The Lecture Contains:

This lecture is adopted from the following book

1. A. Ghatak, L. Mahadevan and M. K. Chaudhury, Measuring the Work of Adhesion between a Soft Confined Film and a Flexible Plate, ***Langmuir*** 2005, ***21,*** 1277-1281

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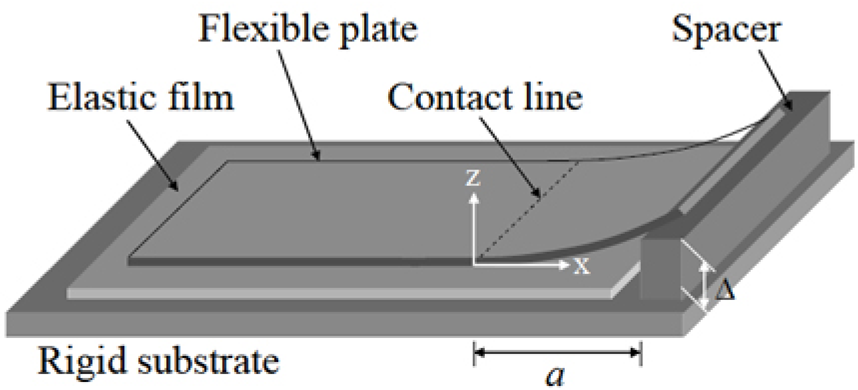
Module 3 : Equilibrium of rods and plates

Lecture 19 : Peeling a thin flexible plate off an elastic adhesive bonded to a flexible substrate



Many engineering and practical applications involve a thin layer of soft adhesive sandwiched between two rigid and flexible plates.

Question arises how much load is required to separate these two adherents.

Here we will consider the simple situation in which a thin flexible plate is lifted off a thin layer of adhesive which remains bonded to a rigid substrate. The lifting load is applied at the hanging end of the plate because of which the contact line between the plate and the film moves away from or towards the point of application of the load till equilibrium of forces is established. The adhesive layer is assumed to be incompressible and elastic without any viscous effect.

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Stress equilibrium relations and boundary conditions

The stress equilibrium relations for such a film in plane strain approximation is written in terms of the pressure field as,

$$p_x = \mu\left(u_{xx} + u_{zz}\right),\ p_z = \mu\left(w_{xx} + w_{zz}\right) \quad (19.1)$$

Where, $u(x,z)$ and $w(x,z)$are the deformation components along x and z directions respectively and μ is the shear modulus of the adhesive material. We can use the plane strain approximation because the film is infinitely long along the y direction and the deformations remain independent of y. The double derivatives of displacements represented by u_{ij}define the strain gradients. The incompressibility condition is written as

$$u_x + w_z = 0 \quad (19.2)$$

These equations are solved using the following boundary conditions:

(a) Displacements along x and z are zero at the film substrate interface:

$$u(x, z = 0) = 0, \quad u(x, z = h) = 0,\ w(x, z = 0) = 0 \quad (19.3a)$$

(b) The normal stress is continuous at the interface of the film and the plate, so that

$$\sigma_{zz}\big|_{z=h} = -p + 2\mu\frac{\partial w}{\partial z} \text{ for } x < 0 \quad (19.3b)$$

$$0 = p \text{ for } 0 < x < a < 0 \quad (19.3c)$$

For small bending of plate in one dimension, the normal stress is equal to the bending stress which can be expressed in terms of the stiffness or flexural rigidity D of plate as

$$\sigma_{zz} = -D\xi_{xxxx}$$

Here $\xi(x) = w(x,h)$ is the vertical displacement of the film at $z = h$ and a is the distance of the line of application of the peeling force from the contact line at $x = 0$.

We can simplify the above set of equations and boundary conditions because of large separation of characteristic length-scales along the x and z directions respectively , i.e. L and h: $h << L$. For example, $u_{xx} \sim 1/L, u_{zz} \sim L/h^2, w_{xx} \sim h/L^2, w_{zz} \sim 1/h$. Therefore, we can neglect all terms except u_{zz}, so that the equations of equilibrium simplify to

$$p_x = \mu u_{zz} \text{ and } p_z = 0 \quad (19.4)$$

Similarly the boundary conditions can be simplified to

$$p = D\xi_{xxxx} \text{ for } x < 0$$
$$p = 0 \text{ for } 0 < x < a \qquad (19.5)$$

Integration of equation (19.4) yields

$$u(x,z) = \frac{p_x}{\mu}\frac{z^2}{2} + Az + B$$

In which we put the boundary conditions as in equation 19.3a and 19.4

$$u(x,z) = \frac{D}{2\mu}\xi_{xxxxx}\left(z^2 - hz\right) \qquad (19.6)$$

Substitution of 19.6 in equation 19.2 followed by integration gives rise to the following expression for vertical displacement of the cover plate in the region $x < 0$ where it is attached with the adhesive film

$$w(x,z) = \frac{D}{2\mu}\xi_{xxxxxx}\left(\frac{z^3}{3} - \frac{hz^2}{2}\right) + C$$

Using boundary condition 19.2 and noting that $w(x, z = h) = \xi(x)$, we obtain,

$$\xi_{xxxxxx} = \frac{12\mu}{Dh^3}\xi \text{ for } x < 0 \qquad (19.7)$$

And in the region $0 < x < a$, where the film is not in contact with film,

$$\xi_{xxxx} = 0 \text{ for } 0 < x < a \qquad (19.8)$$

Equations 19.7 and 19.8 together demand ten constants which are to be determined using ten boundary conditions:

Since the deformation of the flexible plate film must vanish far away from the contact line,

$$\xi|_{x\to-\infty} = 0,\ \xi_x|_{x\to-\infty} = 0,\ \xi_{xx}|_{x\to-\infty} = 0 \qquad (19.9)$$

Continuity of the displacement, slope, bending moment, vertical shear force and the pressure at the contact line results in

$$\xi|_{0-} = \xi|_{0+},\ \xi_x|_{0-} = \xi_x|_{0+},\ \xi_{xx}|_{0-} = \xi_{xx}|_{0+},$$
$$\xi_{xxx}|_{0-} = \xi_{xxx}|_{0+},\ \xi_{xxxx}|_{0} = 0 \qquad (19.10)$$

Finally, at $x = a$, where the flexible plate is freely pivoted while being lifted vertically by an amount Δ, the boundary conditions are

, (19.11)

$$\xi\big|_{x=a} = \Delta \quad \xi_{xx}\big|_{x=a} = 0$$

Thus equations 19.9 to 19.11 present nine boundary conditions. One additional boundary condition is obtained by considering that at the line of contact between the flexible plate and film the stress concentration is maximized, which implies that

$$1 \;\; p_x\big|_{x=0} = 0 \tag{19.12}$$

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Module 3 : Equilibrium of rods and plates

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Solution of equations 19.7 and 19.8 along with boundary conditions 19.9 to 19.12 yield the following set of expressions for displacement of the film in x and z directions respectively

$$u(x,z)=\frac{6z(z-h)F'}{kh^3}e^{kx/2}\left(ake^{kx/2}+\frac{3ak+4}{\sqrt{3}}\sin\left(\frac{\sqrt{3}kx}{2}\right)-ak\cos\left(\frac{\sqrt{3}kx}{2}\right)\right) \quad (19.13a)$$

$$w(x,z)=\frac{z^2(3h-2z)F'}{h^3}e^{kx/2}\left(ake^{kx/2}+\frac{3ak+2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}kx}{2}\right)+(ak+2)\cos\left(\frac{\sqrt{3}kx}{2}\right)\right) \quad (19.13b)$$

Where $F'=3\Delta/\left(6+12ak+9(ak)^2+2(ak)^3\right)$ and $k^{-1}=\left(Dh^3/12\mu\right)^{1/6}$ are two different characteristic length scales in the problem: the first accounts for the response of the plate to the displacement of its pivoted end, while the second length scale measures the relative deformability of the plate and the film in terms of the size of the effective contact zone. Thus the solutions of displacements u and w are obtained as oscillatory with exponentially decreasing amplitude away from the contact line. At $z=h$, the vertical displacement of the film and that of the contacting plate is expressed as:

$$\xi(x)=F'e^{kx/2}\left(ake^{kx/2}+\frac{3ak+2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}kx}{2}\right)+(ak+2)\cos\left(\frac{\sqrt{3}kx}{2}\right)\right) \text{ for } x<0$$

$$\xi(x)=F'\left(2(ak+1)+(3ak+2)kx+ak(kx)^2-(kx)^3/3\right) \quad (19.14)$$

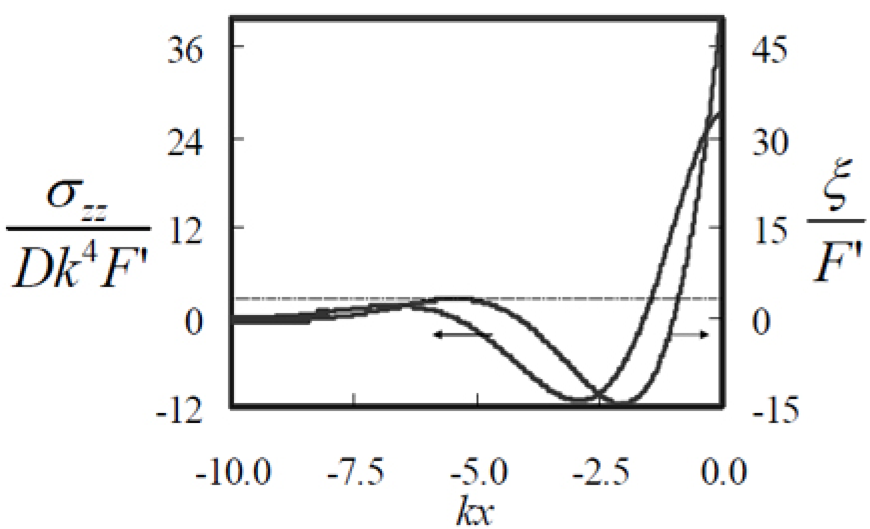

Figure 19.2

The normal stress in the film can be obtained as

$$\sigma_{zz}(x,h) = -D\xi_{xxxx}$$
$$= -\frac{D\Delta k^4 e^{kx/2}}{6+12(ak)+9(ak)^2+2(ak)^3}\left(3ake^{kx/2} - 6(1+ak)\cos\left(\frac{\sqrt{3}kx}{2}\right) + 2\sqrt{3}\sin\left(\frac{\sqrt{3}kx}{2}\right)\right)$$

(19.15)

The above figure shows the normal stress and vertical displacement in the film as a function of distance from the contact line. The wavelength of oscillation is found to be $\lambda = \mathbf{5.6}k^{-1}$.

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Calculation of work of adhesion

In order to obtain the work of adhesion between the adhesive film and the flexible plate, we can first write the total energy of the system in terms of the distance of the contact line from the point of application of the load and then . The total energy consists of the elastic energy of the film, bending energy of the plate and the interfacial energy:

$$\prod(a) = \int_{-\infty}^{a} \frac{D}{2}\left(\frac{d^2\xi}{dx^2}\right)^2 dx + \int_{-\infty}^{0}\int_{0}^{h} \frac{\mu}{4}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 dz\,dx + Wa \qquad (19.16)$$

The first term in the above equation defines the bending energy of the plate per unit width: flexural rigidity x (curvature)^2 ; the second term defines the elastic energy of the film and the third term is the interfacial energy per unit width that must be provided in order to create the new surface area between the plate and the adhesive film. Here W defines the thermodynamic work of adhesion. We can then find out the distance a for which the total energy of the system is minimized: $\partial\prod(a)/\partial a = 0$. Such an analysis finally leads to the following expression for the work of adhesion:

$$W = \frac{9D\Delta^2}{2a^4} g(ak), \text{ where}$$

$$g(ak) = \frac{8(ak)^4\left(12 + 46(ak) + 72(ak)^2 + 56(ak)^3 + 21(ak)^4 + 3(ak)^5\right)}{3\left(6 + 12(ak) + 9(ak)^2 + 2(ak)^3\right)^3} \qquad (19.17)$$

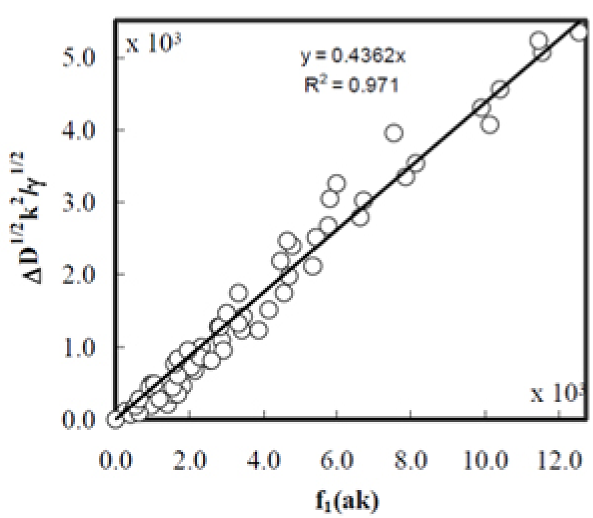

Figure 19.3

The work of adhesion can be calculated using equation 19.17, which can be rewritten as

(19.18)

$$\frac{\Delta D^{1/2} k^2}{\gamma^{1/2}} = \left(\frac{W}{12\gamma}\right)^{1/2} f_1(ak)$$

Where $f_1(ak) = \left(8(ak)^4 / 3g(ak)\right)^{1/2}$ and $\gamma = 20$ mJ/m^2 is the surface energy of the poly(dimethylsiloxane) (PDMS) films. The above Figure shows the experimental data from several combinations of elastic film, flexible contacting plate scaled as in equation 19.18. From the slope of this curve the work of adhesion can be obtained as 45 mJ/m^2 .

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