Module 1 : Brief Introduction

Lecture 3 : Work and Energy

The Lecture Contains:

This lecture is adopted from the following book

1. "A Treatise on the Mathematical theory of Elasticity" by A.E.H. Love

Work and Energy:

When a body initially in equilibrium with its surroundings is subjected to external body and surface forces it deforms and eventually reaches a new equilibrium.

Here we want to estimate the work done by the external agency to bring about the change in the configuration of the body. From first law of thermodynamics the energy budget can be written as:

Incremental work done on the body (dW) **+** incremental heat supplied to it (dQ)**=** increase in its kinetic energy (**Δ**KE)+ increase in its intrinsic energy

$$\Rightarrow \; dW + dQ = \Delta K.E. + dU \qquad (3.1)$$

K.E.

For small displacements the kinetic energy per unit volume can be expressed by $\frac{1}{2}\rho\left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \upsilon}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right\}$, where ρ is the density in the unstrained state. The rate at which the kinetic energy increases can be expressed as,

$$\iiint \rho\left(\frac{\partial^2 u}{\partial t^2}\frac{\partial u}{\partial t}+\frac{\partial^2 \upsilon}{\partial t^2}\frac{\partial \upsilon}{\partial t}+\frac{\partial^2 w}{\partial t^2}\frac{\partial w}{\partial t}\right)dx\,dy\,dz$$

in which we can substitute the expressions from equation 2.12 to obtain,

$$\iiint\left[\left(\rho X+\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}\right)\frac{\partial u}{\partial t}+\ldots+\ldots\right]dx\,dy\,dz \qquad (3.2)$$

dW

The body is acted on by both external body forces (X, Y, Z) and the surface forces (X_ν, Y_ν, Z_ν). The rate at which work is done by the body forces can be represented as,

$$\iiint \rho\left(X\frac{\partial u}{\partial t}+Y\frac{\partial \upsilon}{\partial t}+Z\frac{\partial w}{\partial t}\right)dx\,dy\,dz \tag{3.3}$$

where the integration is taken through the volume of the unstrained state of the body.

Similarly, the work done by the surface forces is expressed as,

$$\iint\left(X_\nu\frac{\partial u}{\partial t}+Y_\nu\frac{\partial \upsilon}{\partial t}+Z_\nu\frac{\partial w}{\partial t}\right)dS \tag{3.4}$$

where the integration is taken over the surface of the body in the unstrained state.

Putting the expressions for from equation 2.7 we have,

$$\iint\begin{bmatrix}\left(X_x\cos(x,\nu)+X_y\cos(y,\nu)+X_z\cos(z,\nu)\right)\dfrac{\partial u}{\partial t}+\\ \left(Y_x\cos(x,\nu)+Y_y\cos(y,\nu)+Y_z\cos(z,\nu)\right)\dfrac{\partial \upsilon}{\partial t}\\ \left(Z_x\cos(x,\nu)+Z_y\cos(y,\nu)+Z_z\cos(z,\nu)\right)\dfrac{\partial w}{\partial t}\end{bmatrix}dS \tag{3.5}$$

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dW (contd...)

Now using Green's transformation for surface to volume integral,

$$\iiint\left[\begin{array}{l}\dfrac{\partial}{\partial x}\left(X_x\dfrac{\partial u}{\partial t}+Y_x\dfrac{\partial \upsilon}{\partial t}+Z_x\dfrac{\partial w}{\partial t}\right)+\dfrac{\partial}{\partial y}\left(X_y\dfrac{\partial u}{\partial t}+Y_y\dfrac{\partial \upsilon}{\partial t}+X_y\dfrac{\partial w}{\partial t}\right)+\\ \dfrac{\partial}{\partial z}\left(X_z\dfrac{\partial u}{\partial t}+Y_z\dfrac{\partial \upsilon}{\partial t}+Z_z\dfrac{\partial w}{\partial t}\right)\end{array}\right]dx\,dy\,dz$$

$$=\iiint\left[\begin{array}{l}\left(\dfrac{\partial X_x}{\partial x}+\dfrac{\partial X_y}{\partial y}+\dfrac{\partial X_z}{\partial z}\right)\dfrac{\partial u}{\partial t}+\left(\dfrac{\partial Y_x}{\partial x}+\dfrac{\partial Y_y}{\partial y}+\dfrac{\partial Y_z}{\partial x}\right)\dfrac{\partial \upsilon}{\partial t}+\\ \left(\dfrac{\partial Z_x}{\partial x}+\dfrac{\partial Z_y}{\partial y}+\dfrac{\partial Z_z}{\partial z}\right)\dfrac{\partial w}{\partial t}\end{array}\right]dx\,dy\,dz+$$

$$\iiint\left[\begin{array}{l}X_x\dfrac{\partial}{\partial t}\left(\dfrac{\partial u}{\partial x}\right)+Y_y\dfrac{\partial}{\partial t}\left(\dfrac{\partial v}{\partial y}\right)+Z_z\dfrac{\partial}{\partial t}\left(\dfrac{\partial w}{\partial z}\right)+X_y\dfrac{\partial}{\partial t}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right)+\\ Y_z\dfrac{\partial}{\partial t}\left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right)+Z_x\dfrac{\partial}{\partial t}\left(\dfrac{\partial w}{\partial x}+\dfrac{\partial u}{\partial z}\right)\end{array}\right]dx\,dy\,dz \tag{3.6}$$

$$=\iiint\left[\begin{array}{l}\left(\dfrac{\partial X_x}{\partial x}+\dfrac{\partial X_y}{\partial y}+\dfrac{\partial X_z}{\partial z}\right)\dfrac{\partial u}{\partial t}+\left(\dfrac{\partial Y_x}{\partial x}+\dfrac{\partial Y_y}{\partial y}+\dfrac{\partial Y_z}{\partial x}\right)\dfrac{\partial \upsilon}{\partial t}+\\ \left(\dfrac{\partial Z_x}{\partial x}+\dfrac{\partial Z_y}{\partial y}+\dfrac{\partial Z_z}{\partial z}\right)\dfrac{\partial w}{\partial t}\end{array}\right]dx\,dy\,dz+$$

$$\iiint\left[X_x\frac{\partial e_{xx}}{\partial t}+Y_y\frac{\partial e_{yy}}{\partial t}+Z_z\frac{\partial e_{zz}}{\partial t}+2X_y\frac{\partial e_{xy}}{\partial t}+2Y_z\frac{\partial e_{yz}}{\partial t}+2Z_x\frac{\partial e_{zx}}{\partial t}\right]dx\,dy\,dz$$

Hence the total work done by the external forces is the summation of r.h.s of equation 3.5 and 3.6.

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dQ:

Say the change in the configuration takes place adiabatically, which results in

$$dQ = 0 \tag{3.7}$$

dU:

Change in intrinsic energy of the body consists of two components: change associated with its temperature (dU_h) and that associated with the change in configuration (dU_c).

If we consider that the action takes place in the standard state, change in intrinsic energy associated with D_T is zero.

Putting together all these in equation 3.1 and considering the change in energy over a time *dt* results in the following expression for dU_c :

$$\begin{gathered}\iiint \delta U_c\,dx\,dy\,dz = \iiint \left[X_x \delta e_{xx} + Y_y \delta e_{yy} + Z_z \delta e_{zz} + X_y \delta e_{xy} + Y_z \delta e_{yz} + Z_x \delta e_{zx}\right] dx\,dy\,dz \\ \text{or, } \delta U_c = X_x \delta e_{xx} + Y_y \delta e_{yy} + Z_z \delta e_{zz} + X_y \delta e_{xy} + Y_z \delta e_{yz} + Z_x \delta e_{zx}\end{gathered} \tag{3.8}$$

Note that the right hand side of equation 3.8 is an exact differential so that there exists a function W which has the following properties,

$$X_x = \frac{\partial W}{\partial e_{xx}}, \quad \ldots, \quad Y_x = \frac{\partial W}{\partial e_{yx}}, \ldots \tag{3.9}$$

W is the potential energy, per unit volume stored up in the body by the strain and is called the strain energy function .

Hooke's law

Each of the six components of stress at any part of the body is a linear function of the six components of strain at the point.

Thus, following relations between stress and strains hold,

$$\begin{aligned} &X_x = c_{11}e_{xx} + c_{12}e_{yy} + c_{13}e_{zz} + c_{14}e_{yz} + c_{15}e_{zx} + c_{16}e_{xy} \\ &\ldots\ldots \\ &Y_z = c_{41}e_{xx} + c_{42}e_{yy} + c_{43}e_{zz} + c_{44}e_{yz} + c_{45}e_{zx} + c_{46}e_{xy} \\ &\ldots\ldots \end{aligned} \qquad (3.10)$$

$c_{11}, c_{12} \ldots c_{66}$ are 36 coefficients which are called elastic constants of the substance.

For isotropic systems, the formulae connecting stress components to the strain components should be independent of the directions. Following simplification results by considering that the x, y and z directions are equivalent:

$$\begin{aligned} X_x &= c_{11}e_{xx} + c_{12}\left(e_{yy} + e_{zz}\right) + c_{14}e_{yz} + c_{15}\left(e_{zx} + e_{xy}\right) \\ Y_y &= c_{11}e_{yy} + c_{12}\left(e_{xx} + e_{zz}\right) + c_{14}e_{zx} + c_{15}\left(e_{yz} + e_{xy}\right) \\ Z_z &= c_{11}e_{zz} + c_{12}\left(e_{xx} + e_{yy}\right) + c_{14}e_{xy} + c_{15}\left(e_{yz} + e_{zx}\right) \\ Y_z &= c_{41}e_{xx} + c_{42}\left(e_{yy} + e_{zz}\right) + c_{44}e_{yz} + c_{45}\left(e_{zx} + e_{xy}\right) \\ Z_x &= c_{41}e_{yy} + c_{42}\left(e_{zz} + e_{xx}\right) + c_{44}e_{xy} + c_{45}\left(e_{yz} + e_{xy}\right) \\ X_y &= c_{41}e_{zz} + c_{42}\left(e_{xx} + e_{yy}\right) + c_{44}e_{xy} + c_{45}\left(e_{zx} + e_{yz}\right) \end{aligned} \qquad (3.11)$$

Thus the stress strain relationships do not alter when any of the two axes are interchanged.

Furthermore they should not change when the direction of any axis is reversed. However when say the x axis is reversed the signs of e_{xy}, e_{zx}, X_y and Z_x also get reversed while the signs of the remaining components remain unchanged.

Hence, the constants $c_{14}, c_{15}, c_{41}, c_{42}, c_{45}$ must vanish, so that,

$$\begin{aligned} &X_x = c_{11}e_{xx} + c_{12}\left(e_{yy} + e_{zz}\right),\ Y_y = c_{11}e_{yy} + c_{12}\left(e_{xx} + e_{zz}\right),\ Z_z = c_{11}e_{zz} + c_{12}\left(e_{xx} + e_{yy}\right) \\ &Y_z = c_{44}e_{yz},\ Z_x = c_{44}e_{zx},\ X_y = c_{44}e_{xy} \end{aligned} \qquad (3.12)$$

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Hooke's law (contd...)

Furthermore, the stress strain relations must remain unaltered when the axes are rotated to a new position. Say x and y axes are rotated about the z axes through angle θ to a new location x', $y'$and z'. Then the relation $X'_{y'} = c_{44} e_{x'y'}$, from equations 2.18b and 1.29 results in

$$\begin{aligned} & l_1 l_2 X_x + m_1 m_2 Y_y + n_1 n_2 Z_z + (l_2 m_1 + m_2 l_1) X_y + (n_2 m_1 + n_1 m_2) Y_z + (n_2 l_1 + n_1 l_2) Z_x \\ & = c_{44} e_{x'y'} = c_{44} \begin{pmatrix} 2e_{xx} l_1 l_2 + 2e_{yy} m_1 m_2 + 2e_{zz} n_1 n_2 + \\ e_{yz}(m_1 n_2 + m_2 n_1) + e_{zx}(n_1 l_2 + n_2 l_1) + e_{xy}(l_1 m_2 + l_2 m_1) \end{pmatrix} \end{aligned}$$

which results in $c_{11} - c_{12} = 2c_{44}$, and the expression for X_x ...and Y_z ... in equation 26 simplifies to,

$$\begin{aligned} X_x &= c_{11} e_{xx} + (c_{11} - 2c_{44})(e_{yy} + e_{zz}) = c_{11}(e_{xx} + e_{yy} + e_{zz}) - 2c_{44}(e_{yy} + e_{zz}) \\ Y_z &= c_{44} e_{yz} \end{aligned} \tag{3.14}$$

Historically, c_{11} and c_{44} were substituted as

$$c_{11} = \lambda + 2\mu \quad \text{and} \quad c_{44} = 2\mu$$

which resulted in the following expressions for the stresses in terms of the strains,

$$\begin{aligned} & X_x = \lambda\Delta + 2\mu e_{xx}, \quad Y_y = \lambda\Delta + 2\mu e_{yy}, \quad Z_z = \lambda\Delta + 2\mu e_{zz} \\ & Y_z = 2\mu e_{yz}, \quad Z_x = 2\mu e_{zx}, \quad X_y = 2\mu e_{xy} \end{aligned} \tag{3.15}$$

The constants, λ and μ are called the Lame's constants.

$$\begin{aligned} & e_{xx} = \frac{\partial u}{\partial x}, \; e_{yy} = \frac{\partial v}{\partial y}, \; e_{zz} = \frac{\partial w}{\partial z}, \; e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \; e_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \; e_{zx} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ & \Delta = e_{xx} + e_{yy} + e_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \end{aligned}$$

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Strain Energy

From Equation 3.8 and 3.9,

$$
\begin{aligned}
dW &= \frac{\partial W}{\partial e_{xx}} de_{xx} + \frac{\partial W}{\partial e_{yy}} de_{yy} + \frac{\partial W}{\partial e_{zz}} de_{zz} + 2\frac{\partial W}{\partial e_{xy}} de_{xy} + 2\frac{\partial W}{\partial e_{yz}} de_{yz} + 2\frac{\partial W}{\partial e_{xz}} de_{xz} \\
&= \left(\lambda\Delta + 2\mu e_{xx}\right) de_{xx} + \left(\lambda\Delta + 2\mu e_{yy}\right) de_{yy} + \left(\lambda\Delta + 2\mu e_{zz}\right) de_{zz} + 2\mu e_{xy} de_{xy} + 2\mu e_{xz} de_{xz} + 2\mu e_{yz} de_{yz}
\end{aligned}
$$

Integrating, we obtain the strain energy per unit volume of the body and is written as the summation of the squared sum of the diagonal components and the sum of the square of all the components of the strain tensor:

$$
W = \frac{1}{2}\lambda e_{ii}^2 + \mu e_{ik}^2 = \frac{1}{2}\lambda\left(e_{xx} + e_{yy} + e_{zz}\right)^2 + \mu\left(e_{xx}^2 + e_{yy}^2 + e_{zz}^2 + 2e_{xy}^2 + 2e_{yz}^2 + 2e_{zx}^2\right) \qquad (3.16)
$$

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Resolution of any stress system into uniform tension and shearing stress:

Note that the quantity $X_x + Y_y + Z_z$ is invariant with respect to transformations from one set of rectangular axes to another, so that we can call the quantity $\frac{1}{3}\left(X_x + Y_y + Z_z\right)$ the “mean tension at a point”.

For a stress system with uniform normal pressure p, this quantity is $-3p$. Using this concept we can resolve any stress system into components characterized by the existence and non-existence of mean tension, e.g.

$$X_x = \frac{1}{3}\left(X_x + Y_y + Z_z\right) + \frac{2}{3}X_x - \frac{1}{3}\left(Y_y + Z_z\right) \qquad (3.17)$$

The stress system $\frac{2}{3}X_x - \frac{1}{3}\left(Y_y + Z_z\right)$ involves no mean-tension, furthermore, we can choose co-ordinates x', y', z' such that the normal tractions corresponding to these axes vanish.

In a word, this stress system involves no mean tension. Hence, we can say that any stress system at a point is equivalent to uniform tension in all directions and tangential traction across three planes which cut each other at right angles.

Consider a body of an arbitrary shape is subjected to a constant pressure *p* which remains constant in all directions, such that,

$$X_x = Y_y = Z_z = -p, \quad Y_z = Z_x = X_y = 0 \qquad (3.18)$$

and the state of strain in the body is such that,

$$e_{xx} = e_{yy} = e_{zz} = -p/(3\lambda + 2\mu), \quad e_{yz} = e_{zx} = e_{xy} = 0 \qquad (3.19)$$

The quantity $p/\left(\lambda + \frac{2}{3}\mu\right)$ is a measure of its cubicle compression and the quantity, $k = \lambda + \frac{2}{3}\mu$ is called the modulus of compression. It is obtained by dividing the mean tension $\frac{1}{3}\left(X_x + Y_y + Z_z\right)$ at a point by cubicle dilation at that point.

Resolution of any stress system into uniform tension and shearing stress (contd...)

A cylinder or prism of any form, subjected to tension T which is uniform over its plane ends and free from traction at the lateral surfaces. Then we can write,

$$X_x = T, \quad Y_y = Z_z = Y_z = Z_x = X_y = 0 \tag{3.20}$$

Algebra:

$$T = \lambda\Delta + 2\mu e_{xx}, \quad 0 = \lambda\Delta + 2\mu e_{yy}, \quad 0 = \lambda\Delta + 2\mu e_{zz}$$

$$T = 3\lambda\Delta + 2\mu\Delta \Rightarrow \Delta = \frac{T}{3\lambda + 2\mu}$$

Then from equation 3.15, state of strain can be written as,

$$e_{xx} = \frac{T(\lambda + \mu)}{\mu(3\lambda + 2\mu)}, e_{yy} = e_{zz} = -\frac{\lambda T}{2\mu(3\lambda + 2\mu)} \tag{3.21}$$

where, we put,

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \quad \sigma = \frac{\lambda}{2(\lambda + \mu)} \tag{3.22}$$

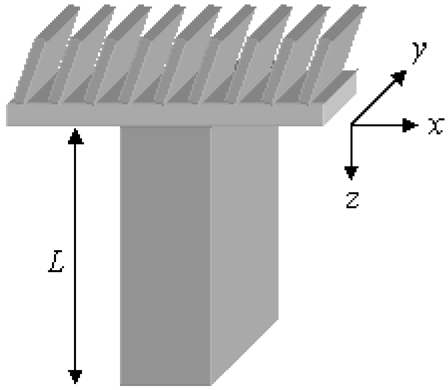
The quantity E obtained by dividing the simple longitudinal tension by the measure of the extension produced by it, is known as the Young's modulus . The number σ is the ratio of lateral contraction to longitudinal extension is called Poisson's ratio . μ is called shear modulus.

Furthermore, note the relations:

$$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}, \quad \mu = \frac{E}{2(1+\sigma)}, \quad k = \frac{E}{3(1-2\sigma)} \tag{3.23}$$

Material	Specific garvity, ρ	Young's modulus, E	Bulk modulus, k	Shear modulus, μ	Poisson's ratio, σ
		N/m 2	N/m 2	N/m 2	
Steel	7.849	2.1x10 12	1.84x10 12	8.2x10 11	0.31
Brass	8.471	1.085x10 12	1.05x10 12	3.66x10 11	0.327
Copper	8.843	1.234x10 12	1.684x10 12		0.378
Glass	2.942	6.03x10 11	4.15x10 11	2.4x10 11	0.258

Uniformly varying stress:



Let a prism of incompressible elastic material be subjected to gravitational pull, so that it is subjected to the following body forces,

$$X = 0, \quad Y = 0, \quad Z = g \tag{3.24}$$

Then all the stress components vanish, except Z_z which from equation 2.12 is obtained as

$$Z_z = -g\rho z \tag{3.25}$$

The strain are given by the equation,

$$e_{zz} = -\frac{g\rho z}{E}, \quad e_{xx} = e_{yy} = \frac{\sigma g\rho z}{E}, \quad e_{yz} = e_{zx} = e_{xy} = 0 \tag{3.26}$$

To displacements can be obtained in the following manner,

$$\frac{\partial w}{\partial z} = -\frac{g\rho z}{E} \text{ ,which gives } w = -\frac{g\rho z^2}{2E} + w_0(x, y) \tag{3.27}$$

From $e_{yz} = e_{zx} = 0$, we have,

$$\frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x} = -\frac{\partial w_0}{\partial x}, \quad \frac{\partial v}{\partial z} = -\frac{\partial w}{\partial y} = -\frac{\partial w_0}{\partial y} \tag{3.28}$$

which on integration results in,

$$u = -z\frac{\partial w_0}{\partial x} + u_0(x, y), \quad v = -z\frac{\partial w_0}{\partial y} + v_0(x, y)$$

Then,

$$\begin{aligned} e_{xx} &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w_0(x, y)}{\partial x^2} + \frac{\partial u_0(x, y)}{\partial x} = \frac{\sigma g\rho z}{E} \\ e_{yy} &= \frac{\partial v}{\partial y} = -z\frac{\partial^2 w_0(x, y)}{\partial y^2} + \frac{\partial v_0(x, y)}{\partial y} = \frac{\sigma g\rho z}{E} \end{aligned} \tag{3.30}$$

Hence,

$$\frac{\partial u_0(x,y)}{\partial x} = \frac{\partial v_0(x,y)}{\partial y} = 0 \quad \text{and} \quad \frac{\partial^2 w_0(x,y)}{\partial x^2} = -\frac{\alpha g \rho}{E}, \quad -\frac{\partial^2 w_0(x,y)}{\partial y^2} = \frac{\alpha g \rho}{E} \tag{3.31}$$

The equation $e_{xy} = 0$ results in

$$\frac{\partial u_0(x,y)}{\partial y} + \frac{\partial v_0(x,y)}{\partial x} = 0 \tag{3.32}$$

The above equations signify that $w_0(x,y)$ should satisfy the following equation

$$w_0(x,y) = -\frac{\alpha g \rho}{2E}\left(x^2 + y^2\right) + \alpha' x + \beta' y + \gamma \tag{3.33}$$

where α', β', γ are constants.

The equations containing $u_0(x,y)\, v_0(x,y)$ show that u_0 is a function of y, say $F_1(y)$ and v_0 is a function of x, say $F_2(x)$. Hence, $F_1(y)$ and $F_2(x)$ can be of the form,

$$F_1(y) = \gamma' y + \alpha \qquad \text{and} \qquad F_2(x) = -\gamma' x + \beta \tag{3.34}$$

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Uniformly varying stress

Finally, the complete solution of u, v and w will be of the form,

$$
\begin{aligned}
u &= \frac{\sigma g\rho}{E} zx - \alpha' z + \gamma' y + \alpha \\
v &= \frac{\sigma g\rho}{E} zy - \beta' z - \gamma' x + \beta \\
w &= \frac{g\rho}{2E}\left(-z^2 + \sigma x^2 + \sigma y^2\right) + \alpha' x + \beta' y + \gamma
\end{aligned}
\tag{3.35}
$$

Notice that the terms containing $\boxed{\alpha, \beta, \alpha', \beta', \gamma'}$ represent a displacement which would be possible in a rigid body.

If the cylinder is not displaced by rotation we can omit α', β', γ'.

If there is no lateral displacement, $\alpha = \beta = 0$.

Since, the plane through $z = 0$ does not get displaced, then $\gamma = 0$. The displacements are then given as:

$$
u = \frac{\sigma g\rho}{E} zx,\; v = \frac{\sigma g\rho}{E} zy,\; w = \frac{g\rho}{2E}\left(-z^2 + \sigma x^2 + \sigma y^2\right) \tag{3.36}
$$

Any cross-section of the cylinder distorts into a paraboloid of revolution about the vertical axis.

Plane strain

In a state of plane strain the displacements u, v are functions of x, y only and displacement w vanishes. All components of strain and stress are independent of z, so that $e_{xz} = e_{yz} = e_{zz} = 0$;

$$
\begin{aligned}
&X_x = (\lambda + 2\mu)e_{xx} + \lambda e_{yy}, \quad Y_y = (\lambda + 2\mu)e_{yy} + \lambda e_{xx}, \quad Z_z = \lambda\left(e_{xx} + e_{yy}\right) \\
&Y_z = 0, \quad Z_x = 0, \quad X_y = 2\mu e_{xy}
\end{aligned}
\tag{3.37}
$$

The situation of plane strain occurs in bodies of long cylindrical form.

Plane stress

The state of plane stress occurs in a planer body.

In general, the stress components Z_x, Z_y, Z_z are equal to zero but the displacements u, v and w are not independent of z.

$$
X_x = \lambda\Delta + 2\mu e_{xx}, \quad Y_y = \lambda\Delta + 2\mu e_{yy}, \quad \lambda\Delta + 2\mu e_{zz} = 0 = e_{yz} = e_{zx}, \quad X_y = 2\mu e_{xy} \tag{3.38}
$$