Module 4: Fluid Dynamics

Lecture 9: Lagrangian and Eulerian approaches; Euler's acceleration formula

- Fluid Dynamics: description of fluid-motion
- Lagrangian approach
- Eulerian approach (a field approach)

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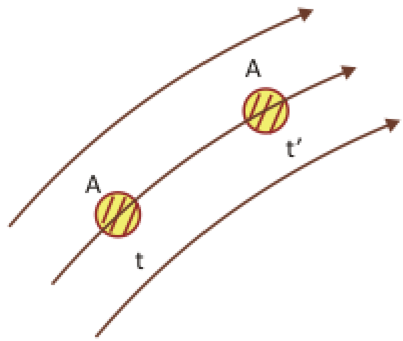
Fluid Dynamics: description of fluid-motion

Consider 2D flow of a fluid.

$\overline{V} = iV_x + jV_y$



(Fig. 9a)

There are two approaches to describe the motion of a fluid and its associated properties.

1. Lagrangian approach
2. Eulerian approach

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Lagrangian approach:

Identify (or label) a material of the fluid; track (or follow) it as it moves, and monitor change in its properties. The properties may be velocity, temperature, density, mass, or concentration, etc in the flow field.

Refer the above-figure. The 'material' or 'particle' of the fluid 'A' at time t has moved to some other location at time t'. Its property, say temperature, is recorded, as the material moves in the flow-field:

$$\left.\begin{matrix} t_1 \rightarrow T_1 \\ t_2 \rightarrow T_2 \\ t_n \rightarrow T_n \end{matrix}\right]$$

Note that the recorded temperatures are associated with the same fluid particle, but at different locations and at different times.

Think of a temperature sensor attached to a balloon, both having negligible mass and floating in the atmosphere and recording the atmosphere-temperature or the temperature of the flow-field. In such case, the following temperature-data are recorded by the sensor:

Location	time	temperature
$(x_1 y_1 z_1)$	t_1	T_1
(x_2, y_2, z_2)	t_2	T_2
$\vdots$	$\vdots$	$\vdots$
(x_n, y_n, z_n)	t_n	T_n

The time change of the temperature in such a measurement is denoted as $\frac{DT}{Dt}$, which is called material derivative or substantial derivative. It reflects time change in the temperature (or any other properties) of the labeled /marked/tagged fluid particles as observed by an observer moving with the fluid. Lagrangian approach is also called "particle based approach".

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Eulerian approach (a field approach)

Identify (or label) a certain fixed location in the flow field and follow change in its property, as different materials pass through that location. In such case, the following property, say temperature is recorded by the sensor :

$$\left.\begin{matrix} t_1 \to T_1 \\ t_2 \to T_2 \\ t_n \to T_n \end{matrix}\right]$$

Note that the recorded temperatures are associated with the fixed location in the flow-fluid, having different fluid elements at different times.

The time- change of the temperature in such a measurement is denoted as $\left.\frac{\partial T}{\partial t}\right]_{(x,y,z)}$ which is called the partial derivativeof the temperature with respect to time. Note that the suffix (x,y,z) implies that the observer records the change in the property at the fixed location (x,y,z).

$\left(\frac{\partial T}{\partial t}\right)$ is also called the local rate of change of that property (temperature in this case).

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The two derivatives are related to each other:

$$\boxed{\frac{\partial n}{\partial t} = \frac{Dn}{Dt} - (V.\nabla)\eta}$$

where, $\eta =$ property (temperature, concentration, velocity, etc)

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\vec{V} = iV_x + jV_y + kV_z$$

$$\frac{\partial n}{\partial t} = \frac{Dn}{Dt} - \left(V_x\frac{\partial \eta}{\partial x} + V_y\frac{\partial \eta}{\partial y} + V_z\frac{\partial \eta}{\partial z}\right)$$

If $\eta = \vec{V}$

$\boxed{\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}.\nabla)\vec{V}}$: Euler's acceleration formula.

$\frac{D\vec{V}}{Dt}$ may be recognized as acceleration or particle-acceleration. It represents a physical acceleration. For example, one can write

$P = ma = m\frac{D\vec{V}}{Dt}$ (Newton's 2nd law of motion)

All it means is that the Newton's 2nd law of motion, which is generally applied for a solid object, can also be applied or written for a fluid element or fluid flow! It also follows that

$\frac{Dm}{Dt} = 0 \Longrightarrow$ Physical mass cannot be created or destroyed !!

We will see later, $\frac{\partial m}{\partial t}$ can be, however, zero or non-zero.

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