The Lecture Contains:

Capillary Rise

$\phi = \pi/2 - \theta;$

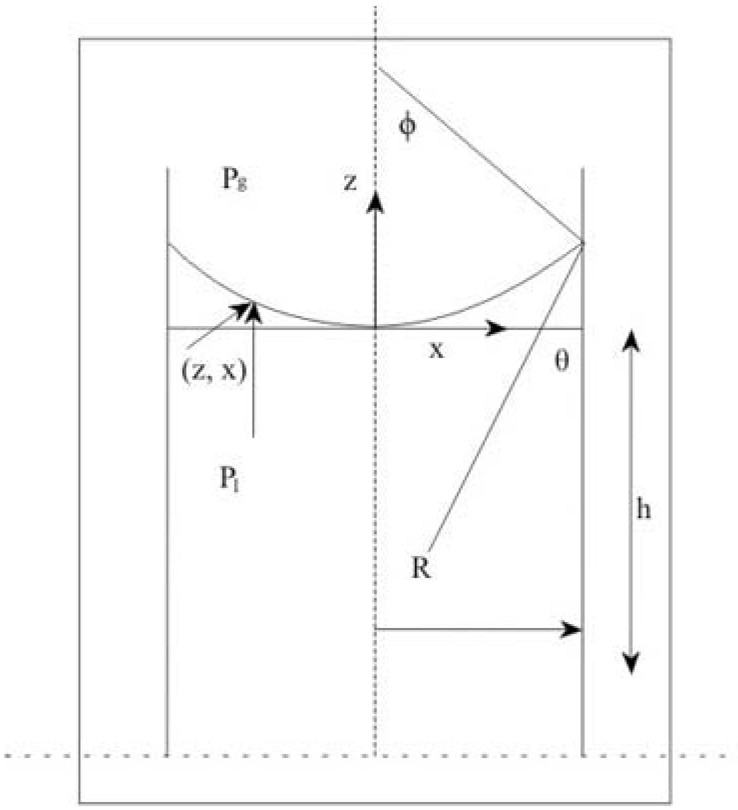

Fig.2.9

$\Delta p = p_g - p_l;$

$p_g = p_l + \rho z g;$

$\Delta p = \rho z g;$

using equation (2.26)

$\rho z g = \gamma \left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

This is a function of 'z' and 'x'. We can solve it both analytically and numerically.

Boundary conditions: (fig:2.9)

1) $x = 0, z = 0$

2) $x = R, \frac{dx}{dz} = \tan\theta$

Weight of total liquid contained

$$W = \rho g \int_0^R 2\pi xzdx$$

But to evaluate this integral we need to know $z = f(x)$.

In this particular case, if we know the value of W then γ can be calculated.

If we neglect the curved region just below the liquid surface and assume the surface to be planar,

$$W \cong \pi R^2 h\rho g$$

$$h \cong \frac{2\gamma\cos\theta}{\rho g R}$$

If the capillary walls are clean and $h \gg R$ then for most of the liquid

$$\theta \cong 0$$

$$W = 2\pi R\gamma\cos\theta = \rho g \int_0^R 2\pi xz dx$$

$$W = 2\pi\gamma \int_0^R \left(\frac{xz''}{(1+z')^{3/2}} + \frac{z'}{(1+z'^2)^{1/2}} \right) dx$$

$$xz''dx = xp\frac{dp}{dz}dx = x\,\frac{dz}{dx}\frac{dp}{dz}dx = xdp;$$

$$z'dx = pdx;$$

Now

$$W = 2\pi\gamma \int \frac{xdp}{(1+p^2)^{3/2}} + \frac{pdx}{(1+p^2)^{1/2}}$$

$$W = 2\pi\gamma \int d\,\left| \frac{xp}{(1+p^2)^{1/2}} \right|$$

$$W = 2\pi\gamma \left[\frac{xp}{(1+p^2)^{1/2}} \right]_{0,p=0}^{R,p=\cot\theta}$$

$$W = 2\pi\gamma \left[\frac{R}{\tan\theta\ (1+\cot^2\theta)^{1/2}} \right]$$

$$W = 2\pi\gamma R\cos\theta \qquad (2.31)$$

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Climbing of liquid on a rod

While we consider the case of a cylinder here, if $R \to \infty$, the system changes to the case of a flat plate.

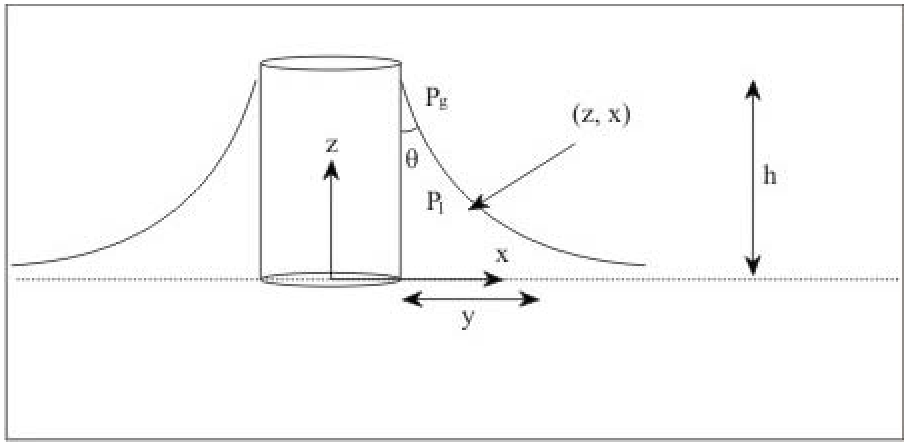

Figure 2.10 :Liquid climbing a rod

$$\rho g z = \gamma \left[\frac{z''}{(i + z'^2)^{3/2}} + \frac{z'}{x(1 + z'^2)} \right]$$

$$R \to \infty \Rightarrow x \to \infty$$

$$\frac{z'}{x(1 + z'^2)} \to 0$$

$$\rho g z \cong \gamma \left[\frac{z''}{(i + z'^2)^{3/2}} \right]$$

$$z'' = \frac{\rho g z \left(1 + z'^2\right)}{\gamma}$$

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Module 2: "Capliarity"

Lecture 7: ""

Let,

$$z' = p$$

$$z' = \frac{1}{2}\frac{dp^2}{dz}$$

$$\int \frac{1}{2}\frac{dp^2}{(1+p^2)^{3/2}} = \int \left(\frac{\rho g}{\gamma}\right) z dz$$

$$-(1+p^2)^{1/2} = \left(\frac{\rho g}{\gamma}\right)\frac{z^2}{2} + c_1$$

$$z = 0 \Rightarrow \frac{dz}{dx} = 0$$

$$\Rightarrow p = 0$$

$$\Rightarrow c_1 = -1$$

$$\Rightarrow z = h, \frac{dx}{dz} = -\tan\theta$$

$$\Rightarrow -\left(\frac{1}{\sqrt{1+\frac{1}{tan^2\theta}}}\right) = \frac{\rho g h^2}{2\gamma} - 1$$

$$\frac{\rho g h^2}{2\gamma} = 1 - \sin\theta \qquad (2.32)$$

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