## The Lecture Contains:

## Spherical Surface

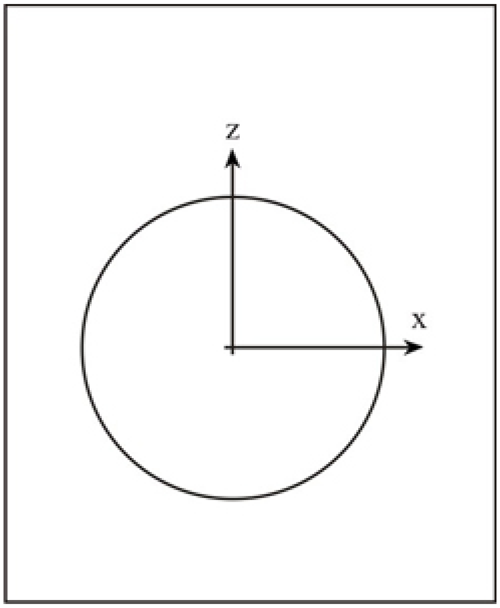


Figure 2.11: A spherical surface

$z^2 + x^2 = R^2$ (fig: 2.11)

$z \geq 0 \rightarrow z = \sqrt{R^2 - x^2}$

$z \leq 0 \rightarrow z = \sqrt{R^2 - x^2}$

Differentiating twice

$1 + z'^2 + zz" = 0$

$$\frac{1}{R_1} = \frac{z"}{(1+z'^2)^{3/2}} = -\left[\frac{1+z'^2}{z(1+z'^2)^{3/2}}\right] = -\frac{1}{z(1+z'^2)^{1/2}} = \frac{z'}{x(1+z'^2)^{1/2}}$$

$$\frac{1}{R_1} = \frac{1}{R_2}$$

= if $\Delta \mathrm{p}$ is constant then a spherical surface can be a solution.

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## Cylindrical Surface

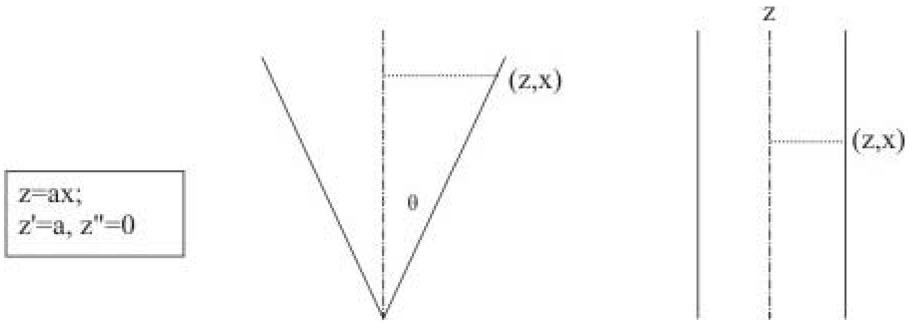


Fig 2.12. A conical surface and a cylindrical surface

This is a limiting case of a conical surface.(fig:2.12)

$$\frac{1}{R_1} = 0,$$

$$z' = a$$

$$\Rightarrow \frac{1}{R_2} = \frac{a}{x(1+a^2)^{\frac{1}{2}}}$$

Further

$$x \to \infty$$

$$\Rightarrow R_2 \to 0$$

$$As\; a \to \infty$$

$$\frac{1}{R_1} = \frac{1}{x}$$

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## Equilibrium shape of a drop on a solid surface (bubbles and drops not moving and axisymmetric)

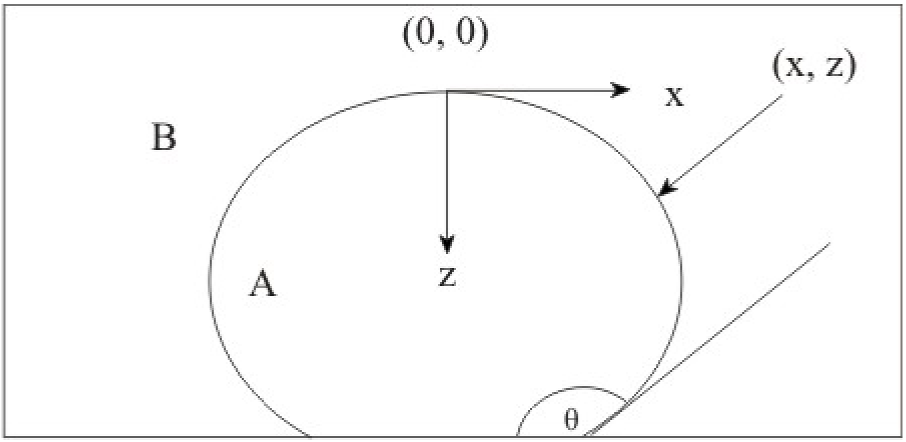


Figure:2.13. Sessile Drop

$\rho_A > \rho_B$

Replacing $z$ by $-z$ in the Young-Laplace equation

Initially (fig:2.13)

At (0,0)

$p = p_{A_0}$

Now

$\Delta p = p_A - p_B$

$p_A = p_{A_0} + \rho_A g z$

$p_B = p_{B_0} + \rho_B g z$

$p_{A_0} \neq p_{B_0}$ (Local curvature $\rightarrow$ jump in pressure)

$p_A - p_B = \left(p_{A_0} - p_{B_0}\right) + (\rho_A - \rho_B) g z$

Any arbitrary curve can be represented by a quadratic relation in the neighborhood of local minima /maxima

$$z = ax^2 + \theta(-----)$$

There would be no odd power term because its presence would cause a loss of symmetry around the $z-\text{axis}$.

$$z = ax^2$$

$$As\ x \to 0$$

$$\frac{1}{R_1} = 2a$$

$$\frac{1}{R_2} = 2a$$

$$\mathrm{R_1 = R_2 = b}$$

$$p_{A_0} - p_{B_0} = \frac{2\gamma}{b}$$

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With an increase in volume, the distortion due to gravity becomes more pronounced.

$$\gamma\left[\frac{z'}{(1+z'^2)^{3/2}}+\frac{z'}{x(1+z'^2)^{1/2}}\right]=\frac{2\gamma}{b}+(\rho_A-\rho_B)gz$$

Boundry Conditions

at $x = 0, z' = 0$

$x = 0, z = 0$

Solving piece-wise up to the point where

$$\frac{dz}{dx} = \tan\theta\,;$$

$\theta \Rightarrow$ Contact angle for the system

Now we can calculate $V$ (volume of the drop)

$$V = \int_0^h \pi x^2 dz$$

We stop integration at the point where the angle becomes equal to the equilibrium contact angle given by Young’s Equation.

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