The Lecture Contains:

Previous Next

Axisymmetric surfaces

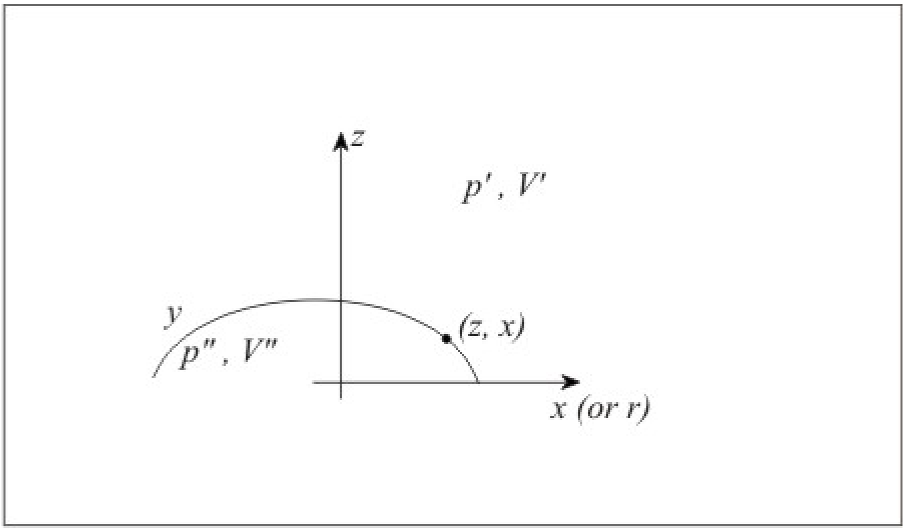

Figure 2.6. Axisymmetric surface (z is the axis of symmetry)

An axisymmetric surface is a surface obtained by rotating a curve around an axis

For an axisymmetric surface (Fig. 2.6),

$$dA = 2\pi x dl = 2\pi x\sqrt{(dx)^2 + (dz)^2} \qquad (2.22)$$

$$dV = (2\pi x dx)z \qquad (2.23)$$

Consider a liquid drop on a surface (Figure 2.6). $p', V', and\ T$ being constant, the Helmholtz free energy (F) is given as

$$F = U - TS + \sum_i \gamma_i A_i$$

Since, $U = -pV + TS + G$

We get,

$$\therefore F = -pV + \sum_i n_i\mu_i + \sum_i \gamma_i A_i$$

$$F = \sum \gamma_i A_i - pV - p'V' + const$$

$$V' + V" = V \Rightarrow V' = V - V"$$

$$F = \sum \gamma_i A_i - pV - p'(V - V") + c$$

Denoting $\Delta p = p' - p"$

$$F = \sum \gamma_i A_i - V''\Delta p$$

$$= \gamma \int_0^R 2\pi x \left[1 + \left(\frac{dz}{dx}\right)^2\right]^{1/2} dx + \int_0^R 2\pi x\,(\Delta p) z dx$$

Δp also includes the body forces.

Here, z as a funtion of x is not known.

F is a functional and variational calculus can be used to find its extrema.

For a functional,

$$F = \int_0^{\kappa} f\,(x, z, z_x) dx \tag{2.24}$$

the condition for equilibrium is given by

$$\frac{\partial f}{\partial z} - \frac{d}{dx}\frac{\partial f}{\partial z_x} = 0 \tag{2.25}$$

In the present example,

$$F = \gamma \int_0^{\kappa} 2\pi x\,(1 + z_x^2)^{1/2} dx + \int_0^{\kappa} \Delta p 2\pi x z dx$$

i.e. $f = \gamma 2\pi x\,(1 + z_x^2)^{1/2} + \Delta p 2\pi x z$

So, for extrema

$$\Delta p = p' - p" = \gamma \left[\frac{z_x}{x(1 + z_x^2)^{1/2}} + \frac{z_{xx}}{x(1 + z_x^2)^{3/2}}\right] \tag{2.26}$$

The first term inside the braces is $1/R_2$ while the other is $1/R_1$.

Previous Next

This is the ***Young-Laplace equation*** for axisymmetric surfaces. It is a non-homogeneous second-order non-linear differential equation. This equation may have a unique solution, no solution or many solutions. A frequently used simplification is that for weakly curved surfaces,

$$\left(\frac{dz}{dx}\right)^2 \ll 1$$

Some special cases

1. Homogeneous Young-Laplace equation $\Delta p = 0$

One possible solution (trivial solution) is a flat surface situation, i.e. R_1 & R_2 both going to infinity. Another solution is a thin film formed between two loops kept at a distance away (Fig. 2.7).

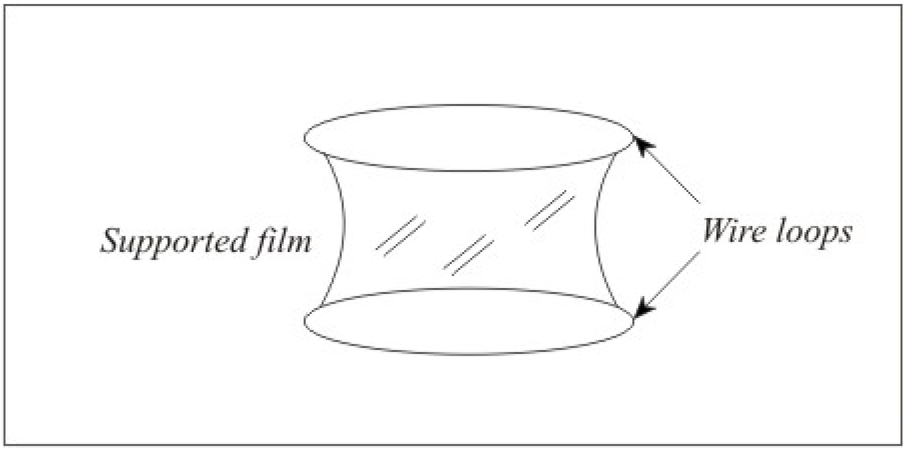

Figure 2.7. A thin film supported by two thin wire loops

2. $\Delta p = constant$

Then $R_1 = R_2 = R$. This means that the surface is spherical.

Previous Next

Young's Equation

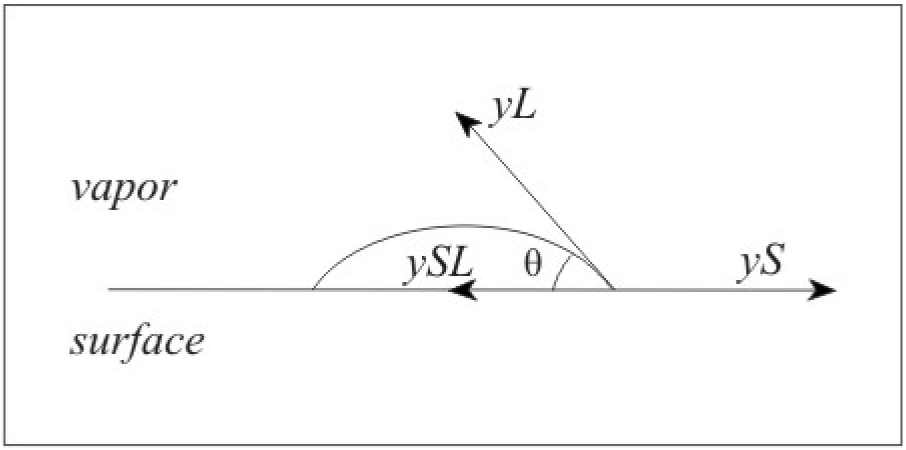

Figure 2.8. A liquid drop on a surface

$$F = \gamma_L \int_0^R 2\pi x\,(1+z_x^2)^{1/2}\,dx + \int_0^R \Delta p 2\pi xzdx + \gamma_{SL}\int_0^R 2\pi xdx + A\gamma_S - \gamma_S \int_0^R 2\pi xdx$$

$$f = 2\pi x\left[\gamma_L(1+Z_x^2)^{1/2} + \Delta pz + \gamma_{SL} - \gamma_S\right]$$

For equilibrium shape at the contact line

$$\left(f - z_x \frac{\partial f}{\partial z_x}\right)_{x=R} = 0$$

$$\gamma_L \frac{1}{(1+z_x^2)^{1/2}} + \gamma_{SL} - \gamma_S = 0 \qquad (2.27)$$

$$\gamma_L \cos\theta + \gamma_{SL} - \gamma_S = 0 \qquad (2.28)$$

This is *Young's Equation*. It represents the balance of interfacial tension in the horizontal direction. Along the vertical, the interfacial tension is balanced by the elastic forces of the underlying surface.

Module 2: "Capliarity"

Lecture 6: ""

The contact angle is a measure of the wettability of the surface by that particular liquid. Higher the contact angle lesser is the wettability. The extreme situations correspond to $\theta = 0°$ when it is perfectly wettable and $\theta = 180°$ when it is non-wettable. A mercury drop is an example of a non-wettable liquid. Therefore, a mercury drop tries to form a perfect sphere but the sphere gets slightly flattened due to gravity.

The Young-Laplace and Young's equation considered together give us the shape of a surface. However, these equations are extremely difficult to solve analytically. Fortunately, various simplifications are possible in most cases which makes it much easier to solve the equations.

$$\gamma_{23} \cos\theta + \gamma_{12} = \gamma_{13} \qquad (2.29)$$

$$\Delta P = \gamma \left[\frac{z'}{(1+z'^2)^{3/2}} + \frac{z'}{x(1+z'^2)^{1/2}} \right] \qquad (2.30)$$