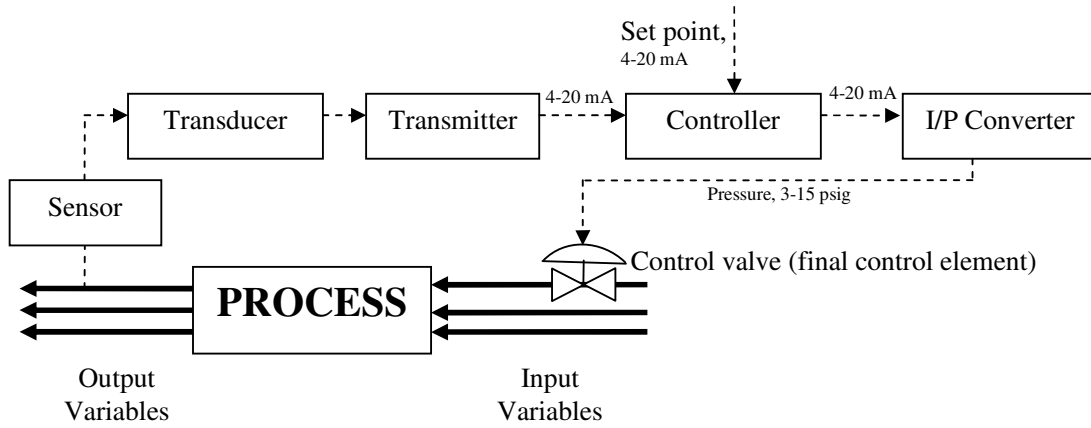


Figure 2.1. Schematic of a process with feed back control scheme

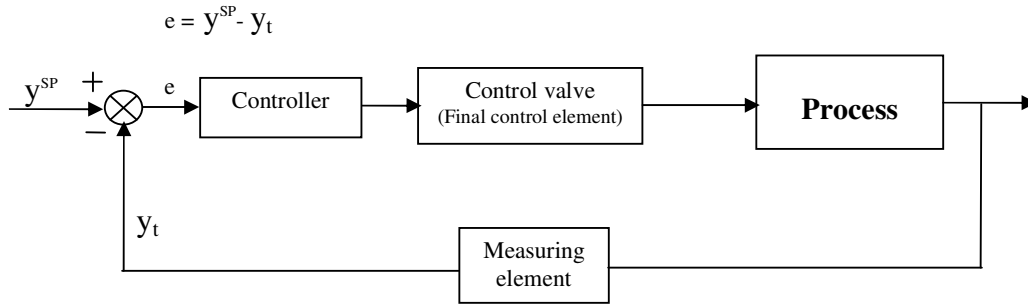


Figure 2.2. Block diagram of a feed back control

2.2. PID Control

2.2.1. The Control Algorithm

Almost all controllers in the process industry use the Proportional Integral Derivative (PID) control algorithm. Even as instrumentation and computation technologies have witnessed a transition from the analog era to the digital revolution, the good old PID control algorithm remains the most widely used algorithm, not withstanding the onslaught of advanced model predictive control algorithms. The positional form of the algorithm states that

$$u_t = K_c \left(e_t + \frac{1}{\tau_I} \int_0^t e(t) dt + \tau_D \frac{de_t}{dt} \right) + bias$$

where u is the controller output (input to the process), e is the error in the controlled variable, and K_c , τ_I and τ_D are controller tuning parameters. The tuning parameters are referred to respectively as the controller gain, reset (or integral time) and derivative time. The bias term in the expression is provided to make the LHS equal the RHS at time $t = 0$ for proper initialisation. The three terms in the algorithm correspond to Proportional, Integral and Derivative action, hence the acronym PID.

The velocity form of the algorithm is more amenable to understanding the effect of each of the P, I and D actions. Differentiating the above equation, we get

$$\frac{du_t}{dt} = K_c \left(\frac{de_t}{dt} + \frac{1}{\tau_i} e_t + \tau_D \frac{d^2 e_t}{dt^2} \right)$$

The controller gain or proportional gain, K_C , determines the fastness of response with larger values resulting in a fast response to deviations from set-point. This can be verified from the first term in the velocity form equation where the rate of change of the control input is directly proportional to the rate of change in the error, K_C being the proportionality constant. The larger the K_C , the larger the change in the control input, the faster the return to set-point.

The integral action is provided to ensure zero offset in the controlled variable. If the controlled variable deviates from its set-point, the controller acts to settle the system at a new steady state. At this new steady state all time derivatives are zero (by definition) implying the LHS in the equation above is zero. The RHS also therefore must be zero which requires that the error term, e_t , must be zero at the final steady state ($t \rightarrow \infty$). The error term in the velocity form above is due to the integral mode so that integral action moves the control input until the error in the controlled variable is driven to zero i.e. ensures a zero offset. P and D action do not guarantee zero offset as at the final steady state, the LHS and RHS terms corresponding to P and D action are zero. For a P or PD controller with no integral action, the velocity form of the algorithm imposes no restriction on the output error at the final steady state. A non-zero offset thus can and does result sans integral action.

The derivative action causes the controller to “think ahead” and is usually introduced to suppress oscillations from the “seeking behaviour” caused by integral action. In effect, the derivative action puts brakes on the control action as the controlled variable approaches the set-point thus avoiding large oscillations around the set-point. Most controllers in the industry are P or PI controllers and the D action is set to zero. This is because the D action amplifies noise so that the controller input signal must be pre-filtered appropriately to reap the benefit of D action. It is easier to simply turn the D action off and properly tune the controller gain and reset time for the desired control performance.

2.2.2. Controller Tuning

Empirical rules have been developed for tuning PID controllers. These tuning rules are based on the idea of ultimate gain and ultimate period. Figure 2.3 plots the closed loop response for a unit step change in the set-point of a first order plus dead time process for a P only controller as the controller gain is increased. Notice that as the controller gain is increased, the steady state offset reduces. Also, the response becomes faster. For larger gains the closed loop response is oscillatory. As the gain is increased further, sustained oscillations result. Any further increase in the controller gain results in an unstable system with the oscillations increasing in magnitude with time. The controller gain for which the closed loop response exhibits sustained oscillations corresponds to the transition from a stable to an unstable closed loop response. This controller gain at which the closed loop system borders on instability is referred to as the ultimate gain, K_U . The period of the sustained oscillations is known as the ultimate period, P_U . The empirical tuning rules recommend the controller gain to be a fraction of the ultimate gain and the reset time and derivative time as fractions (multiples) of P_U . Two popular tuning rules are the Zeigler-Nichols and Tyreus-Luyben tuning rules are tabulated in Table 2.1. For a given ultimate gain and ultimate period, the controller gain is the least for a PI controller. This is due to the “seeking behaviour” caused by integral action for zero offset. The closed loop system thus goes unstable for a lower controller gain implying that it should be lower. The controller gain is the maximum for a

PID controller due to the stabilizing effect of D action. As discussed before, D action is however used rarely in practice due to noise amplification. The PI algorithm is most commonly used in the industry. The tuning rules show that Zeigler-Nichols tuning is more aggressive than the Tyreus-Luyben tuning. Application of the ZN tuning rule can cause process upsets such as a distillation column flooding due to a sudden large increase in the vapour boil-up caused by a controller. The more conservative TL tuning rule is preferred in the process industry for a smooth and bumpless handling of transients avoiding large and sudden changes in the control input.

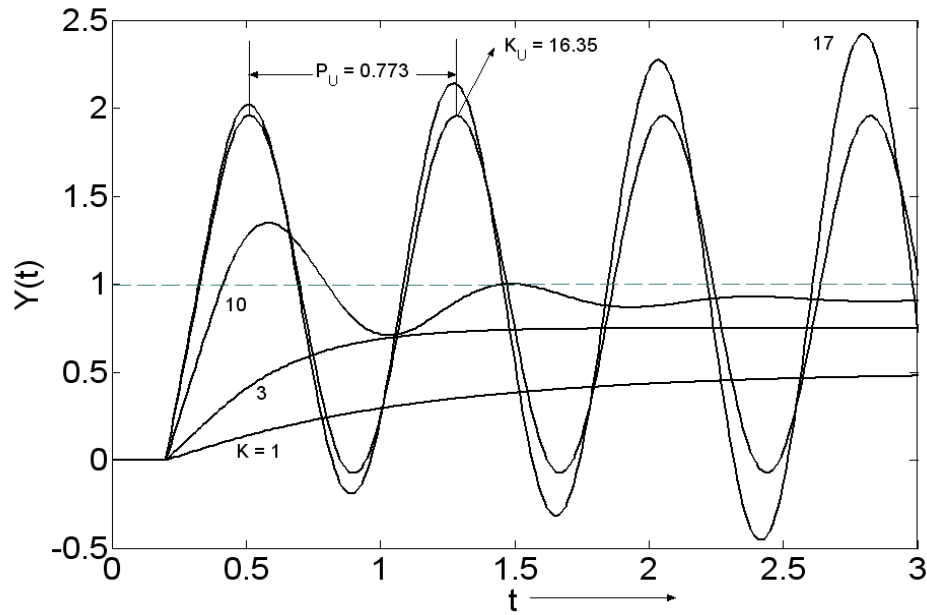


Figure 2.3. Closed loop response of a first order plus dead time process using P controller with different controller gains (K).

Table 2.1

	P	PI	PID
Ziegler-Nichols			
K_C	$K_U/2$	$K_U/2.2$	$K_U/1.7$
τ_i	--	$P_U/1.2$	$P_U/2$
τ_D	--	--	$P_U/8$
Tyreus -Luyben			
K_C	--	$K_U/3.2$	$K_U/2.2$
τ_i	--	$2.2P_U$	$2.2P_U$
τ_D	--	--	$P_U/6.3$

It is appropriate to highlight that a controller is required to handle two types of changes namely, a change in the output set-point and a change in the measured / unmeasured disturbance into the process. The closed loop response for these is respectively referred to as the servo and the regulator response. A disturbance into a process is also sometimes referred to as a load change. Control systems in the process industry are typically designed for effective load rejection. In contrast, set-point tracking is the primary objective in the design of control systems for aerospace systems such as aeroplanes, rockets and missiles.

Figure 2.4 plots the regulator response for a unit step in the load variable with a P, PI and PID controller tuned using the ZN and TL tuning rules for the first order plus dead time process considered earlier. Notice that P only control results in an offset at the final steady state. This offset is larger for TL tuning due to the lower controller gain. The PI and PID regulator responses show no offset at the final steady state due to integral action. Also notice that the aggressive ZN tuning results in a quicker but oscillatory return to the set-point for the PI controller. These oscillations are suppressed by the D action in a PID controller. PID control leads to a faster and smoother return to set-point due to the stabilizing effect of D action. It is also highlighted that the TL tuning leads to a comparatively sluggish but non-oscillatory response due to the more conservative tuning parameters. Large and sudden changes in the control input are not desirable in the process industry to avoid hitting operating constraints (e.g. flooding / weeping in sieve tray towers) during transients. Also, the process equipment changes its dynamic characteristics due to equipment fouling, change in process through-put, wear and tear over time etc so that the need for retuning a control loop is mitigated using conservative controller settings. The TL settings thus represent a good compromise between control performance and robustness.

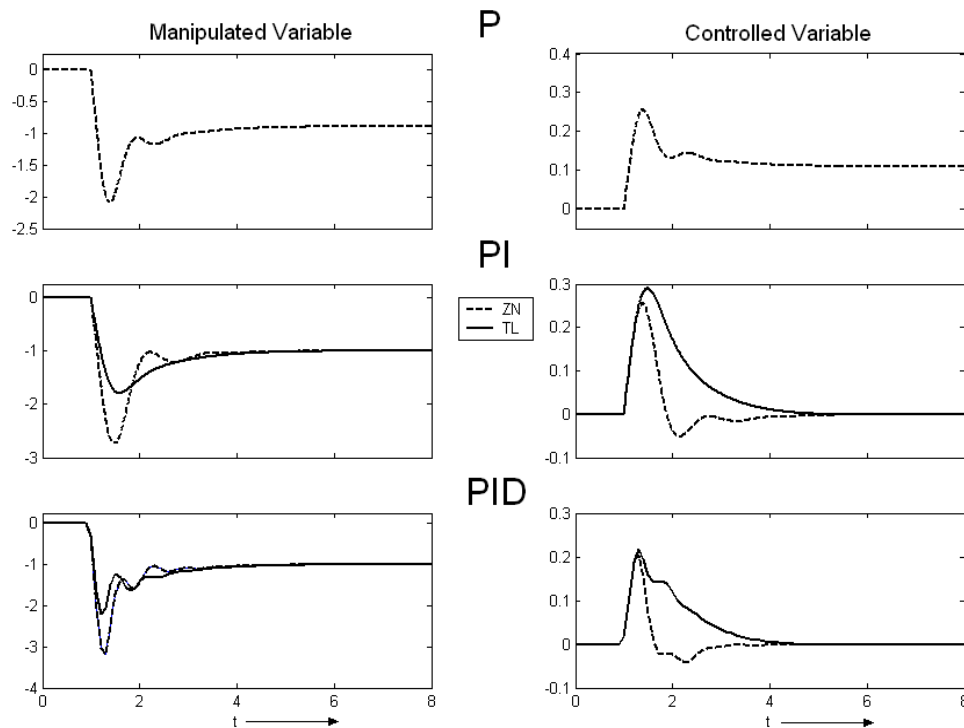


Figure 2.4. Dynamics of manipulated and controlled variables using P, PI and PID controllers with ZN and TL controller parameters for a unit step change in load. (Regulatory response).

2.3. Process Identification

Obtaining the ultimate gain and period of a control loop by increasing the controller gain causes the process to be driven towards instability. Considering the hazardous nature of chemicals processed in any chemical plant, such a methodology for tuning loops must be avoided. Alternative methods are needed that can be used for proper tuning. Two practical methodologies namely, the process reaction curve and auto-tune variation are presented next.

2.3.1. Process Reaction Curve Fitting

The process reaction curve is the open-loop response of the output variable to a step change in the manipulated variable which usually corresponds to a step change in a valve position. Most of the transient responses can be well represented by a first order plus dead time model. The model parameters are obtained as illustrated in Figure 2.5. The model parameters can be obtained by two methods as illustrated in Figure 2.5. In both methods, the ratio of the change in the controlled variable (output) from the initial to the final steady state to the magnitude of the step change gives the process gain K_P . For the controller, both input and output are 4-20 mA signals corresponding to the sensor and final control element span. In most commercial DCS systems, this range is represented as an equivalent 0-100% range. The units of K_P are then % change in controlled variable per % change in manipulated variable.

The two methods differ in the manner in which the dead time, θ , and the first order time constant, τ_P , are obtained. In Method 1, a tangent at the inflection point in the process reaction curve is drawn. Its intersection with the time axis gives the dead time θ . Its intersection with the horizontal line $Y = Y_{ss}$ where Y_{ss} is the final steady state equals $\theta + \tau_P$, from where τ_P is obtained. Equivalently, τ_P is obtained as

$$\tau_P = \frac{K_P}{S}$$

where S is the slope of the tangent drawn at the inflection point.

In Method 2, the time it takes for the response to reach 28.3% and 63.2% of the final steady state are noted. Denote these two times with $t_{28.3\%}$ and $t_{63.2\%}$ respectively. Noting that for a first order lag, 28.3% and 63.2% response completion occurs in $\tau_P/3$ and τ_P time units respectively, we have

$$\theta + \tau_P/3 = t_{28.3\%}$$

$$\theta + \tau_P = t_{63.2\%}$$

Subtracting the two equations to eliminate θ , we have

$$\tau_P = 1.5(t_{63.2\%} - t_{28.3\%})$$

and finally

$$\theta = 1.5 t_{28.3\%} - 0.5 t_{63.2\%}$$

The response of the fitted model using the two methods in shown in Figure 2.11. Method 2 is clearly simpler and fits the actual process reaction curve better.

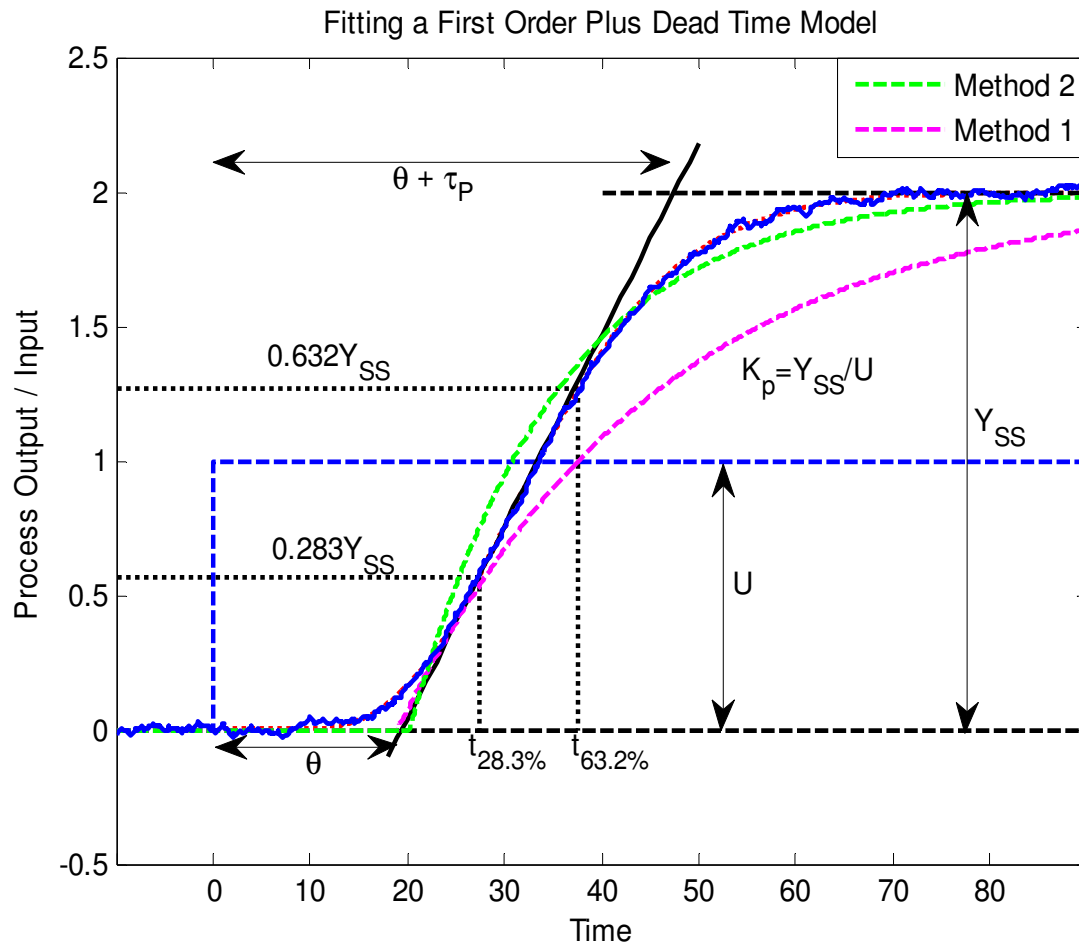


Figure 2.5. Fitting a first order plus dead time model to the process reaction curve

With the fitted model, K_U and P_U can be obtained either by simulation or complex variable analysis. The ZN or TL tunings can then be calculated as in Table 2.1.

2.3.2. Autotuning

Astrom and Hagglund (1984) proposed a powerful auto-tune variation (ATV) method for obtaining the ultimate gain and ultimate period. The method consists of putting a relay at the error signal that toggles the process input by $\pm h\%$ on detecting a zero crossing. This is schematically illustrated in Figure 2.6(a). The action of the relay causes the process input to toggle around the steady state by $\pm h\%$ for every zero crossing in the error signal corresponding to the output crossing the set-point. Sustained oscillations result and the system ends up in a limit cycle as depicted in Figure 2.6(b). The period of oscillations is the ultimate period P_U . The amplitude a of the output oscillations gives the ultimate gain K_U as

$$K_U = \frac{4h}{a\pi}$$

The ATV method has advantages over open loop step methods. The method automatically finds the critical frequency (or period) of the process. Also, large deviations away from the steady state are avoided as this is a closed loop test. Finally, the amplitude at the critical frequency (ultimate period) is obtained so that the identification procedure is more accurate than step / pulse tests.

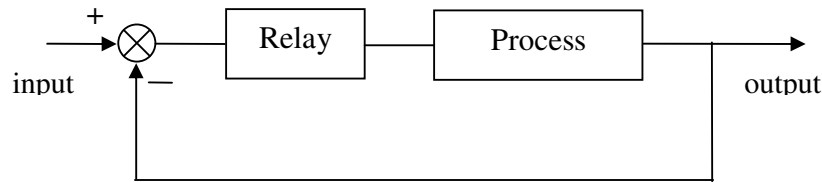


Figure 2.6(a). Block diagram of relay feedback approach

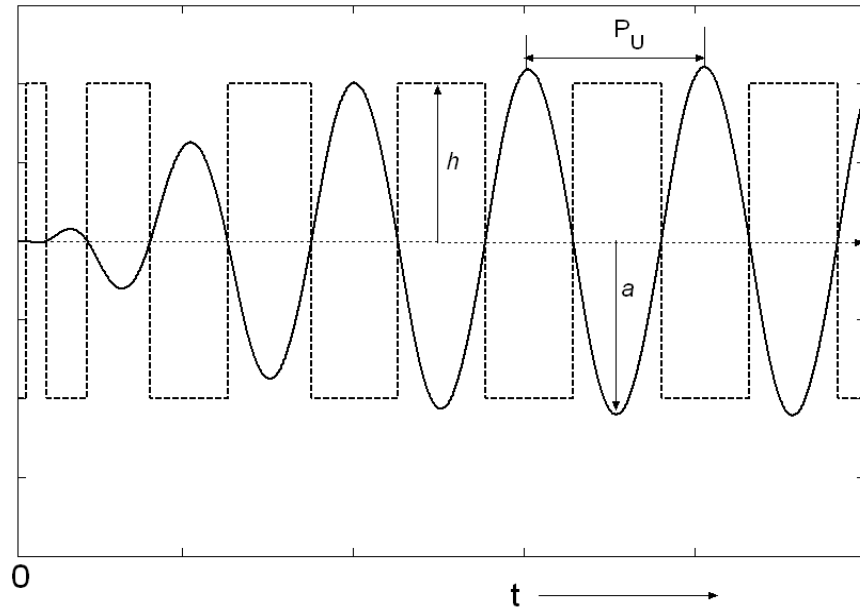


Figure 2.6(b). Relay feed back experiment a process with positive steady state gain

2.4. Controller Modes and Action

In all DCS systems, the controller can be in the indicator, manual, automatic or cascade mode. In the indicator mode, the controller is off and the process variable (controlled variable) is displayed. The control valve position cannot be adjusted by the operator. In the manual mode, the controller is off. The process variable reading is displayed and the operator can manually input the control valve position. Open loop step / pulse tests are performed in the manual mode with the operator giving a step change to the control valve position. In the automatic mode, the controller is on so that the control valve position is now set by the controller. The operator inputs the set-point for the controlled variable. In the cascade mode, the controller receives the set-point for the controlled variable from a master controller (and not the operator).

Depending on the sign of the process gain, the controller action must be specified to be “direct” or “reverse”. Usually a “direct” acting controller increases the controller output as the controlled variable increases above the set-point. A reverse acting controller, on the other hand, decreases the controller output as the controlled variable increases above set-point. For a negative process gain, the controller is “direct” acting while for a positive process gain the controller is “reverse” acting. The definition of “direct” or “reverse” action can vary from one vendor to the other and it is always best to confirm the definition. Another consideration in correctly specifying the controller action is whether the control valve fails open (air-to-close) or fails closed (air-to-open). Process safety considerations dictate if a control valve fails open or fails closed. For example the cooling water valve for removing heat from a reactor would fail open while the steam valve into a reboiler would fail close. If the controller action for a fail open valve is “direct”, the action would be “reverse” for a fail close valve in the same control loop.

In control parlance, the controller gain is many-a-times reported as proportional band. The proportional band is defined as

$$PB = \frac{100}{K_c} \%$$

The higher the proportional band, the lower the controller gain and vice versa.

2.5. Rules of Thumb for Controller Tuning

Almost all control loops in the process industry are one of the following

- Flow control loop
- Pressure control loop
- Level control loop
- Temperature control loop
- Product quality control loop

Some heuristics are discussed for tuning these loops that reflect common industrial practice. Depending on the application, exceptions to these heuristics are always possible.

2.5.1. Flow Loops

Flow is usually controlled using a PI controller. The signal from the flow sensor is noisy due to turbulent flow so that a large proportional band (about 150%) is used. A small reset time (10-20s) is used for good set-point tracking.

2.5.2. Level Loops

Most liquid levels provide surge capacity for filtering out flow disturbances. For example, the reflux drum in a distillation column allows for the reflux into the column to be held constant even as the vapour condensation rate and distillate rate vary. If the drum is not provided, the reflux into the column would fluctuate unnecessarily disturbing the column. The reflux drum thus acts as a surge capacity. In order to filter out flow disturbances, the level should be controlled loosely. The control objective is to maintain the liquid level within acceptable limits. Accordingly, a P controller is used for level control. A proportional band of 50% is commonly used so that the valve fully closes / opens for a 25% change in the level assuming the valve is initially 50% open. Note the use of PI controllers for level control of surge capacities is not recommended as a change in the inlet (outlet) flow would require that the outlet (inlet) flow increase above (decrease below) the inlet flow before becoming equal to the inlet flow in order to bring the level back to its set-point (zero offset). The flow disturbance thus gets magnified downstream (upstream). This magnification would only worsen for a series of interconnected units defeating the very purpose of providing surge capacity for attenuating flow disturbances. There are, of course, exceptions where tight level control is desired. For example, the level in a CSTR should be controlled tightly to maintain the residence time.

2.5.3. Pressure Loops

The dynamics of pressure in a can be very fast (flow like) or slow (level like) depending on the process system. For example, the pressure dynamics are extremely fast for a valve throttling the vapour outlet line from a tank. On the other hand, the dynamics are slow for the cooling water flow adjusting the pressure in a condenser due to the heat transfer and water flow lag. PI controllers are usually used for pressure loops with a small proportional band (10-20%) and integral time (0.2-2 mins) for tight pressure control. Tight pressure control is usually desired in most processing situations. For example, in distillation columns, the pressure must be controlled tightly as large pressure deviations would require compensation of the temperature controller set-points that ensure inferential product quality control. Similarly, most gas phase reactors are designed for near maximum pressure operation for maximum reaction rates so that large pressure deviations are not acceptable.

2.5.4. Temperature Loops

Temperature loops are moderately slow due to sensor lags and heat transfer lags. PI and PID controllers are often used. In most processing situations, tight temperature control is desired so that the proportional band is low (2-20%). The integral time is usually set to about the same value as the process time constant. In situations where derivative action is used for faster closed loop response, the derivative time constant is set to about one-fourth the process time constant or less depending on the transmitter signal noise.

2.5.5. Quality Loops

Composition control loops are usually applied for maintaining the product quality. In terms of relative importance, these loops are probably the most crucial for process profitability. If the product quality shows large variability, the process must be operated at a mean product quality that is significantly better than the quality specification to ensure the production of on-spec or better quality product all the time. This results in a quality giveaway

adversely affecting the process profitability. The quality giveaway can be reduced by ensuring tight product quality control. The concept of quality give-away is illustrated in Figure 2.7.

Typical composition measurements involve large dead-times or lags. For example the dead-time introduced by a gas-chromatograph can vary from a few minutes to an hour. Some compositions may be measured once a shift or once a day through laborious analytical measurements. Of all the measurements, analytical composition measurements are the most expensive and unreliable. The product specifications increasingly require the measurement of ppm / ppb levels of trace impurities so that a logarithmic scale is more appropriate in many situations. Product quality measurements are typically used to make small / incremental adjustments in the set-point of a loop. The frequency of the changes may vary from once a day to once every hour etc. Whenever PID controllers are applicable, a large proportional band is used (100-2000%). A large reset time (0.1 – 2 hrs) must be used due to the lag introduced by the composition measurement as well as the usually slow process dynamics.

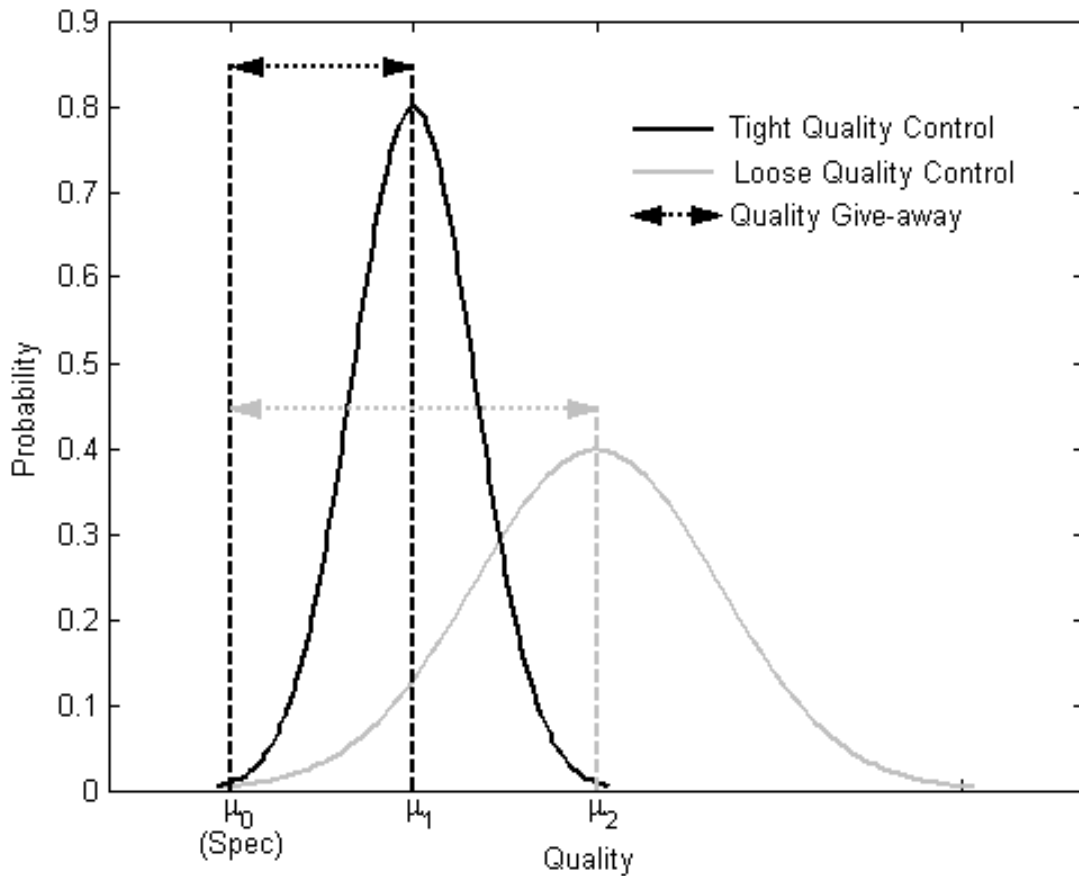


Figure 2.7. The concept of quality give-away

Chapter 3. Advanced Control Structures

The feedback control loop, discussed at length, forms the backbone of control systems applied in the process industry. Some typical feedback control loops are schematically illustrated in Figure 3.1. Over the years, enhancements to the basic feedback control structure that lead to significant improvement in control performance, have been developed. These advanced control structures include ratio control, cascade control, feed-forward control, override control and valve positioning control and are briefly described in the following.

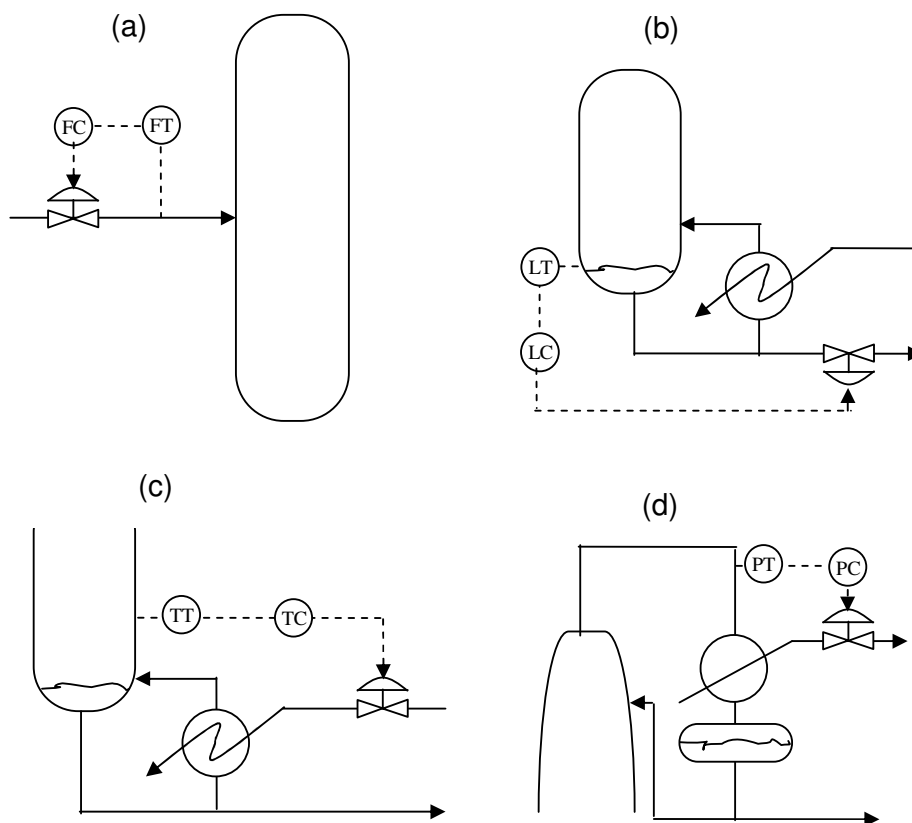


Figure. 3.1. Typical feed back control schemes commonly employed in distillation columns. (a) Feed flow control, (b) Level control in reboiler drum using bottoms flow, (c) Tray temperature control using reboiler duty and (d) Column pressure control using condenser duty.

3.1. Ratio Control

Ratio control, as the name suggests, is used for maintaining the ratio between two streams. The independent stream is referred to as the wild stream. The ratio controller adjusts the flow of the other stream to keep it in ratio to the wild stream. The implementation of ratio control is illustrated in Figure 3.2. The wild stream flow measurement is multiplied by the ratio set-point to obtain the flow set-point for the manipulated stream. The calculated flow set-point is input to the flow controller on the manipulated stream. Ratio control is implemented as a feed-forward strategy (to be discussed later) where two flows are increased

in tandem so that the change in the wild stream is compensated for before it affects the process output. For example, if the feed flow rate into a distillation column increases by 10%, the reboiler duty necessary to maintain the same separation should also increase by about 10%. It therefore makes sense to ratio the reboiler duty to the fresh feed rate so that the necessary change in the reboiler duty is implemented apriori. This leads to tighter product purity control with the change in the feed rate causing only small deviations in the product purity.

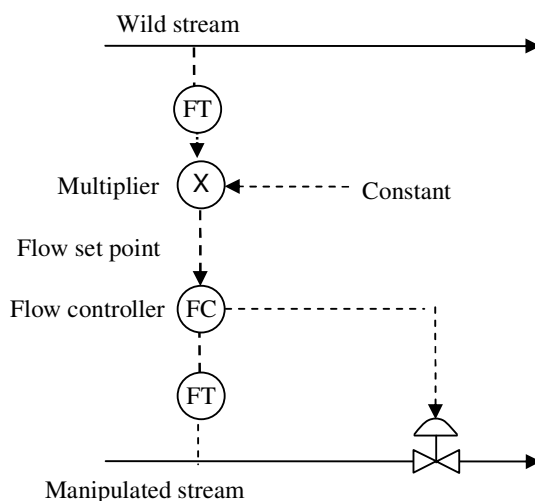


Figure. 3.2. Implementation of ratio control.

3.2. Cascade Control

Cascade control is arguably one of the most useful concepts in chemical process control. The cascade control scheme consists of two control loops, namely the master loop and the slave loop, with the master loop setting the set-point for the slave loop. The concept is best illustrated by an example. Consider a jacketed CSTR where cooling water is recirculated in the jacket to remove the exothermic reaction heat. The typical feedback reactor temperature control scheme and the cascade reactor temperature control scheme is shown in Figure 3.3. In the feedback arrangement, the reactor temperature controller directly adjusts the cooling water valve to maintain the reactor temperature at set-point. In the cascade arrangement, a slave loop is introduced that controls the jacket temperature by manipulating the cooling water valve. The master reactor temperature loop adjusts the jacket temperature set-point.

At first glance, the advantage of cascade arrangement over simple feedback control is not very obvious. To appreciate the same, consider an increase in the coolant temperature as an input disturbance. In the simple feedback scheme, the reactor temperature must rise before the controller opens the cooling water valve to bring the reactor temperature back to set-point. In the cascade control scheme, the jacket temperature controller senses the increase in the cooling water temperature and adjusts the cooling water valve to maintain the jacket temperature. The reactor temperature would thus show comparatively much smaller / negligible deviations from set-point. The slave controller acts to remove local disturbances into the process and prevents its effect on the primary controlled variable. Another subtle

advantage is that the slave controller compensates for the non-linearity in the slave loop so that the master controller ‘sees’ a more linear system. In the current example, the non-linear characteristics of the cooling water valve are compensated for by the slave controller. Since the slave loop has much faster dynamics than the master loop (else the cascade arrangement is infeasible), the master loop does not have to compensate for the valve non-linearity. It therefore sees a less non-linear system compared to simple feedback control resulting in improved control performance. The improvement is however at the expense of installing, tuning and maintaining an additional slave controller.

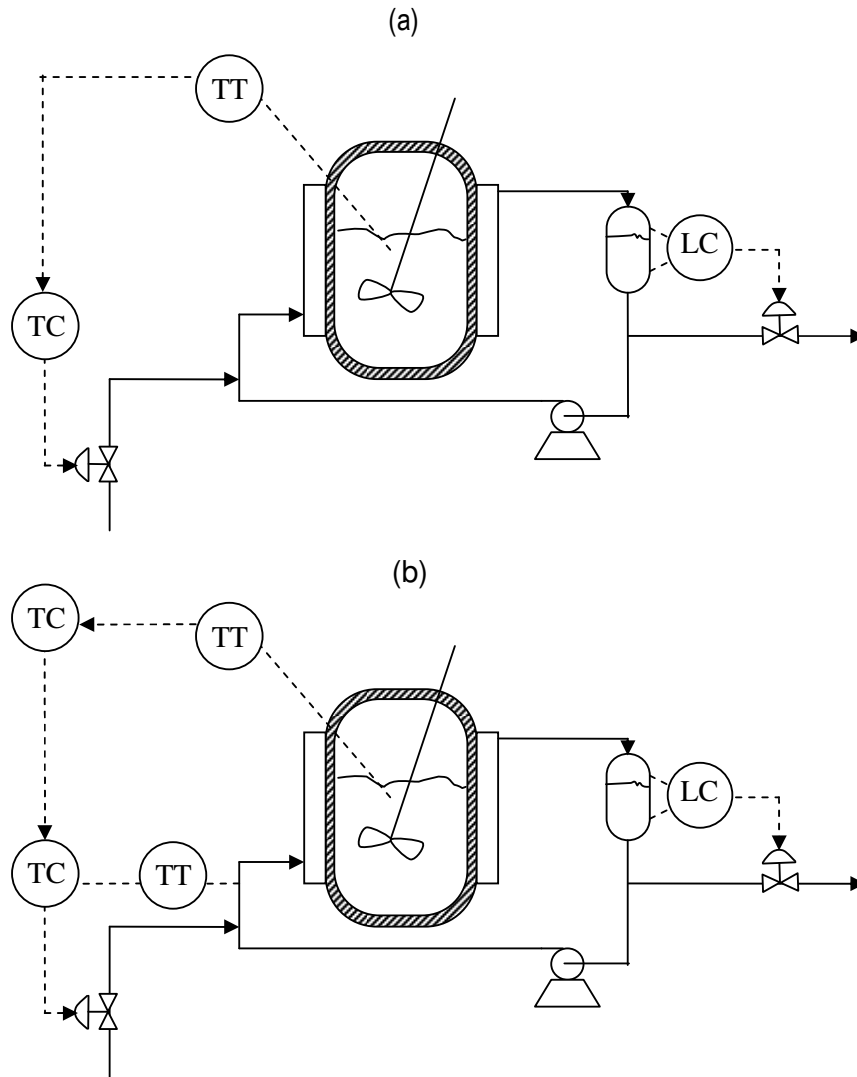


Figure 3.3. Temperature control of an exothermic CSTR. (a) the typical feedback reactor temperature control scheme and (b) the cascade reactor temperature control scheme.

To tune a cascade control structure, the slave loop is first tuned with the master loop in manual. P only controllers with a small proportional band (large controller gain) are commonly used in the slave loop for a fast response to a set-point change from the master controller. Integral action is usually not applied in the slave loop as an offset in the secondary

measurement is acceptable. The tuned slave loop is then put on automatic and the master loop is tuned. Note that for the cascade control system to be stable, the dynamics of the slave loop should be much faster than the master loop allowing the slave loop to keep-up with the set-point changes received from the master loop. A typical rule of thumb is that the time constant for the master loop should be more than thrice that of the slave loop.

Cascade control loops are quite common in the process industry. Some common configurations are shown in Figure 3.4. The interpretation of these configurations is left as an exercise to the reader.

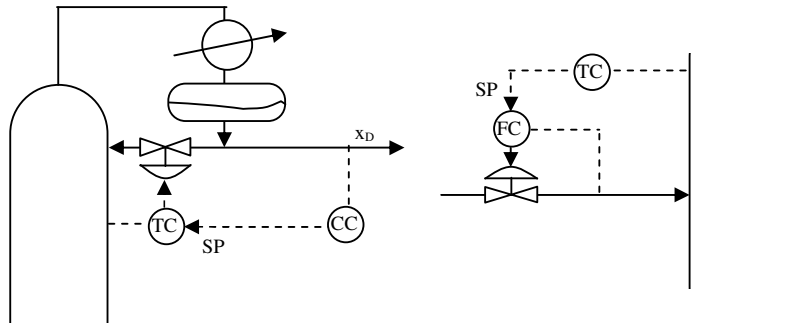


Figure 3.4. Some typical cascade arrangements

3.3. Feed-forward Control

The concept of feed-forward control has already been alluded to earlier. If a measured disturbance enters a process, the control input can be adjusted to compensate for effect of the disturbance on the output. Perfect compensation would cause the controlled output to show no deviations from its set-point even as a disturbance has entered the process. This apriori compensation to mitigate the transient effect of a measured disturbance on the controlled output is referred to as feed-forward control. A very simple example of feed-forward control is driving a car. Adjusting the hot and cold water knobs for the right temperature water from the shower is an example of feedback control. As discussed previously, ratio control compensates for disturbances in a feed-forward manner.

The design of a feed-forward compensator is illustrated using block diagrams in Figure 3.5. G_d represents the disturbance to output transfer function while G_p represents the control input to output transfer function. The control input u must be varied such that

$$G_p \cdot u + G_d \cdot d = 0$$

The control input is adjusted by the feed-forward compensator with the transfer function G_{ff} so that

$$u = G_{ff} \cdot d.$$

Substituting into the previous equation and solving for G_{ff} gives the feed-forward compensator design as

$$G_{ff} = -G_d/G_p$$

Assuming that G_d and G_p are first order plus dead time transfer function, the feed-forward compensator is then a lead-lag plus dead time transfer function. Modern DCS allow lead-lag plus dead time blocks to be configured into the control system.

For a better appreciation of the improvement in control performance using feed-forward compensation, consider a very simple example where

and
$$G_d = 1/(s+1)$$
$$G_p = 1/(5s+1)$$

Then
$$G_{ff} = -(5s+1)/(s+1)$$

Figure 3.6. plots the simulated transient output response for a unit step change in the measured disturbance with and without feed-forward compensation. Since there is no plant-model mismatch, perfect feed-forward compensation is observed with the output showing no deviations from set-point. In a real-life scenario, the presence of a plant-model mismatch may cause small transient deviations. The feed-back controller compensates for these small deviations resulting in an overall tighter closed loop response.

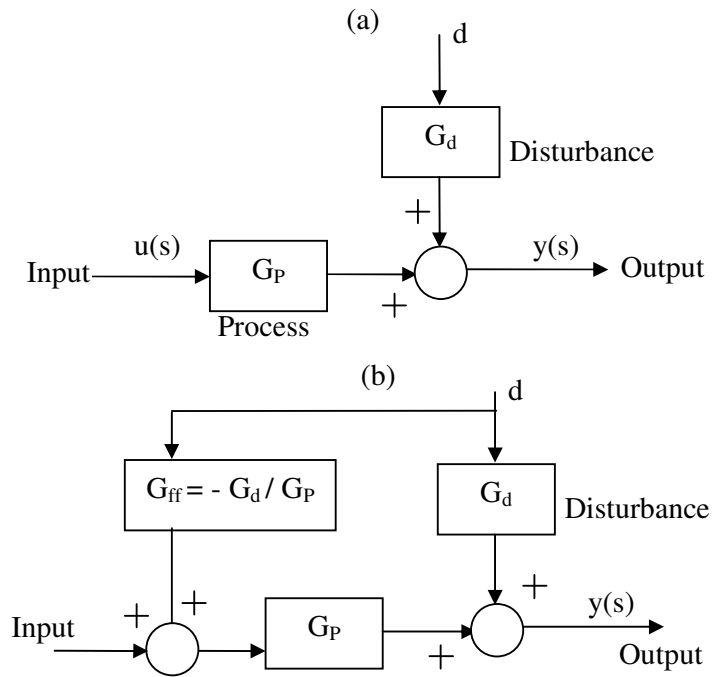


Figure 3.5. Design of feed forward compensator. (a) Process and (b) process with feed forward compensator.

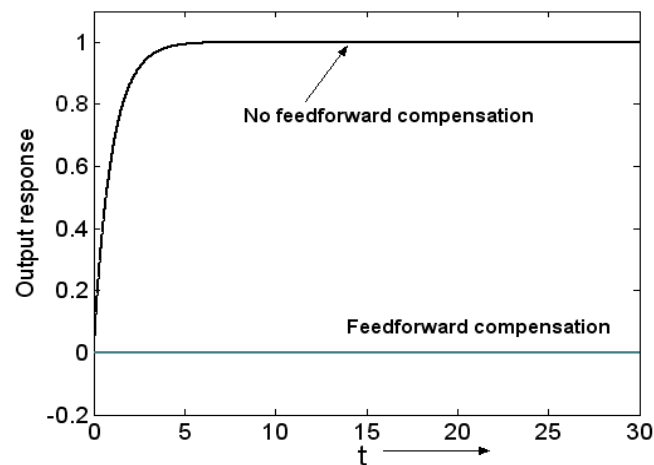


Figure.3.6. Deviation in the output with and without feed forward action

3.4. Override Control

Over-ride control is employed to ensure that an unsafe condition does not arise during process operation. As the name suggests, an over-ride controller over-rides the output of another controller as an unsafe condition develops and acts to move the process away from the unsafe condition. This is an example of multivariable control where the same manipulated variable can be adjusted at any time by one of many controlled variables. An example best illustrates the concept of over-ride or selective control. Consider the bottom section of a distillation column. The bottom sump level is controlled by the bottoms flow rate. During normal operation, the steam rate into the reboiler is manipulated to control a tray temperature. During severe transients, a situation may arise where the bottoms level is low and continues to fall even as the bottoms flow rate is zero. An unsafe situation can arise with the reboiler tubes getting exposed to vapour and fouling. Also, the bottoms pump may lose suction as the reboiler dries up. A sensible operator would put the temperature loop on manual and cut back on the steam rate to ensure the reboiler tubes remain submerged. In effect, the temperature controller output, the signal to the steam valve, gets over-ridden to maintain the liquid level. The over-ride controller automates this action as shown in Figure 3.7. The base level signal is input to a multiplier. A multiplier value of 5 is used so that if the level is above 20%, the multiplier output is above 100%. As the level decreases below 20%, the multiplier output decreases below 100%. If the level continues to decrease, the multiplier output would eventually decrease below the temperature controller output. The low select would then pass on the multiplier signal to the steam valve over-riding the temperature controller. The steam rate would thus decrease. Once the level begins to rise, the multiplier output would increase above the temperature controller output so that the low select would pass the manipulation of the steam valve back to the temperature controller. In addition to the level over-ride controller, the low select may also receive signals from a pressure over-ride controller or a

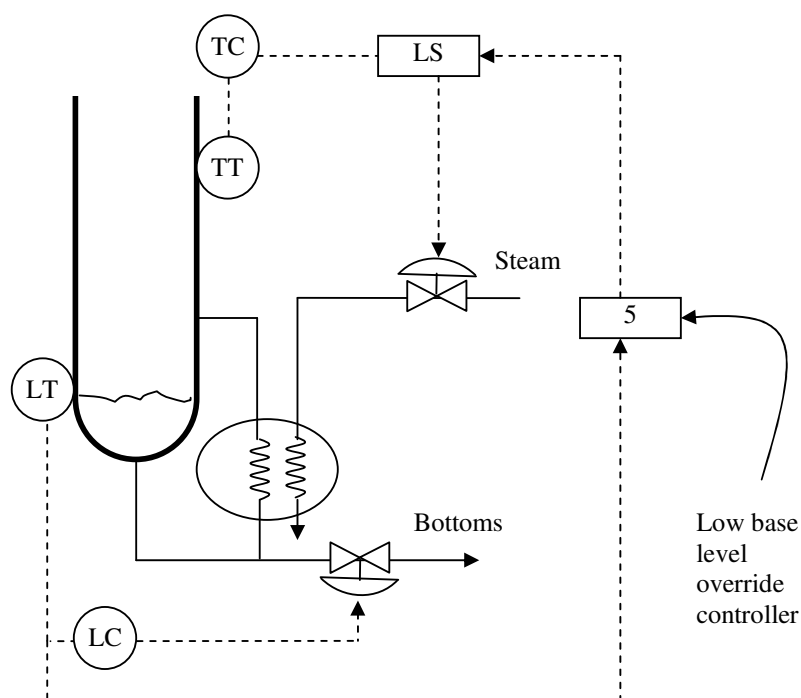


Fig. 3.7. Override control scheme

temperature over-ride controller to reduce the steam flow rate. Pressure over-ride would be needed if the column pressure goes too high. Similarly temperature over-ride may be necessary if the base temperature goes too high.

In temperature or pressure over-rides, a PI controller is needed unlike the P only controller for a level over-ride. This is because a pressure / temperature over-ride is needed only for a very small range of the total transmitter span. A very large proportional gain would then be necessary which can destabilize the closed loop system. Therefore a PI controller with lower gain and fast reset action is used to achieve the tightest control possible.

3.5. Valve Positioning (Optimizing) Control

Valve positioning control was originally proposed by Shinskey as an effective way of minimizing the energy consumption in distillation columns. The pressure in a distillation column is set by the condenser cooling duty. For a given separation, as the column pressure increases, more stages are needed as the x-y VLE plot moves towards the 45 degree line as shown in Figure 3.8. Translated to process operation, the same separation can be achieved at lower reboil as the column operating pressure is reduced. To minimize energy consumption, the column should be operated at lowest possible pressure corresponding to the maximum condenser duty. This can be accomplished by the valve positioning control scheme as illustrated Figure 3.9. The column pressure is typically controlled by adjusting the condenser cooling water valve. The VPC controller takes in the pressure controller output signal and adjusts the pressure set-point. If the valve is not nearly open, the controller reduces the column pressure set-point so that the pressure controller increases the cooling duty to reduce the column pressure. The VPC controller thus ensures that any underutilized cooling capacity is exploited to reduce the column operating pressure. The column pressure thus floats with the condenser duty being near maximum. The VPC controller is tuned to be slow with the fast pressure controller rejecting any pressure disturbances.

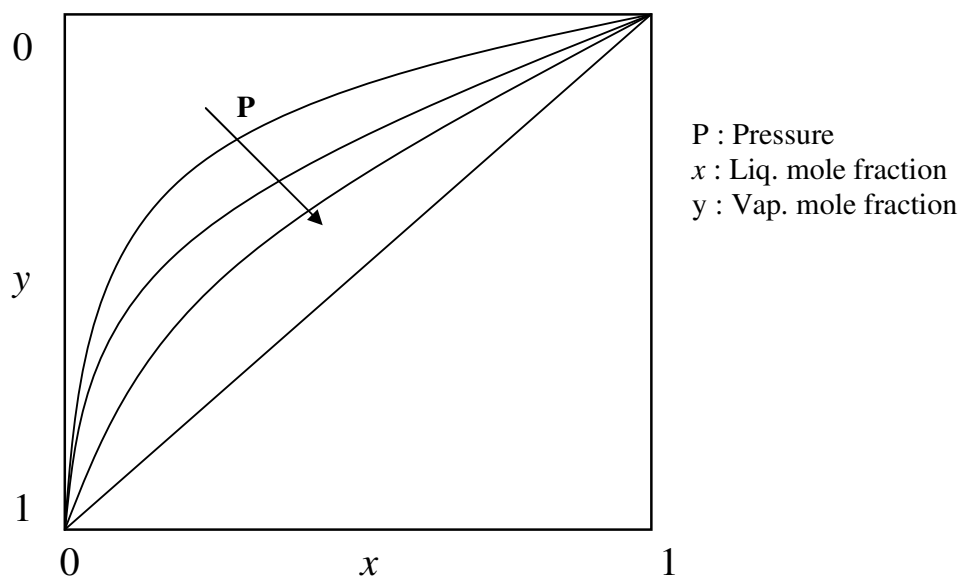


Figure 3.8. Typical x-y diagram with varying Pressures

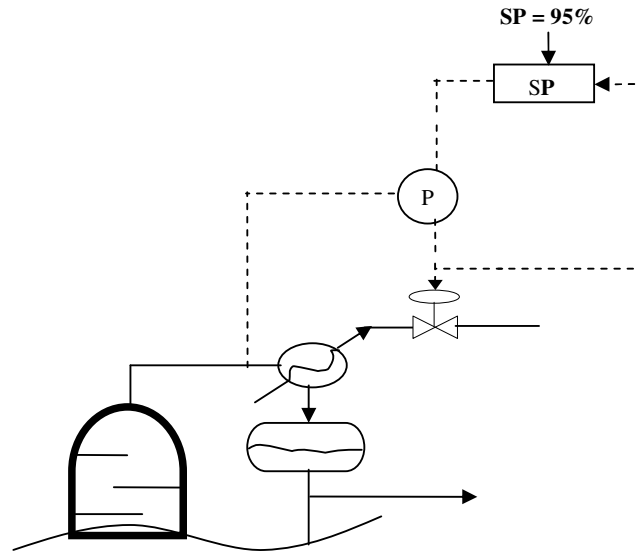


Figure 3.9. Valve positioning control

Another simple VPC application is shown in Figure 3.10. Let us say a high capacity variable speed pump is providing feed to N parallel trains of processes. We would like to minimize the pump electricity consumption while ensuring the desired flow setpoints for each of the parallel trains is achieved. The electricity consumption gets minimized by running the pump at as low an rpm as possible. This gets achieved by ensuring that the most open process feed valve is nearly fully open. The high select passes the position of the most open valve. A valve position below the nearly fully open VPC setpoint (say 80%) indicates unnecessary valve throttling. The VPC then reduces the pump rpm. In response, the flow controllers would open the valves to maintain the flow. The VPC reduces the pump rpm till the most open valve position reaches the VPC setpoint (80%) ensuring the pump operates at as low an rpm as possible while maintaining the desired flow to each of the parallel trains.

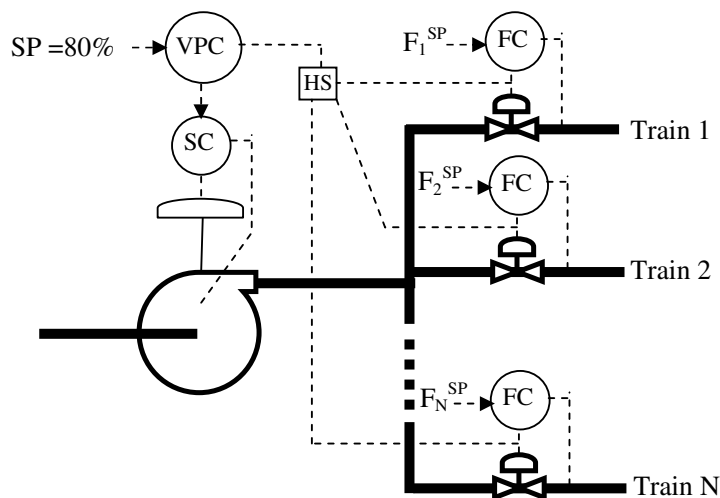


Figure. 3.10. VPC for minimizing variable speed pump electricity

Chapter 4. Multivariable Systems

Single input single output (SISO) systems have been treated till now. Most practical control system design problems are multivariable in nature with multiple inputs multiple outputs (MIMO). A 2 X 2 multivariable system is shown in Figure 4.1. There are two inputs, u_1 and u_2 and two outputs y_1 and y_2 . In the most general case, a step change in an input causes a transient response in both the outputs. The input output relationship may be compactly represented in matrix notation as

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$

and the corresponding block diagram is shown in Figure 4.1.

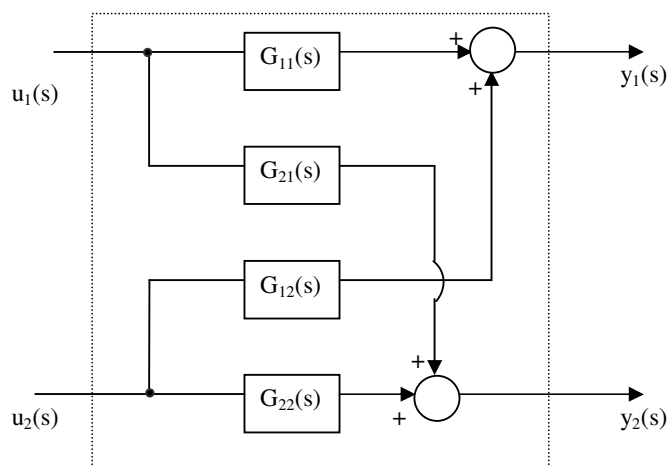


Figure 4.1. A block diagram of a 2 X 2 multi variable system

In general, G_{ij} denotes the transfer function between the j^{th} input and the i^{th} output. The non-diagonal terms with $i \neq j$ are the interaction terms. The simplest way of controlling a multivariable process is to control each of the outputs by manipulating an input using a PID controller. This is referred to as multivariable decentralized control and is illustrated in Figure 4.2. for the example 2x2 system. Controller 1 manipulates u_1 to maintain y_1 and controller 2 adjusts u_2 to maintain y_2 .

In the design of a multivariable decentralized control system, choice exists as to which manipulated variable is used to control an output. For the 2x2 example, there are a total of two control structures with y_1 being controlled by u_1 or u_2 . The number of such possibilities grows exponentially as the number of inputs / outputs increase. In the most general sense, the design of a plant-wide decentralized control system for a complex chemical process is a multivariable problem of high order. The high order problem is naturally broken down into smaller process unit specific controller design problems and controller design for managing plant-wide issues such as inventory balancing. A high order unit specific controller design problem can also be further broken down into a smaller subset of fast loops and slow loops based on the process dynamics. An example is the simplification of the 5x5 controller design problem for a simple distillation column into a 2x2 problem. In a distillation column, the pressure, reflux drum and bottom levels and two temperatures (or compositions) may be controlled. Since the tray temperature dynamics are significantly slower than the pressure /

level dynamics, SISO controllers are applied for the latter reducing the 5x5 problem into a 2x2 design problem for the two temperature controllers. Any complex high order control system design problem can thus be simplified into subsets of simple SISO, 2x2 or in the worst case 3x3 decentralized control system design problems. A systematic unit specific and plant-wide control system design methodology for complete chemical plants will be developed in the subsequent chapters.

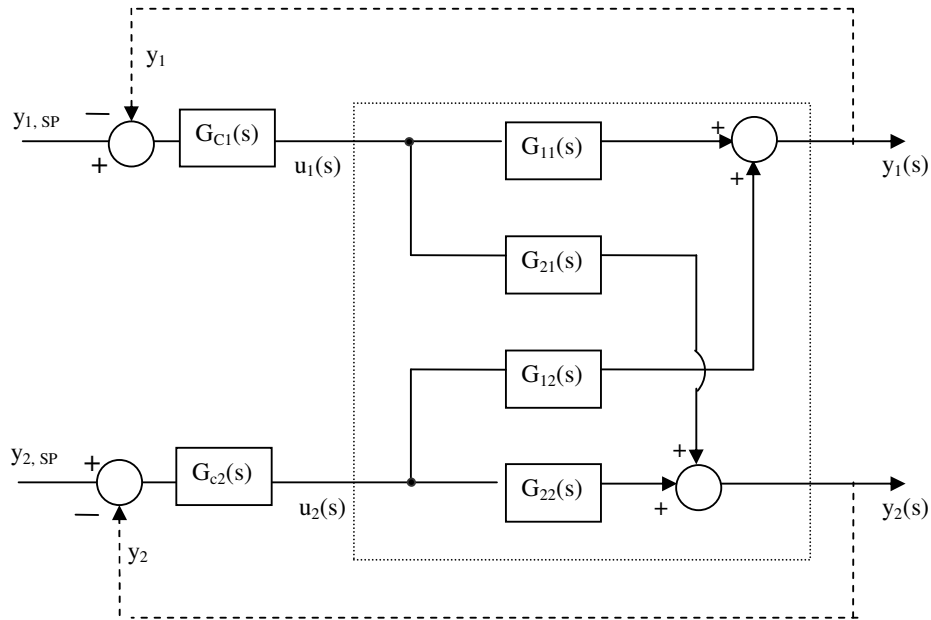


Figure 4.2. Block diagram of a multivariable decentralized control for a 2X2 system

4.1. Interaction Metrics

The selection of the input-output pairing in a decentralized control system is usually made based on engineering considerations which shall be covered in greater detail in subsequent chapters. The individual controllers in a decentralized control system may need to be detuned in order to maintain process stability. This is because the interaction between the loops during closed loop operation can lead to instability. The magnitude of interaction depends on the aggressiveness of the individual controller tunings employed. Detuning or less aggressive tuning mitigates the interaction to ensure closed loop stability. The Niederlinski Index and Relative Gain Array are two commonly used quantitative measures of interaction between control loops. Both are based on the open-loop steady state gain matrix \mathbf{K}_P , where

$$\mathbf{y} = \mathbf{K}_P \mathbf{u}$$

4.1.1. Niederlinski Index

The Niederlinski Index for a control structure where the i^{th} input is used to control the i^{th} output is then defined as

$$NI = \frac{|K_p|}{\prod_i K_{ii}}$$

The NI for any control structure can thus be obtained through appropriate relabeling of the outputs and inputs so that the i^{th} input controls the i^{th} output. If the Niederlinski Index is negative, the closed loop system is guaranteed to be integral closed loop unstable. If the NI is positive, the closed loop system may or may not be stable. In other words, the criteria $NI > 0$ is a necessary but not sufficient condition for closed loop stability. Input-output pairings with small positive or large positive (> 1) NI values indicate ill-conditioning problems and should be avoided. Control structures with NI close to 1 indicate favourable interaction. For example, an NI value of 1 for a 2X2 system indicates that either K_{12} or K_{21} or both are zero implying one-way or no steady state interaction between the loops. The primary use Niederlinski Index is for rejecting unworkable control structures.

4.1.2. Relative Gain Array

The relative gain is another popular metric that measures the interaction of a control loop with other loops as the ratio of the steady state process gain the controller sees with all other loops off to the process gain with all other loops on (all other outputs at their set-points). Mathematically, if the i^{th} output is controlled by the j^{th} input, its relative gain is defined as

$$\lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j} \right)_{u_k = \text{const}, k \neq j}}{\left(\frac{\partial y_i}{\partial u_j} \right)_{y_k = \text{const}, k \neq i}}$$

If the relative gain is negative, the i^{th} output should not be paired with the j^{th} input as the process gain sign would change depending on whether the other loops are on automatic or manual mode. Input-output pairings with relative gain close to 1 may be preferred as the process gain the controller sees is independent of the state of the other loops. The relative gain array is obtained as i and j are varied for respectively all outputs and inputs.

The relative gain array is an effective tool for input-output pairing when the primary control objective is set-point tracking. For set-point tracking, lower interaction between the loops increases the degree of independence of the different control loops so that each can be separately tuned for tight set-point tracking. Interaction is thus undesirable for set-point tracking. For load disturbance rejection, interaction is not necessarily undesirable and may actually favour disturbance rejection. This was demonstrated in an early article by Niederlinski (1971). Since the primary objective in chemical process control is load rejection, the application of RGA for control structure selection makes little sense. Candidate control structures should be proposed based on engineering considerations and unworkable structures further eliminated using the Niederlinski Index. The same arguments can be applied to recommend the use of dynamic decouplers only when the primary control objective is set-point tracking. Dynamic decoupling is not covered here as load rejection is the primary control objective in chemical process control systems.

4.2. Multivariable Decentralized Control

Consider the 2x2 multivariable open loop system in Figure 4.1. We would like to hold both the outputs at their respective setpoints. The simplest way to do it is to implement individual PI controllers for y_1 and y_2 . Without loss of generality, let us assume that y_1 is paired with u_1 and y_2 is paired with u_2 . The multivariable control system is shown in Figure 4.2. Notice that even as u_1 and u_2 affect both y_1 and y_2 through the interaction transfer functions G_{12} and G_{21} , the adjustment made to u_1 is based purely on e_1 and the adjustment made to u_2 is based purely on e_2 . In other words, the y_1 controller moves are based purely on y_1 and does not consider the effect of the control moves made by the y_2 controller. Similarly, the y_2 controller moves are based purely on y_2 and does not consider the effect of control moves made by the y_1 controller. Thus even as the actual system is multivariable, the individual controllers do not take the interaction into consideration. This is referred to as decentralized control.

For the decentralized control system, notice that the interaction terms introduce an additional feedback path as shown in blue in Figure 4.3. This additional feedback tends to further destabilize the closed multivariable control system. If each controller is tuned individually with the other controller on manual (other loop is open) and the Zeigler Nichols tunings applied, then when both the loops are closed, the system response is likely to be highly oscillatory and may even be unstable due to the additional feedback path. In the individual tuning of the controllers, since the other loop is open, this additional feedback path is inactive and therefore not accounted for in the determination of the tuning parameters. Clearly the individual ZN tuning parameters need to be detuned due to the additional feedback path to ensure the overall closed loop response is sufficiently away from instability.

4.2.1. Detuning Multivariable Decentralized Controllers

The obvious next question is that how does one tune a decentralized multivariable controller. Typically, in practical settings, tight control of one of the outputs is much more important than the other. A sequential tuning procedure can then be applied, where the more important output controller is tuned individually so that we get the tightest possible controller tuning. The less important output controller is then tuned with the other loop on automatic. Since the other loop is on, the additional feedback path is active and the necessary detuning due to the same gets accounted for in the tuning parameters of this less important loop. This sequential tuning procedure thus gives the tightest possible control of the more important output at the expense of a highly detuned controller for the less important output. The sequential procedure can be easily extended to more than 2 outputs when the prioritization of the controlled outputs is clear.

There are however situations where the need for tight control of each of the outputs is comparable. The detuning due to multivariable interaction then needs to be taken in all the loops. How does one systematically go about the detuning. For the 2x2 multivariable system, we have for the open loop system

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

or more simply

$$y = G_P u$$

where G_P is the open loop process transfer function matrix. For a decentralized controller, we have

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} G_{c1} & 0 \\ 0 & G_{c2} \end{bmatrix} \begin{bmatrix} y_1^{SP} - y_1 \\ y_2^{SP} - y_2 \end{bmatrix}$$

or in matrix notation

$$\mathbf{u} = \mathbf{G}_C (\mathbf{y}^{SP} - \mathbf{y})$$

where the controller matrix, \mathbf{G}_C , is diagonal for decentralized control. Combining the above two matrix equations, we get

$$\mathbf{y} = \mathbf{G}_P \mathbf{G}_C (\mathbf{y}^{SP} - \mathbf{y})$$

or

$$(\mathbf{I} + \mathbf{G}_P \mathbf{G}_C) \mathbf{y} = \mathbf{G}_P \mathbf{G}_C \mathbf{y}^{SP}$$

or

$$\mathbf{y} = (\mathbf{I} + \mathbf{G}_P \mathbf{G}_C)^{-1} \mathbf{G}_P \mathbf{G}_C \mathbf{y}^{SP}$$

This is the multivariable closed loop servo response equation and its analogy with SISO systems is self evident. Each element of the $(\mathbf{I} + \mathbf{G}_P \mathbf{G}_C)^{-1}$ matrix would have $\det(\mathbf{I} + \mathbf{G}_P \mathbf{G}_C)$ as its denominator. The closed loop multivariable characteristic equation is then

$$\det(\mathbf{I} + \mathbf{G}_P \mathbf{G}_C) = 0$$

Similar to SISO systems, if any of the roots of the multivariable characteristic equation is in the right half plane, the closed loop multivariable system is unstable.

To systematically detune the controllers, an empirical analogy with the Nyquist stability criterion for SISO systems is used. For a SISO system, the closed loop servo response equation is

$$y = [G_P G_C / (1 + G_P G_C)] y^{SP}$$

where G_P is the open loop transfer function and G_C is the controller transfer function. The Nyquist stability criterion then guarantees stability for the closed loops system if the polar plot of the open loop transfer function between y^{SP} and y , ie $G_P G_C$, does not encircle $(-1, 0)$. Gain margin and phase margin are criteria that are commonly used to quantify the distance from $(-1, 0)$ at a particular frequency. To ensure that the distance from $(-1, 0)$ is sufficient at all frequencies, the 2 dB closed loop maximum log modulus criterion is often used, where the closed loop log modulus is defined as

$$L_{CL}(\omega) = 20 \log |G_P G_C / (1 + G_P G_C)|_{s=j\omega}$$

L_{CL} is calculated by putting $s = j\omega$ in the transfer functions, G_P and G_C , and is therefore a function of ω . The SISO PI tuning parameters (K_C and τ_I) are chosen such that the maximum closed loop log modulus (with respect to ω) is 2dB. This ensures that the closed loop servo response is fast and not-too-oscillatory.

To develop a closed loop maximum log modulus criterion for multivariable systems, we note that the SISO closed loop characteristic equation is

$$1 + G_P G_C = 0$$

and the transfer function whose polar plot is used to see encirclements of $(-1,0)$ is then

$$-1 + (1 + G_P G_C)$$

ie

$$-1 + \text{closed loop characteristic equation}$$

For a multivariable system, we then define by analogy

$$\mathbf{W} = -1 + \det(\mathbf{I} + \mathbf{G}_P \mathbf{G}_C)$$

where \mathbf{W} is $-1 + \text{closed loop characteristic equation}$. The multivariable closed loop log modulus (L_{MVCL}) is then defined as

$$L_{MVCL} = 20 \log |W / (1 + W)|.$$

The tuning parameters for the individual controllers should be chosen such that

$$L_{MVCL}^{\text{MAX}} = 2 N_C$$

where N_C is the number of loops.

A simple algorithm for systematic detuning of the individual controller for the 2x2 decentralized control system is then:

1. Obtain individual ZN tuning parameters, $(K_{C1}^{ZN}, \tau_{I1}^{ZN})$ and $(K_{C2}^{ZN}, \tau_{I2}^{ZN})$, for each loop.
2. Detune the individual tuning parameters by a factor f ($f > 1$) to get the revised tuning parameters as $(K_{C1}^{ZN}/f, f \cdot \tau_{I1}^{ZN})$ and $(K_{C2}^{ZN}/f, f \cdot \tau_{I2}^{ZN})$
3. Adjust f such that $L_{MVCL}^{\text{MAX}} = 4$ dB.

The above procedure can be easily extended to an $N \times N$ ($N > 2$) decentralized control system.

As a parting thought, we re-emphasize that in chemical processes, the dominant time constants of different loops can differ by up to two orders of magnitudes. Thus for example, the residence time of a surge drum may be ~ 5 minutes while it may take 2-5 hrs for transients caused by a change in its setpoint to reach back after passing through the different downstream units, the material recycle and the upstream units. Similarly, on a distillation column, while the column pressure time constant with respect to condenser duty is ~ 1 min and the reflux drum / bottom sump level residence times are ~ 5 mins, the tray temperature response times to changes in reflux / boilup rates are much slower (~ 15 -20 mins). Thus even as the dual-ended distillation column control problem is 5×5 (2 levels, 1 pressure and 2 temperatures), the separation in time constants allows the level and pressure controllers to be tuned first followed by the two temperature controllers. The 5×5 problem thus reduces to a 2×2 problem due to the separation in time constants. In industrial practice, most high order multivariable problems reduce to 2×2 or at most 3×3 problems, which are mathematically tractable.

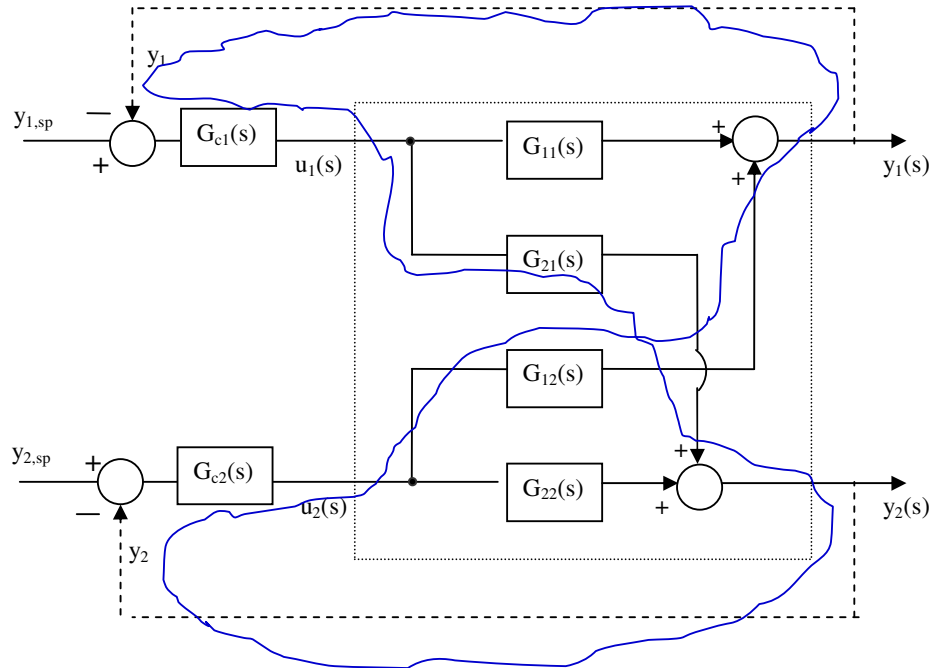


Figure 4.3. Additional feedback path due to multivariable interaction

Illustrative Example:

Consider a 2x2 openloop multivariable system

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{-18.9e^{-3s}}{21s+1} & \frac{12.8e^{-s}}{16.7s+1} \\ \frac{-19.4e^{-3s}}{14.4s+1} & \frac{6.6e^{-7s}}{10.9s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

- Calculate its RGA. Based on the RGA, what input-output pairing would you recommend.
- Calculate the Niederlinski Index for the recommended pairing. What can you say about closed loop integral stability of the recommended pairing.
- Calculate the Niederlinski Index for the other alternative pairing (the one that is not recommended). What can you say about the closed loop integral stability of this other pairing.
- For the recommended pairing, design a feedforward dynamic decoupler showing its complete block diagram and also the physically realizable feedforward compensator transfer functions.

Solution:

- (a) The steady state input-output relationship is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -18.9 & 12.8 \\ -19.4 & 6.6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

so that the steady state gain matrix is

$$K = \begin{bmatrix} -18.9 & 12.8 \\ -19.4 & 6.6 \end{bmatrix}$$

Inverting the matrix, we get

$$K^{-1} = \begin{bmatrix} 0.0534 & -0.1036 \\ 0.1570 & -0.1529 \end{bmatrix}$$

The RGA is then obtained as

$$RGA = K \cdot (K^{-1})^T$$

where the ‘.’ operator denotes element-by-element multiplication. Performing the necessary operations, we get

$$RGA = \begin{bmatrix} -1.0094 & 2.0094 \\ 2.0094 & -1.0094 \end{bmatrix}$$

Notice that the row/column sum of the RGA is 1. This is a property of the RGA (can you prove it?).

Rejecting the IO pairings corresponding to the negative RGA elements, the recommended pairing based on the RGA is y_1 - u_2 and y_2 - u_1 .

- (b) The steady state IO relation for the recommended pairing is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 12.8 & -18.9 \\ 6.6 & -19.4 \end{bmatrix} \begin{bmatrix} u_2 \\ u_1 \end{bmatrix}$$

The Niederlinski Index is then

$$NI = \frac{12.8 \times (-19.4) - 6.6 \times (-18.9)}{12.8 \times (-19.4)} = 0.4977$$

Since $NI > 0$ for the recommended pairing, the multivariable decentralized control system may be integrally stable.

- (c) The other possible pairing is y_1 - u_1 and y_2 - u_2 . For this pairing, the IO relation is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -18.9 & 12.8 \\ -19.4 & 6.6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The Niederlinski Index is then

$$NI = \frac{6.6 \times (-18.9) - 12.8 \times (-19.4)}{(-18.9) \times 6.6} = -0.99$$

Since the NI for this pairing is < 0 , the multivariable decentralized control system is guaranteed to be integrally unstable. This pairing should therefore not be implemented.

(d) If we look at the open loop 2x2 system with the recommended pairing (y_1 - u_2 and y_2 - u_1), a change in u_2 affects both y_1 (its controlled variable, CV) and y_2 (other CV). Similarly, a change in u_1 affects both y_2 (its CV) and y_1 (other CV). When both the control loops are on, the adjustment made by a loop ends up disturbing the other loop. A dynamic decoupler uses feedforward compensation ideas to make appropriate adjustments in the “other” process input so that a change in a process input only affects its CV and not the other CV. The dynamic decoupler block diagram for the recommended pairing is shown in Figure 4.4. We are looking for the feedforward compensator G_I^{ff} (G_{II}^{ff}) so that a change in u_2^* (u_1^*) only affects its CV, y_1 (y_2) with no effect on the other CV y_2 (y_1).

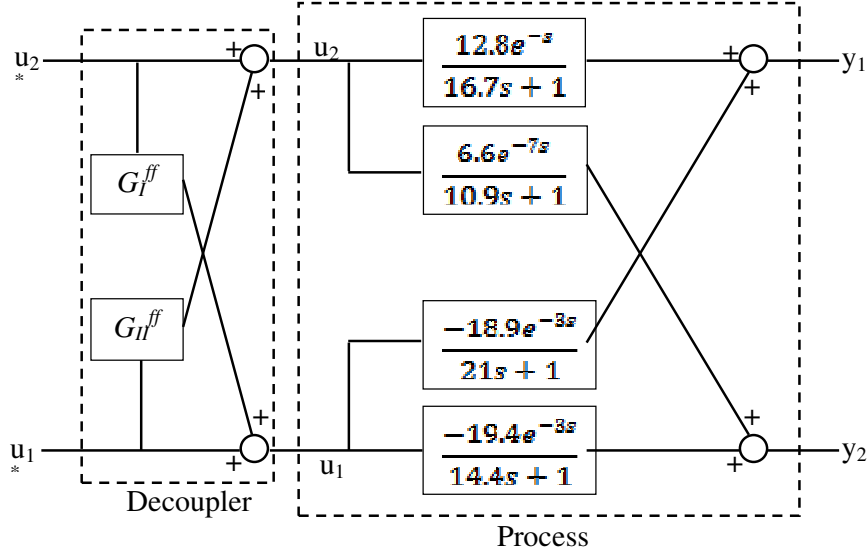


Figure 4.4. 2x2 process example with dynamic decoupler

From the block diagram, the ideal compensator G_I^{ff} would be such that

$$y_2 = G_{22}u_2^* + G_{21}G_I^{ff}u_2^* = 0$$

so that

$$G_I^{ff} = -G_{22}/G_{21}$$

Similarly, we have

$$G_{II}^{ff} = -G_{11}/G_{12}$$

Putting in the appropriate transfer functions, we get

$$G_I^{ff} = -\frac{\frac{6.6e^{-7s}}{10.9s + 1}}{\frac{-19.4e^{-3s}}{14.4s + 1}} = 0.3402 \frac{14.4s + 1}{10.9s + 1} e^{-4s}$$

$$G_{II}^{ff} = -\frac{\frac{-18.9e^{-3s}}{21s + 1}}{\frac{12.8e^{-s}}{16.7s + 1}} = 1.4766 \frac{16.7s + 1}{21s + 1} e^{-2s}$$

The feedforward compensators consist of a gain, a lead-lag and a deadtime. In some cases, it is possible that we get an exponential term of form e^{+Ds} ($D > 0$) implying a negative deadtime. This means that a change in the causal variable leads to a change in the effected variable in the past, which is impossible. The term e^{+Ds} is then physically unrealizable and dropped from the compensator.