Module 2: Fluid Statics

Lecture 5: Pressure variation in two immiscible fluids, manometer, barometer

- Pressure- variation in two-immiscible liquids (one is heavier than the other)
- Barometer (instrument to measure atmospheric pressure)
- Monometer (to measurer pressure of the fluid in a device or container)

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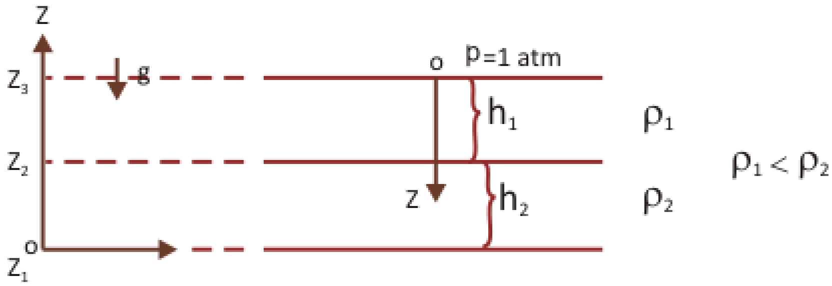
(2) Pressure- variation in two-immiscible liquids (one is heavier than the other)



(Fig. 5a)

Consider two liquids of densities $\rho_1 \& \rho_2$ $(\rho_1 < \rho_2)$. Depth (measured form the top) of the top-fluid is h_1 and that of the bottom-fluid is $(h_1 + h_2)$

Put the co-ordinate axis at the bottom (left). The governing equation for the static fluid is

$-\nabla p + \rho\vec{g} = 0 \quad \vec{g} = -k\hat{g}$

or $\dfrac{dp}{dz} = -\rho g$

On integration, $p(z = z_2) = p(z = z_1) - \rho_2 g(z_2 - z_1)$

Similarly, $p(z = z_3) = p(z = z_1) - \rho_1 g(z_3 - z_2)$

or $p\ (z = z_3) = 1\ \text{atm} = p\ (z = z_1) - \rho_2 g h_2 - \rho_1 g h_1$

One can get similar expression by putting the co-ordinate axis at the top of the surface, with z-axis pointing vertically downward (right):

$-\nabla p + \rho\vec{g} = 0 \quad \vec{g} = -k\hat{g}$

or, $\dfrac{dp}{dz} = -\rho g$

On integration $p\ (h_1) = p_{\ atm} + \rho_1 g h_1$

Similarly, $p\ (h_2) = p\ (h_1) + \rho_2 g h_2$

or, $p(h_2) = p_{atm}(1\ atm) + \rho_1 g h_1 + \rho_2 g h_2$

Note: Both expressions are same!

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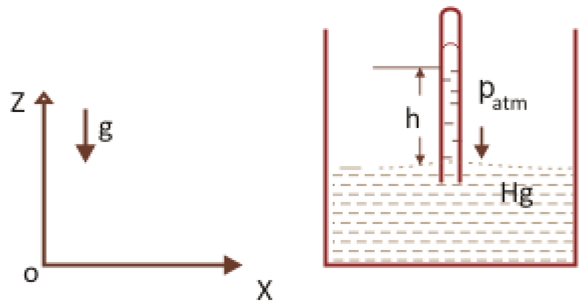
(3) Barometer (Instrument to measure atmospheric pressure)



(Fig. 5b)

- Neglect surface tension or capillary effects
- Height of the liquid in the vertical tube is h measured from the free surface.

$\boxed{-\nabla p + \rho\vec{g} = 0}$ $(\rho = \rho_{Hg})$ $\vec{g} = -k\hat{g}$

$$\frac{dp}{dz} = -\rho g$$

On integration, $p(h) = p(z = 0) - \rho g h$

But, at the same level $\frac{dp}{dx} = \frac{dp}{dy} = 0$ and $p(z = 0) = p_{atm}$

Therefore, $p(h) = p_{atm} - \rho g h$. $p(h)$ is nothing but the vapor pressure of Hg in the space over the liquid Hg in the tube. Generally, $p(h) = p_y \sim 0$ (very small at Room temperature). Hence, $p_{atm} = \rho g h$. One can experimentally measure to show that

$$\left.\begin{array}{c} h \simeq 752 \text{ mm of Hg} \\ \text{or } 10.4 \text{ m of water} \\ \text{at sea level.} \end{array}\right\}$$

There are two problems in using water-barometer

1. Tube is long!
2. Vapor pressure of water is significantly larger than that of Hg, i.e., $p_v \neq 0$ and should be considered in the calculation!!

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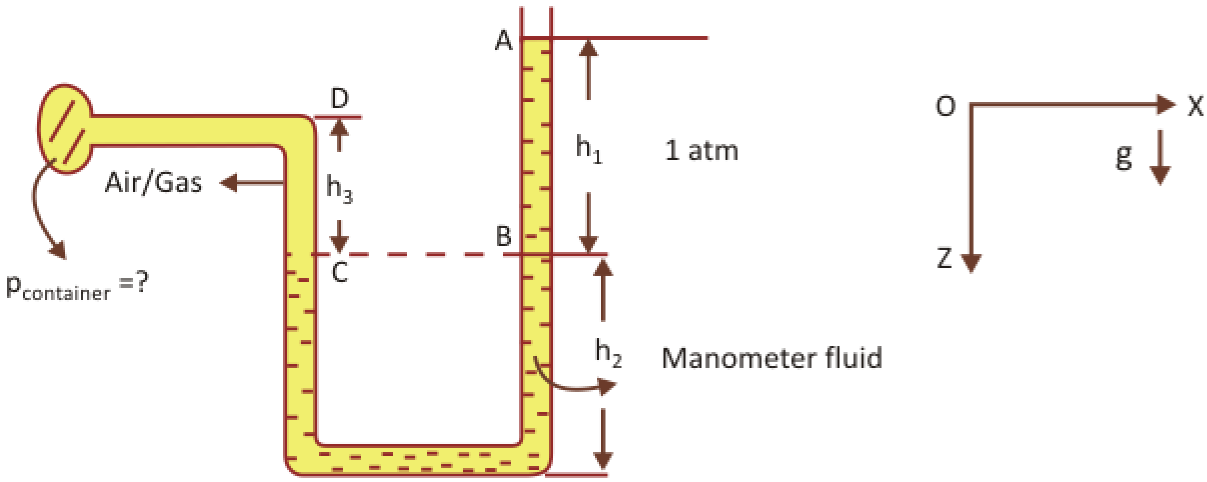
(4) Monometer (To measure pressure of the fluid in a device or container)



(Fig. 5c)

$$-\nabla p + \rho\vec{g} = 0$$

$$\frac{dp}{dz} = -\rho g \qquad \vec{g} = -k\hat{g}$$

On integration between A & B

$$p_B = p_A + \rho g h_1 = 1\text{ atm} + \rho g h_1$$

$$P_B = p_C \text{ because } \frac{dp}{dx} = 0$$

On integration between C and D

$$p_C = p_D + \rho_{air}\, g\, h_3$$

But, $p_D = p_{container}$

Therefore, $1\,atm + \rho g h_1 = p_{container} + \rho_{air}\, g h_3$

Therefore, $p_{container} = 1\,atm + \rho g h_1 - \rho_{air}\, g h_3$

However, $p_{container} = \rho g h_1 \; if \; \rho_{air} \ll p_{manometer\,fluid} \; and \; h_1 > h_2$

(NOTE $1\text{ atm} = 1.013 \times 10^5$ Pa at sea - level)

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