Module 7: Energy conservation

Lecture 21: Minor loss, kinetic energy correction factor

- Energy balance equation: Minor losses

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Energy balance equation: Minor losses

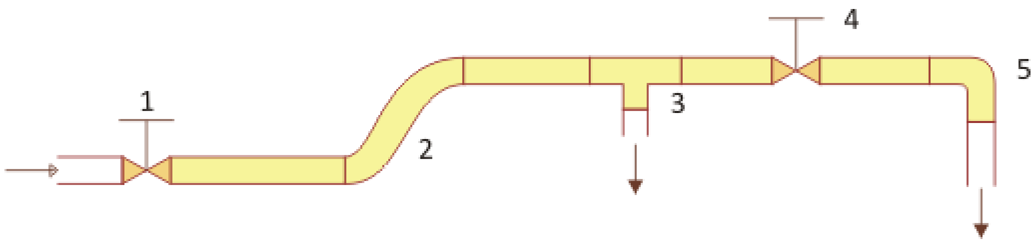
The energy balance equation contains a term, 'w_l' which represents irreversible conversion of viscous work to thermal energy and is always '+ ve':

$$\left[\frac{V^2}{2}+gz+\frac{p}{\rho}\right]_0=\left[\frac{V^2}{2}+gz+\frac{p}{\rho}\right]_i-w_l \text{ (neglecting any shaft work)}$$

(Note: it has been shown that $w_l \equiv \mu(\nabla V)^2$

Minor losses refer to the losses in pipe bends, fittings, valves, and nozzles, etc, whereas major losses refer to the same in a long pipe or tube. We will first minor losses.

Example1 Consider the following system of pipe flow, consisting of several fittings such as pipe bends, and valves:



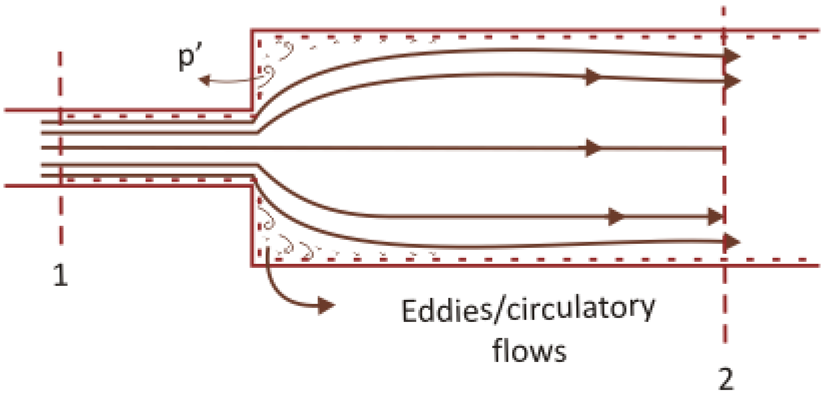
(Fig. 21a)

As the fluid flows through pipe and associated fittings, including valves, there is invariably loss of mechanical energy, which may be mathematically quantified as: $K\left(\frac{\overline{V}^2}{2}\right)$, where, $\overline{V}$ is often calculated at the inlet to the fitting or valves and K is known as loss coefficient and depends on Reynolds number and type of fittings. The manufacturers of valves and fittings provide 'K' value. For example:

Fitting	K
$45^\circ-\text{bend}$	0.4
$90^\circ-\text{bend}$	0.9
Tee	0.4
Globe - valve (fully-open)	10
half open	20
Gate- value (fully- open)	0.3
half open	5

In general, loss in the globe valves is more than that in gate valves, because of the fact that fluid changes its direction sharply in the former. The globe valves are preferred over gate valves because of better flow-control.

Losses also occur in flange or weld joints connecting two pipes or fittings. Losses are also accounted for in diverging or converging nozzles or expanders, or where there are geometrical changes:



(Fig. 21b)

Apply mechanical energy balance between 1 and 2:

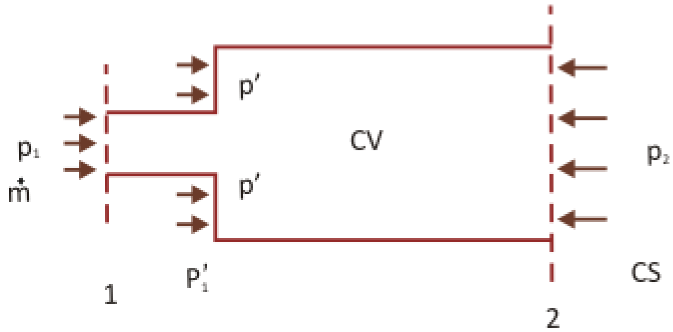
$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + w_l \qquad (\text{J/kg})$$

(assuming velocity profiles are uniform over the cross- sections at 1 and 2)

One can also apply continuity and momentum-balance equations between '1' and '2':

(1) $\rho\, v_1\, A_1 = \rho\, v_2\, A_2 = \dot{m}$

(2) $p_1\, A_1 + p'\,(A_2 - A_1) - p_2\, A_2 = \dot{m}\,(V_2 - V_1)$ (CV is marked with dotted line)



(Fig. 21c)

It is an experimental observation that $p_1^{'} \simeq p_1$.

Therefore, $(p_2 - p_1) = \text{gain in pressure}$

$$= \rho V_2(V_1 - V_2)$$

Or,

$$w_l = \frac{(V_1 - V_2)^2}{2} = \frac{V_1^2}{2}\left[1 - \frac{A_1}{A_2}\right]^2$$

(It is left as an exercise to the students to obtain the above–derivation from combined mass, moments, and energy conservations)

Thus, we have shown that,

$$w_l = K\left(\frac{V_1^2}{2}\right), \text{where } K = \left[1 - \frac{A_1}{A_2}\right]^2$$

For a converging channel, viscous loss, w_l is also estimated as

$$= \frac{V_1^2}{2}\left[(A_2/A_c)^2 - 1\right] = K\left(\frac{V_2^2}{2}\right)$$

Note that V_2 is the velocity in the converging section and A_c is the cross-sectional area at the section where “Vena-Contracta" (or, the minimum flow area) occurs:

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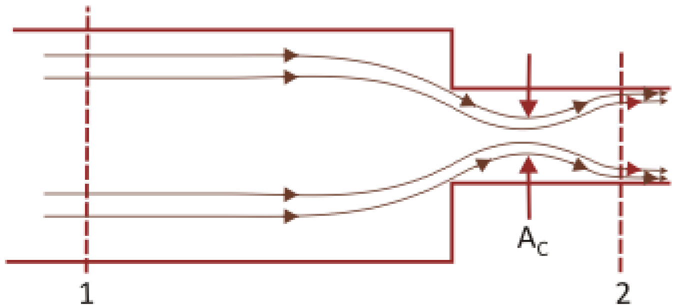


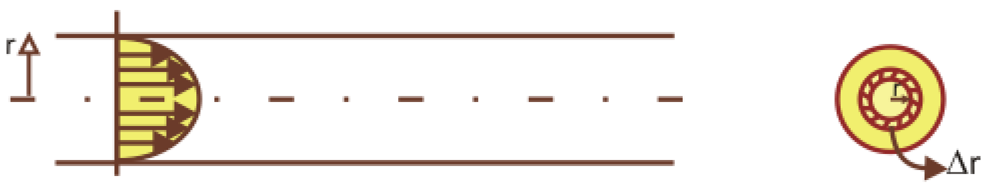
(Fig. 21d)

We revert to “Vena–Contracta” later in the course.

Before we take–up the topic on flow measuring devices, which are related to the application of mechanical energy balance, we address ***kinetic energy correction factor:***

Consider the flow of viscous fluid in a tube. The flow is under laminar conditions and the velocity profile is parabolic:

$$V = V_{max}\left(1 - \frac{r^2}{R^2}\right);\ \overline{V} = \frac{V_{max}}{2} = Q/\pi R^2$$



(Fig. 21e)

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We are interested in evaluating total KE across the section:

$$\frac{1}{2}\int_o^R \underbrace{(v(r)2\pi\, rdr)}_{dQ}\rho\, V_{(r)}^2$$

One can also evaluate KE based on the average velocity, $\overline{V}$ as

$$\frac{1}{2}\left(\overline{V}\, A\, \rho\right)\overline{V}^2$$

where $A = \pi R^2, \overline{V} = \frac{V_{max}}{2}$

Kinetic energy correction factor is defined as

$$\left(\frac{\text{Total KE across A}}{\text{KE based on average velocity}}\right)$$

or

$$Y = \frac{\frac{1}{2}\int_o^R (v(r)2\pi\, rdr)\rho\, V(r)^2}{\frac{1}{2}\left(\overline{V}\, A\, \rho\right)\overline{V}^2}$$

It is left as an exercise to show that $Y = 2$

If the flow is turbulent, Y is approximated between $1.01 \sim 1.15$.

The advantage of defining $'Y'$ is now clear. For a tubular laminar flow, one can write the energy conservator equation in terms of average velocity, $\overline{V}$, without evaluating the complex integral:

$$\left(\frac{p}{\rho} + \underset{\uparrow}{Y}\frac{\overline{V}^2}{2} + gz\right) = \text{constant}$$

KE correction factor

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