# Lecture 20 : Peeling a thin flexible plate off an elastic adhesive bonded to a flexible substrate (contd...)

## The Lecture Contains



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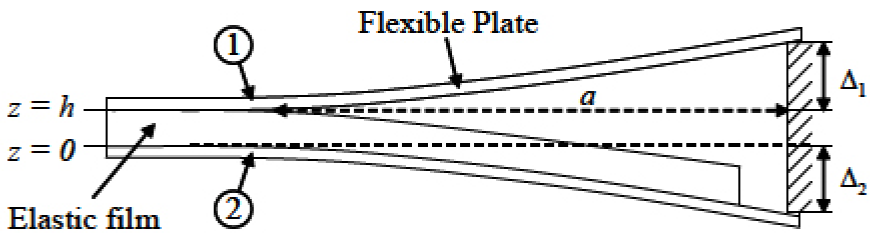


Fig. 1. Schematic of a cantilever plate experiment in which a flexible silanized glass plate (1) of rigidity $D_1$ is peeled from a thin elastic film bonded to a flexible plate (2) of rigidity $D_2$ in a displacement controlled experiment by inserting a spacer between the two flexible plates. $\Delta_1$ and $\Delta_2$ are respectively the vertical displacements of the two plates at the location of the spacer. The distance $a$ of the contact line (the dashed line) from the spacer is a measure of the adhesion strength between the two surfaces.

The schematic representation of our experiment is shown in Fig.1 in which a flexible plate 1 of rigidity $D_1$ is peeled from a thin (40 – 450 µm) elastic film of thickness *h* bonded to another flexible substrate 2 of rigidity $D_2$ . The surface of plate 1 is modified by supporting a self-assembled monolayer of alkyl-siloxane. We consider peeling in displacement controlled mode which can be done by inserting a spacer between the two plates. As a result the two plates undergo vertical displacements $\Delta_1$ and $\Delta_2$ at the pivoted end. The distance $a$, between the spacer and the line of contact of the film and plate 1 is a measure of the adhesion strength of the interface.

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In order to determine the work of adhesion by this experiment, we analyze the contact mechanics of this coupled elastic system. Assuming that the adhesive layer is incompressible, linearly elastic and is loaded in the plane strain, the 2-dimensional stress equilibrium equations are

$$\begin{aligned} P_x &= \mu\left(u_{xx}+u_{zz}\right) \\ P_z &= \mu\left(w_{xx}+w_{zz}\right) \end{aligned} \qquad (20.1)$$

Here $P(x,z)$ is the pressure in the elastic layer, $u(x,z)$ and $w(x,z)$ are the components of displacement field in the $x$ and $z$ directions (figure 1) and $\mu$ is the shear modulus of the layer. The incompressibility relation is written as,

$$u_x + w_z = 0 \qquad (20.2)$$

We solve equation 20.1 and 20.2 by using the following boundary conditions:

(a) We assume perfect bonding at the interface of the adhesive film and the underlying flexible plate (2), so that boundary conditions at this interface are:

$$u(x,0)=0 \quad \& \quad w(x,0)=-\xi_2 \qquad (20.3)$$

Here, $-\xi_2(x)=w(x,z=0)$ denotes the vertical deflection of the flexible plate (2).

(b) Continuity of the normal stress across the interface of the film and the contacting plate (2) i.e. at $z=0$ results in the boundary condition:

$$P\big|_{z=0} = D_2\,\xi_{2\,xxxx} \qquad (20.4)$$

(c) At the interface of the film and the flexible plate (1) we have a more complicated situation since the flexible plate is in contact with the film only over part of the domain. In the region $0<x<a$, there is no traction either on the plate or on the film so that the boundary conditions read

$$P=0\ ,\ \sigma_{zz}\big|_{z=h}=0 \text{ and } \sigma_{xz}\big|_{z=h}=0 \qquad (20.5)$$

Here $\sigma_{ij}$ denote the components of the two-dimensional stress tensor (and not derivatives), and.

(d) In the region $x<0$, the film remains in contact with the plate (1), so that the vertical displacement of the film and that of the flexible plate at this interface are equal, which results in,

$$w(x,h)=\xi_1 \qquad (20.6)$$

(e) Furthermore, continuity of the normal stress across this interface i.e. at $z=h$ yields the boundary condition

$$P\big|_{z=h} = D_1 \xi_{1\,xxxx} \tag{20.7}$$

(f) For the tangential traction, we can consider the following generalized boundary condition:

$$\alpha \sigma_{xz}(x,h) + \frac{(1-\alpha)\mu}{h} u(x,h) = 0 \tag{20.8}$$

where $\alpha$ is a parameters characterizing the nature of the bonding at the interface of the film and the cover plate When $\alpha = 0$ , $u(x,h) = 0$ corresponding to a perfect adhesion at the said interface (Case I), $\alpha = 1$ implies that $\sigma_{xz}(x,h) = 0$ and corresponds to the case of perfect slippage at the interface (Case II). In general $\alpha \in [0,1]$ .

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For a thin film ( $x >> z$), we may use the lubrication approximation so that the equations of equilibrium 20.1 simplify to

$$P_x = \mu u_{zz}$$
$$P_z = 0 \tag{20.9}$$

Physically, this simplification results from the dominant balance between shear stress and the horizontal pressure gradients in these thin layers. Equation 20.9 signifies that the pressure in the film is constant along the thickness coordinate, so that we can equate $P|_{z=0} = P|_{z=h}$, which on integration yields

$$D_2\xi_2 = D_1\xi_1 + \sum_{i=1,3} c_i x^i \tag{20.10}$$

The integration constants $c_i$ are equal to zero since the displacements $\xi_1$ and $\xi_2$ of the plates must vanish at $x \to -\infty$, yielding

$$\xi_2 = D_1\xi_1 / D_2 \tag{20.11}$$

Integrating equations 20.9 in the region $x < 0$ and using the boundary conditions 11.21, yields

$$u(x,z) = \frac{P_x}{2\mu}\left(z^2 - chz\right) \tag{20.12}$$

Here, the constant *c* = 1 for the case of perfect adhesion ( *Case I)* and *c* =2 for the case of perfect slippage ( *Case II* ) . Substituting this result into 20.2 and integrating the equation across the thickness of the film and using the boundary conditions 20.3, 20.6 and 20.7 yields an equation for the vertical displacement of the interface of the elastomeric film in the region $x < 0$ where it is attached to the flexible plate

$$\xi_1 = \frac{D_1 h^3 (3c-2)\xi_{1xxxxxx}}{12\mu} - \xi_2 \tag{20.13}$$

Substituting the expression for $\xi_2$ from equation 11.29 in equation 11.31, we obtain,

$$\xi_{1xxxxxx} = \frac{12\mu}{D_e h^3 (3c-2)}\xi_1 \tag{20.14}$$

Notice that the expression in equation 20.14 matches with that of equation 19.7 derived for the experiment in which a single flexible plate is peeled of a layer of elastic adhesive bonded to a rigid substrate. Here, however, we obtain the definition of an equivalent flexural rigidity $D_e = D_1 D_2 / (D_1 + D_2)$, i.e. the harmonic mean of the rigidities of the two flexible plates which implies that the experiment with two flexible plates is equivalent to one with a single plate having an equivalent flexural rigidity.