

## Module 6: "Forces in Colloidal Systems"

### Lecture 30:

The Lecture Contains:

☰ Van der Waals energy of interaction between curved bodies

- Spherical particles
- Sphere and half space
- Parallel cylinders

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## Van der Waals energy of interaction between curved bodies

Below are presented some results on the van der Waals interaction energies for macroscopic bodies in various geometries<sup>1</sup>. The mathematical analysis is involved, so only final results are shown:

## Spherical particles

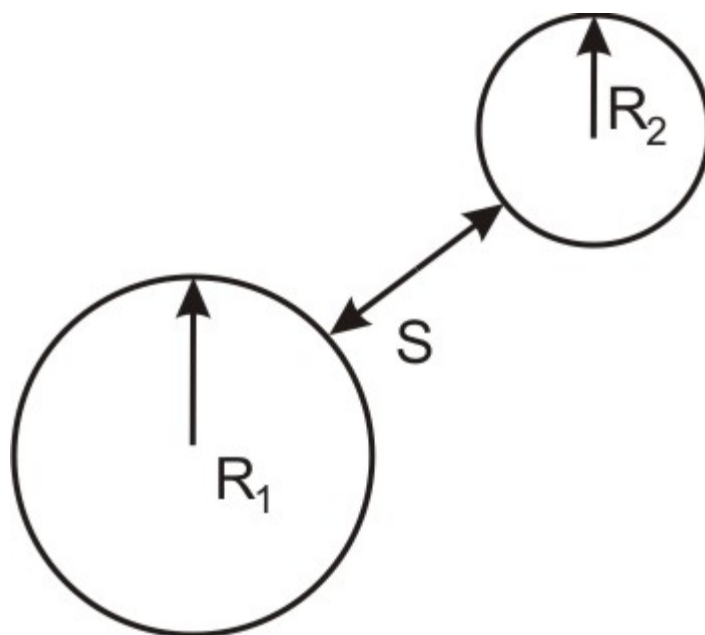


Fig. 8.15:

For two spheres of radii  $R_1$  and  $R_2$  separated by a closest separation distance  $s$  (see Figure 8.15), the interaction energy can be written as

$\Phi = \text{energy}$

$$= -\frac{A_{\text{Effective}}}{6} \left[ \frac{2R_1R_2}{s^2 + 2R_1s + 2R_2s} + \frac{2R_1R_2}{s^2 + 2R_1s + 2R_2s + 4R_1R_2} + \ln \left( \frac{s^2 + 2R_1s + 2R_2s}{s^2 + 2R_1s + 2R_2s + 4R_1R_2} \right) \right]$$

*Vacuum*,  $A_{\text{Effective}} \rightarrow A_{12}$

*Medium*,  $A_{\text{Effective}} \rightarrow A_{132}$

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Apart from  $A$ ,  $\phi$  depends on  $R_1$ ,  $R_2$  and  $s$ . It can be shown using simple algebraic manipulation that these variables can be reduced to a smaller number:

$$\phi = \phi\left(\frac{s}{R_1}, \frac{s}{R_2}, A_{132}\right)$$

This implies that if  $s/R_1$  and  $s/R_2$  are constant for different particle system then  $\phi$  remains constant.

In real systems, whether or not this energy of interaction is sizeable depends on its relative magnitude as compared to the thermal energy per Brownian particle,  $kT$ . If  $\phi \ll kT$  then particles do not remain bound to each other for long through the action of LW forces

In the limit  $R_1, R_2 \gg s$  the expression reduces to a far simpler form:

$$\Rightarrow \phi = \frac{-AR_1R_2}{6s(R_1 + R_2)}$$

While in the limit  $R_1, R_2 \ll s$ :

$$\phi = -\frac{16AR_1^3R_2^3}{9s^6}$$

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## Sphere and half space

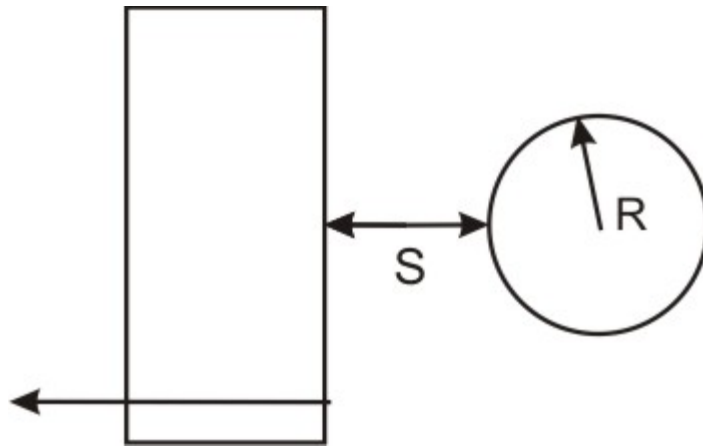


Fig. 8.16:

In the limit  $R \gg s$

$$\Phi = -\frac{AR}{6s} \left[ 1 + \frac{s}{2R + s} + \dots \right]$$

In the limit  $R \ll s$   $\Phi \sim -\frac{1}{s^3}$  which is similar to that between a single molecule of one medium with a semi-infinite slab of the other as derived earlier.

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Parallel cylinders

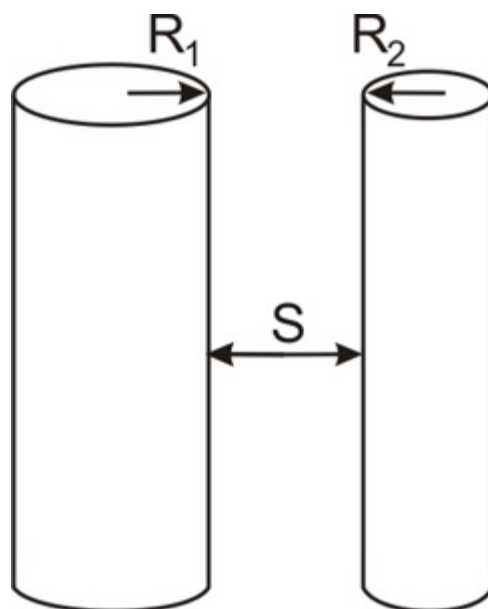


Fig. 8.17:

In the limit  $R_1, R_2 \gg s$  :

$$V = - \left[ \frac{R_1 R_2}{R_1 + R_2} \right]^{0.5} \frac{A}{12\sqrt{2}s^{1.5}}$$

where  $V$  is the interaction energy potential per unit length of the cylinders

In the limit  $R_1, R_2 \ll s$  :

$$V \sim - \frac{\text{Length}}{s^5}$$