# Lecture 33 : Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin. (continued)

## The Lecture Contains



1. Flexural rigidity of microtubules ... fluctuation in shape", Gittes, F. G., et al, ***The Journal of Cell Biology***, volume 120, number 4, 1993, 923-934.

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Let's say that the filament is straight and is composed of $N$ equal segments of length $\Delta s_0$ . But due to experimental error, there is a deviation from the straight line; at the $k^{th}$ point, the deviation is $e_k$ and at $k+1^{th}$ point it is $e_{k+1}$ . Then the $k^{th}$ segment has the length:

$$\Delta s_k = \left[\Delta s_0^2 + \left(\varepsilon_{k+1} - \varepsilon_k\right)^2\right]^{1/2} \cong \Delta s_0 \qquad (33.1)$$

and angle

$$\theta_k = \tan^{-1}\left[\left(\varepsilon_{k+1} - \varepsilon_k\right)/\Delta s_0\right] \cong \left(\varepsilon_{k+1} - \varepsilon_k\right)/\Delta s_0 \qquad (33.2)$$

Where, we have neglected the terms of order $\varepsilon_k^2$ and higher. If we substitute these expressions in equation 32.9, then these errors lead to amplitude of Fourier modes for the error,

$$a_n^{noise} \cong \sqrt{\frac{2}{L}} \sum_{k=1}^{N} \left(\frac{\varepsilon_{k+1} - \varepsilon_k}{\Delta s_0}\right) \Delta s_0 \cos\left(\frac{n\pi}{N}(k - 1/2)\right) \qquad (33.3)$$

Where, $s_k^{mid} = \dfrac{(k-1/2)L}{N}$ is the mid point of the $k^{th}$ segment. The variance $\left\langle a_n^2 \right\rangle^{noise}$ is calculated by regrouping terms on the right hand side of equation 33.3, in terms of each distinct $\varepsilon_k$ and by considering the fact that $\varepsilon_k$ are independent, identical random variables and using,

$$\sum_{k=1}^{N-1} \sin^2\left(\frac{n\pi k}{N}\right) = \frac{N}{2}$$

$$\left\langle a_n^2 \right\rangle^{noise} \cong \frac{4}{L} \left\langle \varepsilon_k^2 \right\rangle \left[1 + (N-1)\sin^2\left(\frac{n\pi}{2N}\right)\right] \qquad (33.4)$$

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The measured variance is obtained as,

$$\mathrm{var}(a_n)^{measured} = \left(\frac{L}{n\pi}\right)^2 \frac{1}{L_p} + \frac{4}{L}\left\langle \varepsilon_k^2 \right\rangle \left[1 + (N-1)\sin^2\left(\frac{n\pi}{2N}\right)\right] \qquad (33.5)$$

The equation shows that the contribution due to the thermal fluctuation decreases with the increase in number of modes $n$ and that due to the

$$\frac{1}{L_p} = \frac{kT}{EI} = \left(\frac{n\pi}{L}\right)^2 \left(\mathrm{var}(a_n)^{measured} - \left\langle a_n^2 \right\rangle^{noise}\right) \qquad (33.6)$$

Measurement error also results in over estimation of the length of the filament. This overestimation too inserts slight bias in the estimation of the persistence length. If $L'$ is as measured using equation 32.10, then we have

$$L' = \sum_{k=1}^{N} \Delta s_k = \sum_{k=1}^{N} \Delta s_0 \left[1 + \frac{1}{2}\left(\frac{\varepsilon_{k+1} - \varepsilon_k}{\Delta s_0}\right)^2\right] \qquad (33.7)$$

Averaging this expression over the N+1 independent $\varepsilon_k$ and putting $L = N\Delta s_0$, and using lowest order in $\left\langle \varepsilon_k^2 \right\rangle$,

$$L / L' = 1 - \frac{\left\langle \varepsilon_k^2 \right\rangle}{\Delta s_0^2} \qquad (33.8)$$

Because we use $L'$ in equation 16, $L_p$ is overestimated by a factor $(L / L')^3$.

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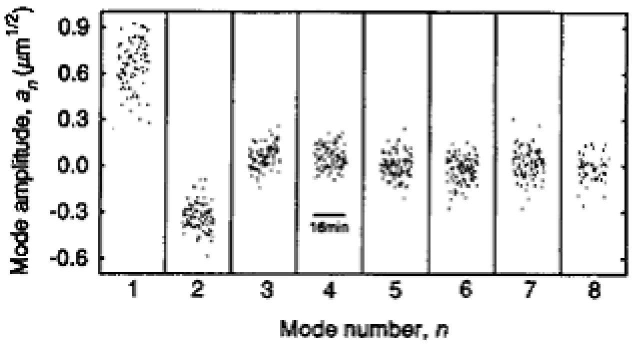
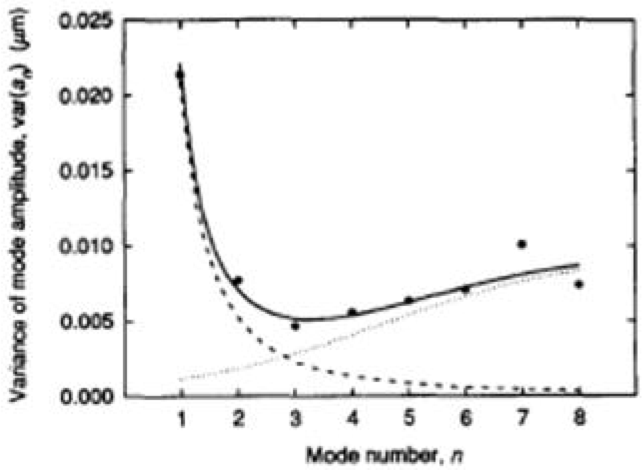
Module 4 : Nonlinear elasticity

Lecture 33 : Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin. (continued)

In experiment, a single microtubule is constrained to move in only two dimensions, so that the entire filament is in focus and also it is simple to analyze its shape.




Using this technique, the flexural rigidity of microtubules was calculated to be,˜2.19±0.14×10$^{-23}$ Nm$^2$. If the inner and outer diameters of the microtubule are 18 and 30 nm respectively, then its area moment of inertia is: 6.155 ×10$^{-32}$ m$^4$. Then making the assumption that the filament is isotropic, its Young's modulus can be calculated as, 0.356 ×10$^9$ N/m$^2$ or 0.356 GPa. The persistence length of microtubule is obtained as 5200±200 µm.

These numbers for actin filaments: Flexural rigidity: 7.29 ± 0.44×10$^{-26}$ Nm$^2$. Young's modulus: 2.6 GPa and the persistence length is obtained as 17.7 ± 1.1 µm

What its means is that the compliance of the cell is primarily due to the bending and sliding and filaments rather that due to stretching.

Young's modulus for several other proteinaceous filaments:

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|---|---|
| Silk (*Bombyx mori*): | 5-10 GPa |
| Keratin in wool: | 4 GPa |
| Collagen: | 0-2.5 GPa± |

Rubber like proteins: elastin, resilin and abductin: 0.6 MPa

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