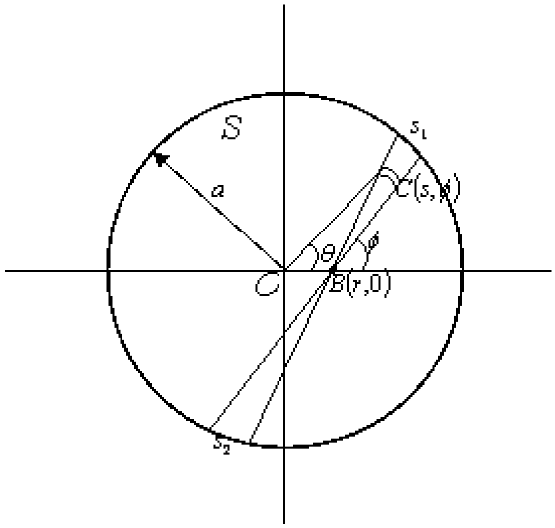
The Lecture Contains:

This lecture is adopted from the following book:

1. "Contact Mechanics" by K.L.Johnson

Pressure Applied to a circular region: Uniform Pressure: P_0



The depression of the surface can be obtained from equation 5.18 as,

$$v\big|_{z=0} = \frac{1-\sigma^2}{\pi E} p_0 \iint_{S\phi} ds\, d\phi \tag{6.1}$$

If we consider that the point B lies within the circle, then it is easy to show that

$$s_{1,2} = -r\cos\phi \pm \left\{ r^2\cos^2\phi + \left(a^2 - r^2\right)\right\}^{1/2} \tag{6.2}$$

Thus

$$v\big|_{z=0} = \frac{1-\sigma^2}{\pi E} p_0 \int_0^{\pi} 2\left\{a^2 - r^2\sin^2\phi\right\}^{1/2} d\phi = \frac{4\left(1-\sigma^2\right)p_0 a}{\pi E}\int_0^{\pi/2}\left\{1-\frac{r^2}{a^2}\sin^2\phi\right\}^{1/2} d\phi = \frac{4\left(1-\sigma^2\right)p_0 a}{\pi E} E\left(r/a\right) \tag{6.3}$$

$E(r/a)$ is called the complete elliptic integral of the second kind with modulus (r/a).

At the center of the circle : $r = 0, E(0) = \pi/2$, thus

$$v\big|_{z=0} = \frac{2\left(1-\sigma^2\right)p_0 a}{E} \tag{6.4}$$

At the edge of the circle $r = a, E(1) = 1$, then

$$v\big|_{z=0} = \frac{4\left(1-\sigma^2\right)p_0 a}{\pi E} \tag{6.5}$$

Mean displacement of the loaded circle is

$$v\big|_{z=0} = \frac{16\left(1-\sigma^2\right)p_0 a}{3\pi E} \tag{6.6}$$

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Since we are considering axial symmetry, the displacement tangential to the surface must be radial.

According to equation 4.48, the tangential displacement at B due to an element of load at C is,

$$\frac{(1-2\sigma)(1+\sigma)}{2\pi E}\frac{ps\,ds\,d\phi}{s} \tag{6.7}$$

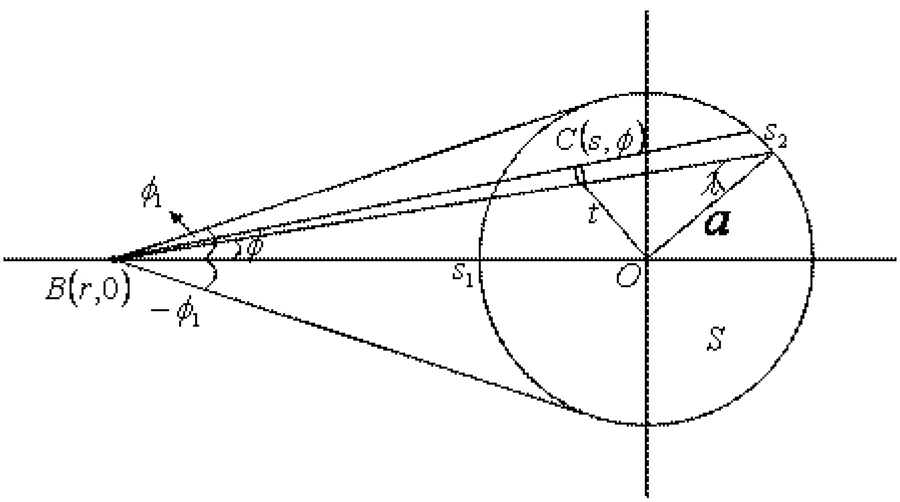
Thus the radial component of this displacement is

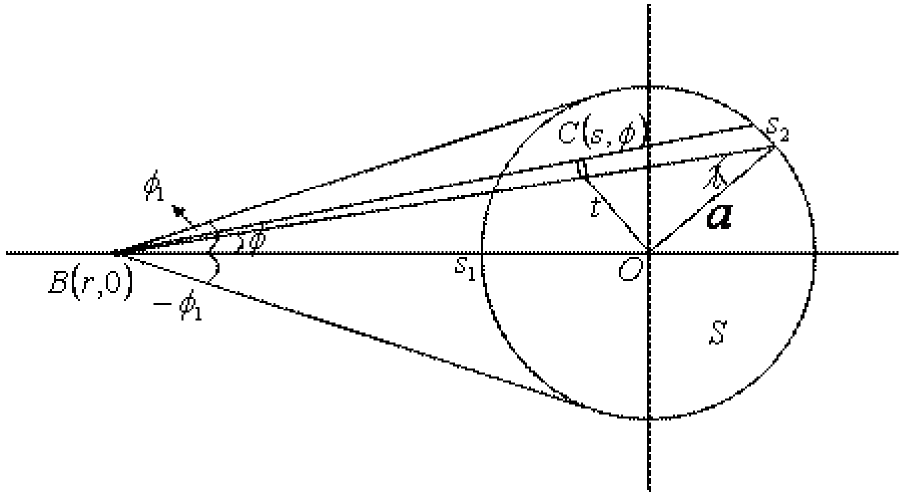
$$u\big|_{z=0} = \frac{(1-2\sigma)(1+\sigma)}{2\pi E}\frac{ps\,ds\,d\phi}{s}\cos\phi \tag{6.8}$$

The total displacement the whole load is then,

$$\begin{aligned} u\big|_{z=0} &= \frac{(1-2\sigma)(1+\sigma)p}{2\pi E}\int_0^{2\pi}\left\{-r\cos\phi+\left(a^2-r^2\sin^2\phi\right)^{1/2}\right\}\cos\phi\,d\phi \\ &= -\frac{(1-2\sigma)(1+\sigma)pr}{2E} \qquad\qquad r\le a \end{aligned} \tag{6.9}$$

The second term in the above integral vanishes when integrated over the limits 0 to 2p.





Now we turn our attention to a point B which is outside the circle, in this case the limits on ϕ are $\pm\,\phi_1$, so that,

$$v\big|_{z=0} = \frac{1-\sigma^2}{\pi E} p_0 a \int_0^{\phi_1} 2\left\{1-\frac{r^2}{a^2}\sin^2\phi\right\}^{1/2} d\phi \tag{6.10}$$

We change the variable to the angle λ , so that $a\sin\lambda = r\sin\phi$, then the expression for $v\big|_{z=0}$ is obtained as,

$$v\big|_{z=0} = \frac{1-\sigma^2}{\pi E} 4 p_0 \int_0^{\pi/2} \frac{a^2\cos^2\lambda}{r\left\{1-\frac{a^2}{r^2}\sin^2\lambda\right\}^{1/2}} d\lambda$$

$$= \frac{1-\sigma^2}{\pi E} 4 p_0 r \int_0^{\pi/2} \left(\left\{1-\frac{a^2}{r^r}\sin^2\lambda\right\}^{1/2} - \frac{1-\frac{a^2}{r^2}}{\left\{1-\frac{a^2}{r^r}\sin^2\lambda\right\}^{1/2}} \right) d\lambda \tag{6.11}$$

$$= \frac{1-\sigma^2}{\pi E} 4 p_0 r \left(E(a/r) - \left(1-\frac{a^2}{r^2}\right) K(a/r) \right) \qquad r > a$$

$K(a/r)$ and $E(r/a)$ are called the complete elliptic integral of the first and second kind.

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The tangential displacement at B is radial and is given by,

$$u\big|_{z=0} = -\frac{2(1-2\sigma)(1+\sigma)p}{\pi E}\int_0^{\phi_1}\left\{\left(a^2 - r^2\sin^2\phi\right)^{1/2}\right\}\cos\phi\, d\phi \qquad (6.12)$$

Changing the variable to $a\sin\lambda = r\sin\phi$, $a\sin\lambda = r\sin\phi$, $u\big|_{z=0}$ is obtained as,

$$u\big|_{z=0} = -\frac{2(1-2\sigma)(1+\sigma)p_0}{\pi E}\frac{a^2}{r}\int_0^{\pi/2}\cos^2\lambda\, d\lambda = -\frac{(1-2\sigma)(1+\sigma)p_0}{2E}\frac{a^2}{r} \qquad r > a \qquad (6.13)$$

Since $p\pi a^2$ is equal to the load P acting on the whole area, we note that the tangential displacement outside the loaded is the same as though the whole load is concentrated at the center of the circle. Stresses at the surface within the circle may now be found from the strain,

$$e_r\big|_{z=0} = \frac{\partial u\big|_{z=0}}{\partial r} = e_\theta = \frac{u\big|_{z=0}}{r} = -\frac{(1-2\sigma)(1+\sigma)p}{2E} \qquad (6.14)$$

Then the radial stress is,

$$R_r\big|_{z=0} = -\frac{1}{2}(1+2\sigma)p, \quad Z_z\big|_{z=0} = -p \qquad (6.15)$$

Assignment: Find out the stress components inside the elastic half space at a point (r,θ,z)

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