Module 2: "Capliarity"

Lecture 9: ""

The Lecture Contains:

- Hanging/Pendent Bubble
- Stability of water Cylinder (No Flow)

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HANGING/PENDANT BUBBLE

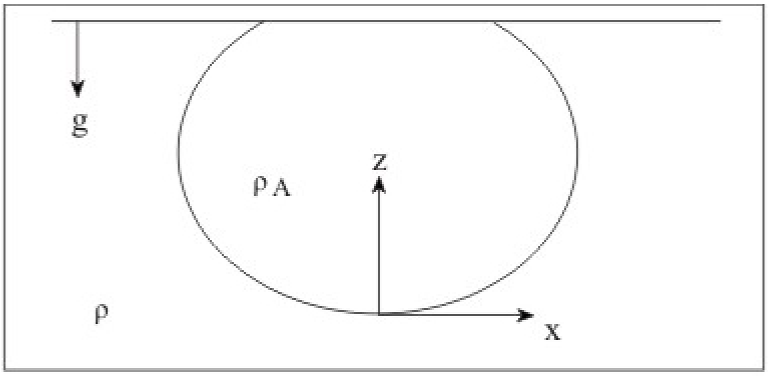

Figure:2.14 Pendant bubble

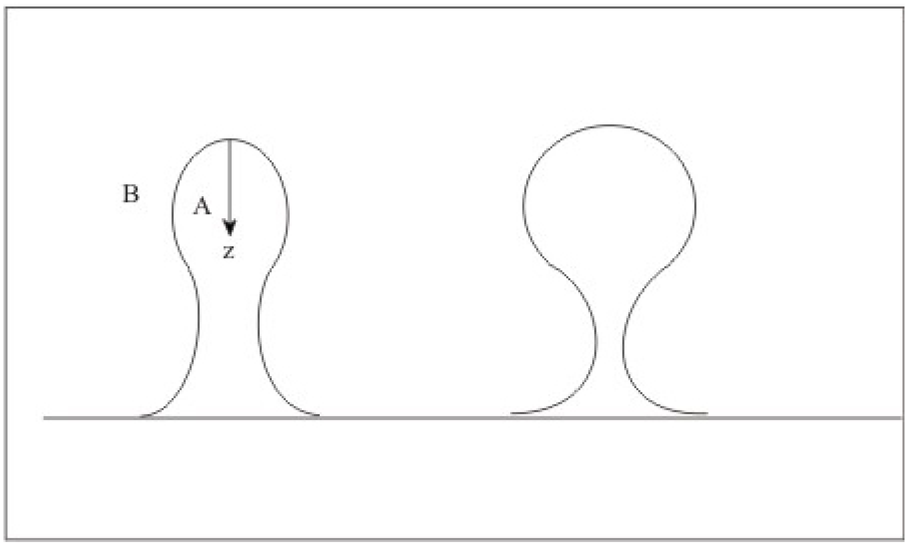

Figure:2.15. Necking

For a pendant bubble

$\rho_A < \rho_B$ (mirror image of the last case)

$\Delta\rho = \rho_B - \rho_A > 0$

Buoyancy would pull the drop upwards and a neck (fig:2.15) would form as the difference in density increases. If volume is more than a critical limit, the bubble would detach from the surface.

This phenomenon occurs only for contact angles $\theta < 90°$; hydrophobic surface.

Considering the case of a pendant drop, we take b as the length scale.

$$Z = \frac{z}{b}$$

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$$X = \frac{x}{b}$$

Using Young-Laplace equation,

$$2 + \left[\frac{\Delta\rho g b^2}{\gamma}\right] Z = \left[\frac{Z''}{(1+Z'^2)^{3/2}} + \frac{Z'}{X(1+Z'^2)^{1/2}}\right]$$

$$Z'' = \frac{d^2 Z}{dX^2}$$

$$Z' = \frac{dZ}{dX}$$

$\left[\frac{\Delta\rho g b^2}{\gamma}\right] \Rightarrow$ Non dimensional number which is denoted by $(\mathrm{Ca})^{-1}$

Where 'Ca' is called capillary number

If $(\mathrm{Ca})^{-1} \ll 1$ then we can neglect the second term on the left hand side.

STABILITY OF A WATER CYLINDER (NO FLOW)

The surface solution will be in the form of

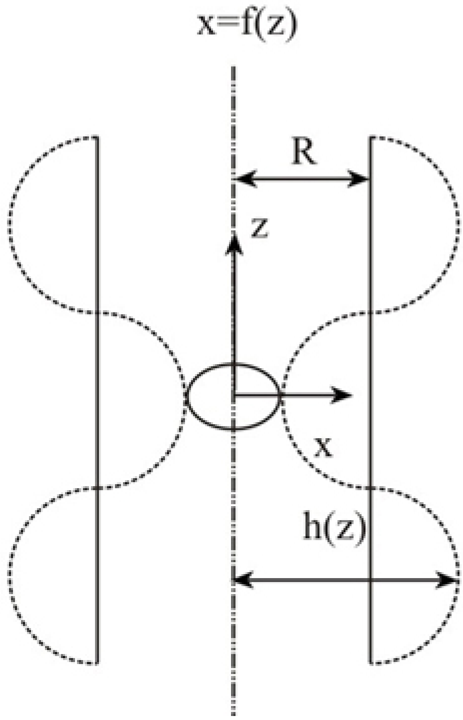

Fig 2.16

z = f(x)

For convenience we write the equation in the form x = f(z)

$$\frac{d^2z}{dx^2}=\frac{d}{dx}\left(\frac{df}{dz}\right)^{-1}=\frac{d}{dz}\left(\frac{df}{dz}\right)^{-1}\frac{dz}{dx}=-\left(\frac{df}{dz}\right)^{-2}\frac{d^2f}{dz^2}\frac{dz}{dx}=-\left(\frac{df}{dz}\right)^{-3}\frac{d^2f}{dz^2}$$

$$\frac{1}{R_1}=\frac{z"}{(1+z'^2)^{3/2}}=-\frac{x"}{(1+x'^2)^{3/2}}$$

$$\frac{1}{R_2}=\frac{z"}{x(1+z'^2)^{1/2}}=-\frac{x"}{x\,(1+x'^2)^{1/2}}$$

$$x=R+h(z)$$

$$\frac{1}{R_1}=-\frac{x"}{(1+x'^2)^{3/2}}=-\frac{h_{zz}}{(1+h_z^2)^{3/2}}$$

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if the slope is small

i.e. $h_z \ll 1$

The scale on which a film is spread is very large in comparison to the thickness scale.

$$\frac{1}{R_1} = -h_{zz} \text{ (for small slope)}$$

$$\frac{1}{R_2} = \frac{1}{(R+h)(1+x'^2)^{1/2}} \to 0 \text{ if } R \to \infty$$

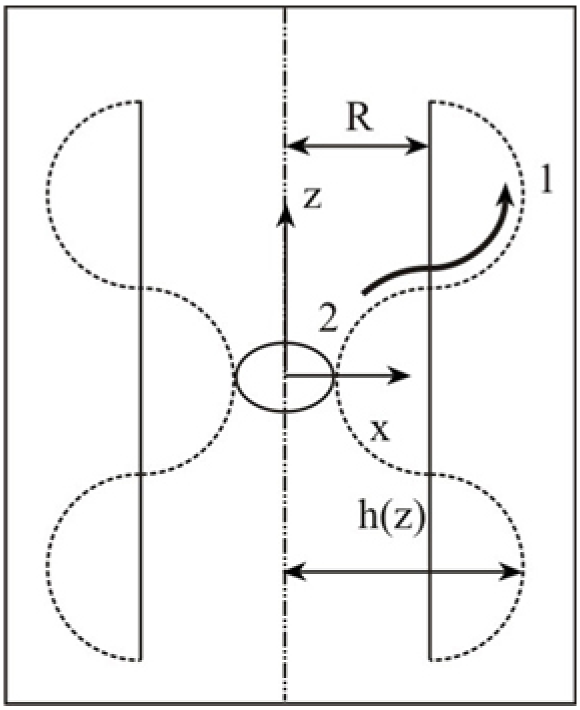

Fig 2.17:Disturbance of a water cylinder