Module 8: Flow at low and high Reynolds numbers

Lecture 27: Flow at high Reynolds numbers (Boundary layer theory)

- Flow at high Reynolds number

Flow at high Reynolds number

(Boundary layer theory)

Re-visit the non–dimensionalized NS equation:

$$\boxed{V_x^* \frac{\partial V_{x^*}}{\partial X^*} = -{}^{1}/_{Eu}\ \nabla^* p^* + {}^{1}/_{Fr}\ g^* + \frac{1}{Re}\nabla^{*2}\ V_x^{*2}}$$

At high Reynolds number, viscous effects are considered negligible in the flow away from a solid surface. In other words, fluid is supposed to be inviscid or $\mu \rightarrow o$.

In such case the equation is simplified to

$$\boxed{V_x^* \frac{\partial V_{x^*}}{\partial X^*} = -{}^{1}/_{Eu}\ \nabla^* p^* + {}^{1}/_{Fr}\ g^*}$$, or in the dimensional form

$$\rho\ \vec{V}.\nabla\ \vec{V} = -\nabla p + \rho\vec{g}$$

Neglecting the gravitational effect, one can state that the inertial force is balanced by pressure force. Such flow is termed as potential flow.

Assuming 1-D flow, the equation can be integrated to obtain

$$P/\rho + \frac{V^2}{2} + gz = const$$

Re-call. We have previously obtained the above equation as Bernoulli’s equation.

Previous Next

- It is important to note that the potential theory or inviscid flow–condition predicts ‘zero–drag’ on a solid surface. The common examples of potential flow are source/sink, free vortex, doublet, and Rankine’s half–body. We skip the analysis of such flows.

- To overcome the paradox of zero drag on a solid surface, Prandtl (1903) came–up with the boundary–layer theory. As per the main postulate of the theory, the viscous term cannot be dropped from the NS equation for the high–Reynolds number flow. Physically, viscous effects are important at or near the solid surface, even at high Reynolds number.

- By choosing the appropriate characteristic variables, it can be shown that one of the viscous terms, namely $\mu \frac{\partial^2 v_X}{\partial Y^2}$ must be retained. This is possible if we assume that the characteristic length in y–direction is much smaller than that in the x–direction.

Re–defining variables as $X^* = X/L$ and $Y^* = {}^{Y}/_{\delta}$

where, $\delta << L$, one obtains the following dimensionless form of the NS equation:

$$V.^* \nabla^* V^* = -\nabla^* p^* + \frac{1}{Re,_L}\left(\frac{\partial^2 V_x{}^*}{\partial Y^{*2}}\right) + \frac{1}{Re,_L}\left(\frac{L}{\delta}\right)^2 \frac{\partial^2 V_x{}^*}{\partial Y^{*2}} + g^*$$

At the limit of $Re,_L \rightarrow \infty$, the 1st viscous term is neglected. However, the 2nd term is retained because

$${}^{1}/_{Re,_L}\left(\frac{L}{\delta}\right)^2 \simeq 1$$

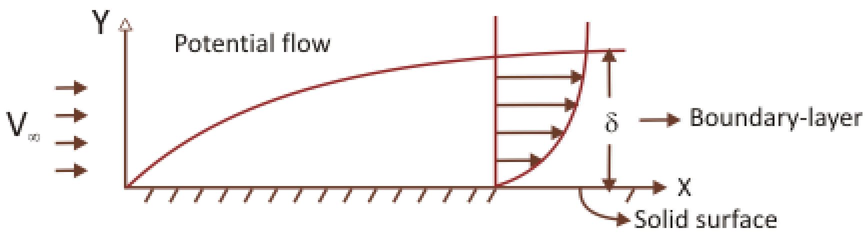
$${}^{\delta}/_{L} \simeq \sqrt{{}^{1}/_{Re,_L}}$$

The dimensional form of the NS equation (for x-direction) may now be written for boundary–layer flow (at high Reynolds number) as:

$$\rho\left(V_X \frac{\partial V_X}{\partial X} + V_Y \frac{\partial V_X}{\partial Y} = \right) - \frac{\partial p}{\partial X} + \mu \frac{\partial^2 V_{\dot{X}}}{\partial Y^{\cdot 2}} + \rho g_X \quad : X - \text{momentum}$$

Previous Next

Blausius has proposed an approximate solution to the above equation for the flow near the solid surface. From the solution, $V_X(y)$, the gradient $\left.\frac{\partial v_X}{\partial Y}\right|_{Y=o}$ at or near the surface may be calculated to obtain drag $\tau_w = \mu \left.\left(\frac{\partial v_X}{\partial Y}\right)\right|_{Y=o}$



(Fig. 27a)

For the flow over a flat plate, the local shear stress has been calculated as

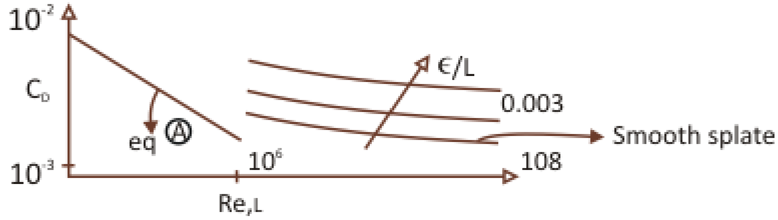
$$\tau_w(x) = 0.332 \frac{\rho V_\infty^2}{\sqrt{Re_{,x}}}$$

where, $Re_{,x} = \frac{V_\infty X \rho_f}{\mu_f}$

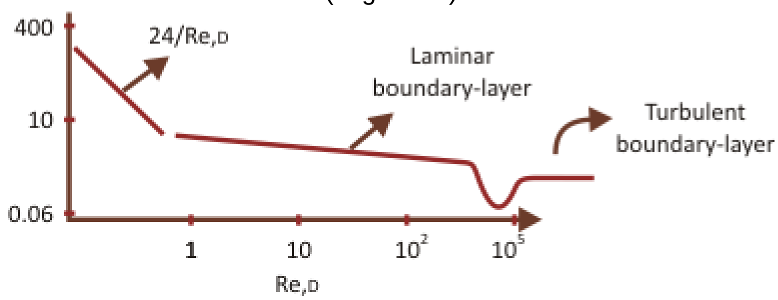
Defining drag coefficient, C_D as $\frac{F_D/A}{\frac{1}{2}\rho V_\infty^2}$, where F_D is the drag on the plate (or any solid surface) and 'A' is the plate-area, C_D is calculated as

$$\frac{1.328}{\sqrt{Re_{,L}}} \text{ -----(A)}$$

At higher Reynolds number, turbulence is observed. For a wide-range of Reynolds number, plots are available to calculate C_D for the flow over a flat plate or past a sphere.



(Fig. 27b)



(Fig. 27c)

In general,

$F_D = C_D A_P \frac{1}{2} \rho v_\infty^2$, where C_D, the drag–coefficient can be read from the plots. Empirical equations are also available.

- There are several features of fluid flow at high Reynolds number, such as boundary layer separation, eddies, and adverse pressure gradients. These topics and the other related topics such as bluff body, streamline body, Von–Karman Street, etc, are excluded from discussion in this course.

Previous Next