Module 2: "Capliarity"

Lecture 5: ""

The Lecture Contains:

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Capillarity

Capillarity is defined as the degree to which a material or object containing minute openings or passages, when immersed in a liquid, will draw the surface of the liquid above the hydrostatic level. We can use the concepts of capillarity to predict the equilibrium shape of an interface.
Consider a bubble (Fig. 2.4).

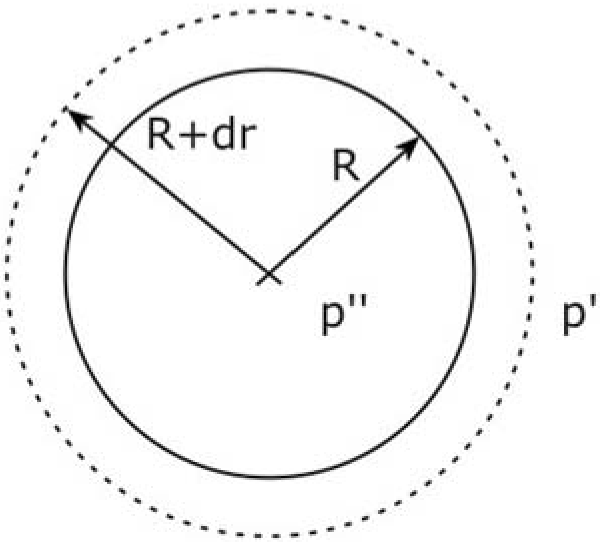

Figure 2.4. A bubble in equilibrium with its surroundings

p'' is the pressure in the dispersed phase

p' is the pressure in the surrounding medium and,

R is the radius of the bubble

At the interface, the pressures (p'' and p') on two sides are equal only for flat surfaces, i.e. when R goes to infinity. For a curved surface, these will not be equal.

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When the bubble surface is stretched, net work done in the process gets stored as the interfacial energy.

$$\text{Work done} = (p" - p')dV$$

$$= \Delta p d\left(\frac{4}{3}\pi R^3\right)$$

$$= \Delta p 4\pi R^2 dR \qquad (2.18)$$

$$\Delta\,(Interfacial\,energy) = \gamma d\,(4\pi R^2) = 8\pi R\gamma dR \qquad (2.19)$$

From Eq.(2.18) and (2.19) we get,

$$\therefore\ \Delta p = p" - p' = \frac{2\gamma}{R} \qquad (2.20)$$

When γ is nearly zero or R approaches infinity, the pressure difference is nearly zero.
But interfacial tension becomes important when R becomes small.

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For any surface, Eq. (2.20) can be rewritten in a more generalized form as

$$\Delta p = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{2.21}$$

Here, *R1* and *R2* are the principal radii of curvature.

Mathematical derivation

$$z = f(x, y)$$

$$dA = (x + dx)(y + dy) - xy$$

$$\cong xdy + ydx + \text{higher order terms}$$

$$\Delta(\text{surface energy}) = \gamma(xdy + ydx)$$

$$\text{Work done} = \Delta p\, x\, y\, dz$$

$$\Rightarrow \Delta p\, x\, y\, dz = \gamma\,(xdy + ydx)$$

Using the properties of similar triangles we get,

$$\frac{x + dx}{R_1 + dz} = \frac{x}{R_1}, \text{ and}$$

$$\frac{y + dy}{R_2 + dz} = \frac{y}{R_2}$$

$$\Rightarrow dx = \frac{xdz}{R_1}, \text{ and}$$

$$dy = \frac{ydz}{R_2}$$

$$\therefore\ \Delta pxyz = xy\gamma dz \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$or\ \Delta p = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Stabilizing role of Surface Tension

In Fig. 2.5, from the shape of the curves at 1 and 2, and by using eqn. 2.21 ,we can see that $p_1 > p_g$ and $p_g > p_2$. Therefore $p_1 > p_2$. Due to this pressure gradient, the fluid will flow from 1 to 2 and this restores the equilibrium shape of the surface.

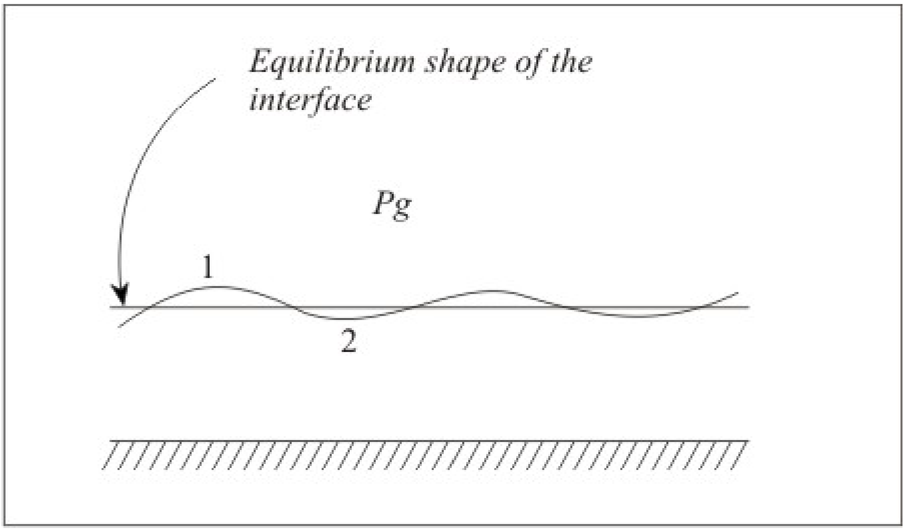

Figure 2.5. Stabilizing effect of surface tension

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