The Lecture Contains:

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Study of the effect produced by a disturbance on a water cylinder

$X = R + h(z)(\text{fig } 2.17) = R + h_0 + \xi \sin kz$

Where $\xi \ll h_0$

$$\frac{1}{R_1} = -\frac{x''}{(1+x'^2)^{3/2}} = \frac{\xi k^2 Sin(kz)}{[1+(k\xi Coskz)^2]^{3/2}}$$

$\xi \to 0$ i.e. small thermal perturbation

$$\frac{1}{R_1} \cong \xi k^2 \mathrm{Sin}(kz)$$

$$\frac{1}{R_2} = \frac{1}{(R + h_0 + \xi \mathrm{Sin}(kz))(1)} \cong \frac{1}{(R+h_0)}\left[1 - \frac{\xi \mathrm{Sin}(kz)}{(R+h_0)}\right]$$

$$p_1 - p_g = \gamma \left[\xi k^2 Sin\,(kz) - \frac{\xi Sin(kz)}{(R+h_0)^2} + \frac{1}{(R+h_0)}\right]$$

If $\xi = 0$

$R_1 \to \infty$

$R_2 = (R + h_0)$

Let

$1/(R+h_0)^2 - k^2 = B$

Let $B > 0$

$$k = \frac{2\pi}{\lambda} < \frac{1}{R+h_0}$$

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If two points ‘1’ and ‘2’ are chosen (as shown in the fig:2.17)

then $p_2 > p_1$ if $B > 0$

In such a case the pressure is out of phase with the disturbance.
Then the fluid will flow from ‘2’ to ‘1’
And the cylindrical surface would break in to drops.

$\Longrightarrow$ For $B > 0$ cylendrical surfaces are unstable.

So for $B > 0$, the condition for λ is

$$\lambda > 2\pi(R + h_0) \Rightarrow \text{Rayleigh instability} \qquad (2.33)$$

For such 'λ' cylindrical surfaces would be unstable.

If R increases, spacing between drops increases.

As pointed out earlier, the Young-Laplace equation can have multiple solutions for the same set of parameters or have no solution at all. In case of multiple solutions, relative stability of the solutions has to be studied.

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Example. A thin film supported by two wire loops (Fig. 2.19).

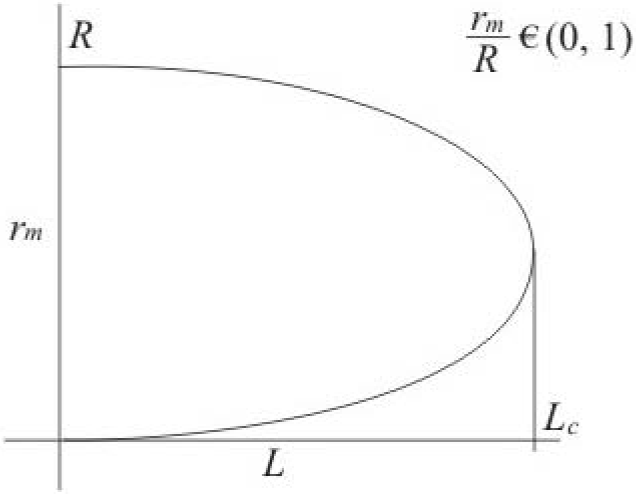

Figure 2.18. A thin film supported by two wire loops.

The thin film here has an axisymmetric surface (or meniscus).

V, the volume of liquid comprising the thin film, is negligible, therefore the body forces can be neglected.

Also, $\Delta P = 0$. Thus in the energy functional, the $P \times V$ term vanishes.

$$\Rightarrow \frac{1}{R_1} + \frac{1}{R_2} = 0$$

$$\Rightarrow z" \left(1 + z'^2\right)^{-3/2} + \frac{z'}{x}\left(1 + z'^2\right)^{-1/2} = 0$$

$$\Rightarrow \frac{1}{x}\frac{d}{dx}\left[\frac{xz'}{\left(1 + z'^2\right)^{1/2}}\right] = 0$$

Upon integration, we get

$$\frac{xz'}{\left(1 + z'^2\right)^{1/2}} = c_1$$

At $z = 0, x = r_m$ (not known a priori), and $z' \to \infty$.

$$\Rightarrow \; c_1 = r_m$$

$$\Rightarrow z' = \frac{dz}{dx} = \frac{\left(\frac{r_m}{x}\right)}{\left[1 - \left(\frac{r_m}{x}\right)^2\right]^{1/2}} = \frac{r_m}{\sqrt{x^2 - r_m^2}}$$

Upon further integration, one gets

$$z = r_m \ln\left[x + \sqrt{x^2 - r_m^2}\right] + c_2$$

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Since $z = 0, x = r_m$,

$c_2 = -r_m \ln r_m$

$$z = r_m \ln\left[x + \sqrt{x^2 - r_m^2}\right] - r_m \ln r_m$$

$$z = r_m \ln\left[\frac{x}{r_m} + \sqrt{\frac{x^2}{r_m^2} - 1}\right] \qquad (2.34)$$

This surface is also called a catenoid surface. It is a minimum or a maximum surface area.

$Obtaining\ r_m$

Note that R_1 is negative whereas R_2 is positive.

The boundary conditions are,

at $z = L$ (also $-L$), $x = R$ (radius of the loop).

Using this boundary condition we get,

$$L = r_m \ln\left[\frac{R}{r_m} + \sqrt{\frac{R^2}{r_m^2} - 1}\right] \qquad (2.35)$$

From this equation the value of r_m can be obtained.

$r_m = f(L, R)$

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Note

1. If r_m is small, i.e.

$$L \rightarrow r_m \ln\left(\frac{2R}{r_m}\right)$$

i.e. as $r_m \rightarrow 0, L \rightarrow 0$

2. Also as $r_m \rightarrow R, L \rightarrow 0$

r_m cannot be larger than R because the surface has to be curved inwards.

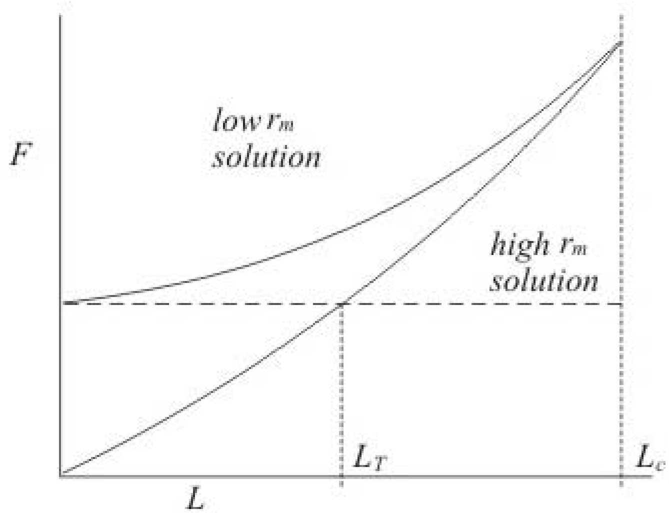

Figure 2.19. The solution

For $R = 1$, the possible solutions are

Table 2.2.

r_m	L
0.01	0.053
0.1	0.299
0.5	0.658
0.5524	0.6627
0.7	0.627
0.8	0.555
0.999	0.045

A point to note at this juncture is that there is no solution for $L > L_c$. Physically, this means that if the two loops are located far away from each other no film will be formed. From the table we can see that in this case $L_c = .6627$

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Analysis of relative stability

At a particular length L there can be 2 possible solutions. We need to find which one of them is more stable. We calculate the energy functional for this purpose.

The Helmholtz energy is given as

$$\frac{F}{\gamma} = \int 2\pi x\sqrt{1+z'^2}\,dx$$

$$= \int x\sqrt{\frac{r_m}{x^2 - r_m^2} + 1}\,dx$$

$$= \int_{r_m}^{R} \frac{x^2}{x^2 - r_m^2}\,dx$$

This represents the top half of the surface.

$$\frac{F}{2\pi\gamma} = \frac{R}{2}\sqrt{R^2 - r_m^2} + \frac{r_m^2}{2}\ln\left[R + \sqrt{R^2 - r_m^2}\right]$$

$$\frac{F}{2\pi\gamma} = \frac{R}{2}\sqrt{R^2 - r_m^2} + \frac{Lr_m}{2} \qquad (2.36)$$

This has been plotted against L Fig. 2.20). Two solutions corresponding to a high r_m and a low r_m are obtained. We need to find that among the two branches, which one corresponds to local energy minimum..

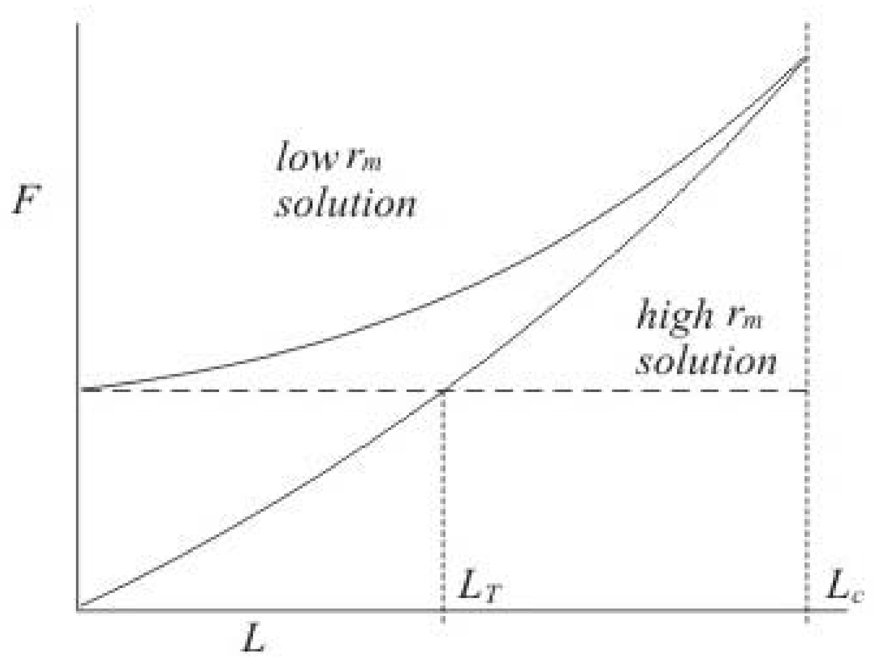

Figure 2.20. Energy vs distance between the wire loops

For the upper branch, $r_m \to 0, \dfrac{F}{2\pi\gamma} \to 0$, whereas for the lower branch $\dfrac{F}{2\pi\gamma} \to some\ finite\ value$. The latter corresponds to a situation where the film is spread over a single wire loop. Further, for $L > L_T$ the film would collapse.

For *R* = 1,

$0.6627 (= L_c) > L > 0.528$	globally stable
$L < 0.528$	high r_m solution is stable
$L > 0.528$	high r_m solution is metastable
for any L	low r_m solution is unstable

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