s

The Lecture Contains

1. "Mechanics of Incremental Deformations" by M. A. Biot

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Buckling of a thick slab

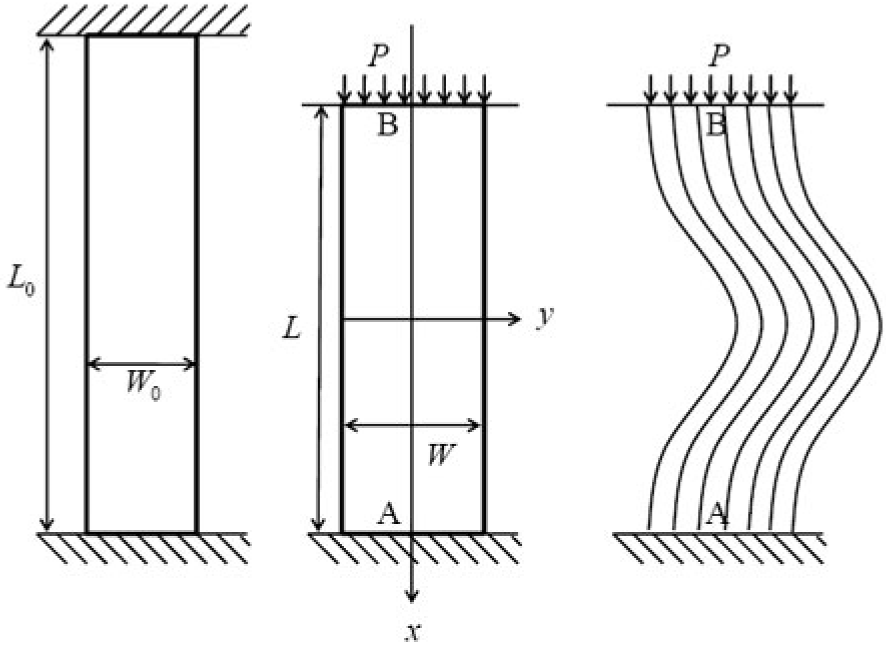

Figure 30.1

An incompressible, elastic slab of length L_0, thickness W_0 and of infinite width is placed between two rigid blocks on which the slab remains strongly bonded. The slab is compressed axially by applying stress P at its end, because of which, the length of the slab decreases to L and its thickness increases to W. The elastic slab behaves like a rubber like material and follows the stress-strain relations as in equation 13.55. Depending on the geometry we can consider two different situations. For example, we can consider that the slab is restrained from deforming along the direction perpendicular to the plane of the paper, so that, $\lambda_3 = 1$ and due to incompressibility, $\lambda_2 = \frac{1}{\lambda_1}$. The compressive stress can then be related as,

$$P = \mu_0\left(\frac{1}{\lambda_1^2} - \lambda_1^2\right) \tag{30.1}$$

And the incremental modulus is

$$\mu = \frac{\mu_0}{2}\left(\lambda_1^2 + \frac{1}{\lambda_1^2}\right) \tag{30.2}$$

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The extension ratio λ_1 ($\lambda_1 < 1$) is a measure of the compressive strain along x, then the length and thickness of the slab in the compressed state respectively are:

$$L = L_0\lambda_1 \, , \, W = \frac{W_0}{\lambda_1} \tag{30.3}$$

We can consider also a situation that the slab freely deform perpendicular to the xy plane, so that

$$S_{33} = 0 \tag{30.4}$$

And the initial compressive stress and the incremental modulus are related as

$$P = \mu_0\left(\frac{1}{\lambda_1} - \lambda_1^2\right) \text{ and } \mu = \frac{\mu_0}{2}\left(\lambda_1^2 + \frac{1}{\lambda_1}\right) \tag{30.5}$$

The length and width of the compressed slab are then written as

$$L = L_0\sqrt{\lambda_1} \, , \, W = \frac{W_0}{\sqrt{\lambda_1}} \tag{30.6}$$

Let us now say that the initial stressed condition of the elastic block is now perturbed by applying incremental deformation in plane strain that is parallel to the xy plane. The equations for incremental deformations are then same as that presented earlier, i.e. equations 29.14 to 29.18:

The solution of these equations, which will correspond to the buckling of the block, can be written as,

$$\begin{aligned} \phi &= \frac{1}{l^2}\left(C_1 \cosh ly + C_2 \cosh kly\right)\sin lx \\ s &= C_2 Pk \sinh kly \ \cos lx \end{aligned} \tag{30.7}$$

In which, C_1 and C_2 are constants. We have as before,

$$k = \sqrt{\frac{1-\zeta}{1+\zeta}} \quad \zeta = \frac{P}{2\mu} \tag{30.8}$$

The boundary conditions at the free surface of the slab is written as

$$\Delta f_x = \Delta f_y = 0 \text{ at } y = \pm W/2 \tag{30.9}$$

The expressions for Δf_x and Δf_y are written directly from the previous section, i.e. equation 29.22 Substitution of the expressions for ϕ and s from 30.7 into 30.9 yields,

$$\begin{aligned} &2C_1 \cosh \gamma + C_2\left(1+k^2\right)\cosh k\gamma = 0 \\ &C_1\left(1+k^2\right)\sinh \gamma + 2C_2 k \sinh k\gamma = 0 \end{aligned} \text{t} \tag{30.10}$$

Where, $\gamma = \frac{lW}{2} = \frac{\pi W}{L}$

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Finally we obtain the following characteristic equation,

$$4k\tanh k\gamma - \left(1+k^2\right)^2 \tanh \gamma = 0 \qquad (30.11)$$

We can consider the asymptotic situation of $\gamma \to \infty$, implying that thickness of the slab increases to a half space, at this situation, the characteristic equation reduces to

$$4k - \left(1+k^2\right)^2 = 0 \text{ or } (1+\zeta)^2 k - 1 = 0 \qquad (30.12)$$

Notice that equation 30.12 coincides with 29.22, implying that for very short slab the buckling deformation of the slab degenerates to surface instability.

The other extreme is the limit of very slender slab for which, $\gamma \to 0$, for which we can simplify the characteristic equation by using

$$\begin{aligned} \tanh k\gamma &= k\gamma - \frac{1}{3}k^3\gamma^3 \\ \tanh \gamma &= \gamma - \frac{1}{3}\gamma^3 \end{aligned} \qquad (13.13)$$

in which we have neglected higher order terms. Substitution of these expressions into 30.11 results in,

$$\zeta = \frac{2\gamma^2}{3+\gamma^2} \qquad (13.14)$$

Notice that if we simplify the above equation to the first order it yields

$$\zeta = \frac{2\gamma^2}{3} \text{ or } P = \frac{1}{3}\mu l^2 h^2 \qquad (30.15$$

It can be shown that this expression matches with that is derived for Euler buckling instability for thin plates. For example, from lecture 14, under small bending approximation, the transverse deflection v of a thin plate of thickness h, under axial load P is written as,

$$\frac{E}{1-\nu^2}\frac{h^3}{12}\frac{d^2v}{dx^2} + Ph\,v = 0 \qquad (30.16)$$

For an incompressible material, $\nu = 0.5$, the above equation simplifies to,

$$\frac{\mu h^3}{3}\frac{d^2v}{dx^2} + Ph\,v = 0 \qquad (30.17)$$

If the deflection is assumed to be sinusoidal, $v = V\cos lx$, the buckling load is deduced as,

(30.18)

$$P = \frac{1}{3}\mu d^2 h^2$$

This relation for buckling load matches with deduced earlier in equation 30.15

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