The Lecture Contains

📧 Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin.

1. Flexural rigidity of microtubules ... fluctuation in shape", Gittes, F. G., et al, *The Journal of Cell Biology*, **volume 120, number 4, 1993, 923-934.**

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Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin.

Microtubules are hollow fibers with outer and inner diameters of 30 and 18 nm respectively and length of about 100mm. They play central role in cell physiology. They form the moving cores of cilia and flagella, they are the tracks along which intracellular proteins move and they actively take part in the intracellular signal transduction. What mechanical properties the microtubules possess in order that it can perform many different structural roles. Here we will show how we can estimate the flexural rigidity of cellular fibers like microtubules.
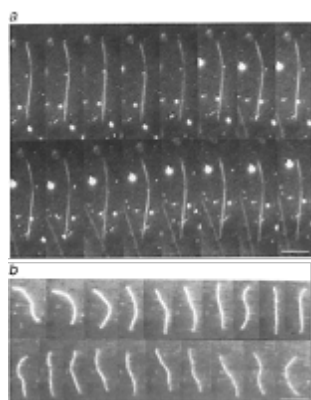

Figure 32.1

From our discussion on bending of thin flexible rods, we can say that we can estimate flexural rigidity by clamping a microtubule of known length at one end while applying a known force to the other and measuring its deflection. However, here we will describe a technique to measure the bending of an unconstrained microtubule by thermal forces.

Module 4 : Nonlinear elasticity

Lecture 32 : Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin.

Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin.

We know that due to thermal fluctuations soft filaments like microtubules and actin bends and constantly changes its shape. However, microtubule is much more rigid, so that the fluctuations due to thermal forces are too small for microtubules than that of actin. The thermal bending of the microtubule is characterized by representing its shape as summation of cosine functions of increasing frequency, that is by Fourier decomposing the profile of the rod. For example if we represent the profile of the filament in terms of tangent at every point s, (θ, s) then we can write:

$$\theta(s) = \sum_{n=0}^{\infty} \theta_n(s) = \sqrt{\frac{2}{L}} \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi s}{L}\right) \qquad (32.1)$$

where L is the length of the flexible rod and a_n is the amplitude of nth Fourier mode. The cosine functions are chosen for computational convenience, but we could as well choose sine and cosine functions. Now due to thermal fluctuations, the shape of the rod changes, so that the amplitude of each mode also and they change independently of each other. For small bending, the bending energy per unit length of the filament is given by

$$\frac{dU}{ds} = \frac{EI}{2}\left(\frac{d\theta}{ds} - \frac{d\theta^0}{ds}\right)^2 \qquad (32.2)$$

Where θ_0 is the shape of the relaxed rod. Equation 32.2 on integration yields

$$U = \frac{EI}{2}\int_0^L \left(\frac{d\theta}{ds} - \frac{d\theta^0}{ds}\right)^2 ds \qquad (32.3)$$

Here, EI is the flexural rigidity of the rod and I is the moment of inertia defined as,

$$I = \iint y^2 dA \quad I = \frac{\pi}{4}\left(r_0^4 - r_i^4\right) \qquad (32.4)$$

Equation 32.2 and 32.3 emphasize the point that the concept of flexural rigidity developed so far for macroscopic filaments is also applicable for microscopic filaments despite their inherent complication in molecular structure.

Substituting the expression for $\theta(s)$ from equation 1 in equation 32.3, one obtains,

$$\qquad (32.5)$$

$$\frac{d\theta(s)}{ds} = -\sqrt{\frac{2}{L}} \sum_{n=0}^{\infty} a_n \left(\frac{n\pi}{L}\right) \sin\left(\frac{n\pi s}{L}\right), \quad \frac{d\theta^0(s)}{ds} = -\sqrt{\frac{2}{L}} \sum_{n=0}^{\infty} a_n^0 \left(\frac{n\pi}{L}\right) \sin\left(\frac{n\pi s}{L}\right)$$

$$U = \frac{EI}{2} \int_0^L \left(\frac{d\theta(s)}{ds} - \frac{d\theta^0(s)}{ds}\right)^2 ds = \frac{EI}{2} \frac{2}{L} \sum_{n=0}^{\infty} (a_n - a_n^0)^2 \left(\frac{n\pi}{L}\right)^2 \int_0^L \sin^2\left(\frac{n\pi s}{L}\right) ds$$

$$= \frac{EI}{2} \sum_{n=0}^{\infty} (a_n - a_n^0)^2 \left(\frac{n\pi}{L}\right)^2$$

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Estimation of flexural rigidity of proteinaceous filaments like microtubules and Actin.

Now the inverse Fourier transform of equation 1 gives the amplitude of the modes,
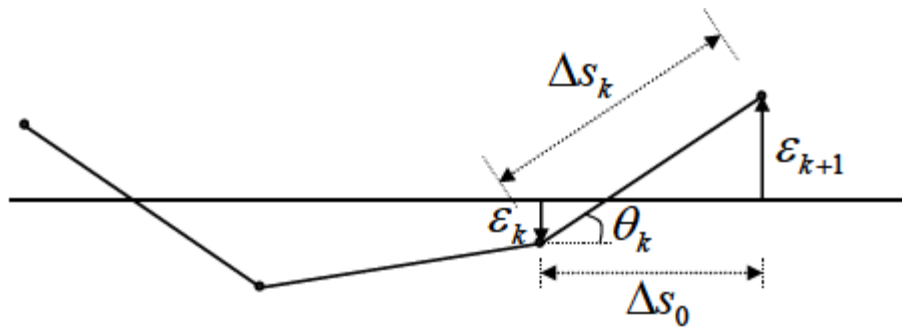
$$a_n = \sqrt{\frac{2}{L}} \int_0^L ds\, \theta(s) \cos\left(\frac{n\pi s}{L}\right)$$

Which are estimated by the following approximate relation:

$$a_n \cong \sqrt{\frac{2}{L}} \sum_{k=1}^N \theta_k \Delta s_k \cos\left(\frac{n\pi s_k^{mid}}{L}\right), \qquad n = 1,\ldots,N-1 \qquad (32.9)$$

where

$$L = \sum_{k=1}^N \Delta s_k \text{ and } s_k^{mid} = \Delta s_1 + \Delta s_2 + \Delta s_3 + \ldots + \Delta s_{k-1} + (1/2)\Delta s_k \quad (32.10)$$

The experimental estimation of the amplitude of the different modes are however not free of error, in fact errors creep in due to poor resolution of the microscope and the video recording system. As a result the measured position of a point deviates from its actual position by a random distance ε_k. This error, which varies from filament to filament depending on the image quality adds directly to the true variance of the amplitude of each mode and thus to the estimate of the flexural rigidity EI. For small amplitude fluctuations, the measurement error ε_k that contributes to a_n remains independent of a_n itself. Then we can estimate it with

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