Module 3: Hydrostatic forces on submerged bodies

Lecture 6: Calculation of vertical component

- Hydrostatic forces on submerged surfaces
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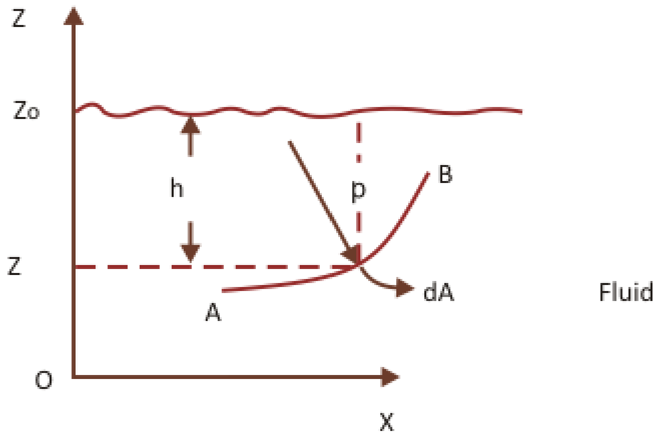
Hydrostatic forces on submerged surfaces

Consider a surface AB of an object submerged in a liquid (not necessarily water; the name hydrostatic force is, therefore, misnomer!). Top of the liquid-surface is exposed to atmosphere. The co-ordinate axis has been placed at the bottom of the liquid.



(Fig. 6a)

Pressure 'p' acts normal to the elemental surface, dA at a distance 'z'

$d\vec{F}$ (differential force) acting on the surface

$$= p\, d\vec{A}$$

$$= (p_{atm} + \rho g h) d\vec{A}, \quad \left.\begin{matrix} \text{where } h = z_o - z \\ = \text{depth of the surface} \end{matrix}\right\}$$

$$= (p_{atm} + \rho g h)\, \hat{n}\, dA,$$

$d\vec{F_g} = \rho g h\, \hat{n}\, dA$. This is the force acting only due to hydrostatic (liquid) pressure. Therefore,

$$\boxed{\vec{F_g} = \int_A \rho g h\, \hat{n}\, dA}$$

.Such direct integration can be performed only for special (well-defined) geometries such as an arc of a circle because the normal to all arcs of the circle will pass through the center of the circle, or a horizontal surface because in such case $\hat{n} = -g\,\hat{k}$.

It is easier to work on the vertical and horizontal components of the force separately.

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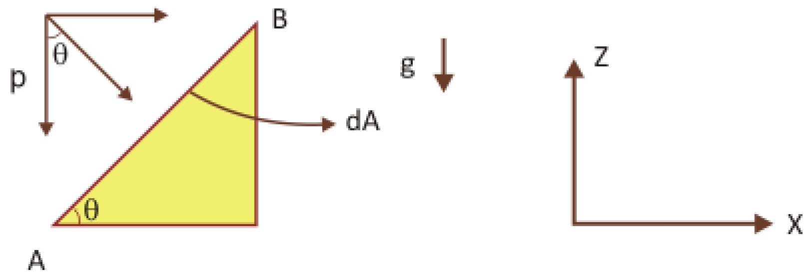
Lecture 6: Calculation of vertical component

Vertical component

Re-consider ‘AB’



(Fig. 6b)

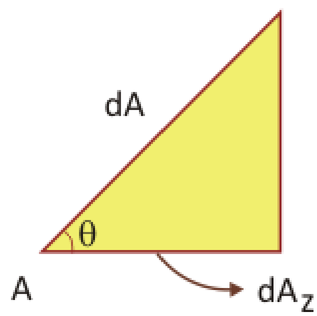
The vertical force

$d\overrightarrow{F_z}$ (in the positive direction of Z)

$= d\overrightarrow{F_g}\ \cos\theta$

$= \rho g h\ \hat{n}\ dA \cos\theta$

$= \rho g h\ dA_z\ \hat{n}$ where $dA_z = dA\ \cos\theta$



(Fig. 6c)

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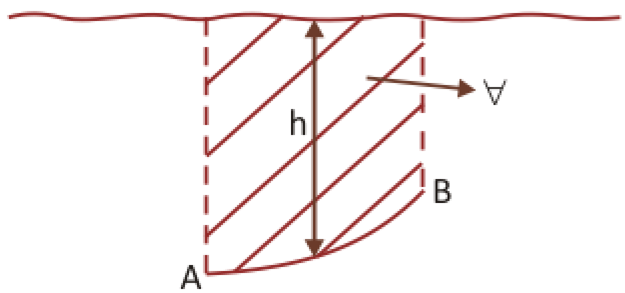
$\left|d\vec{F_z}\right| = \rho g h \, dA_z$. Therefore,

$$\vec{F_z} = \rho g \int_{A_z} h dA_z$$

$$= \rho g \forall, \left\{ \begin{array}{c} \text{where } \forall \text{ is the volume of the} \\ \text{liquid trapped between the} \\ \text{submerged surface and the top} \\ \text{surface of the liquid} \end{array} \right.$$



(Fig. 6d)

Consider the following submerged surfaces. In all cases the vertical force (component of the pressure force) is nothing but $\rho g \forall$, where $\forall$ is the volume of the liquid between the surface and the top surface of the liquid. Think again. $\rho g \forall$, is nothing but the weight of the liquid on the submerged surface.

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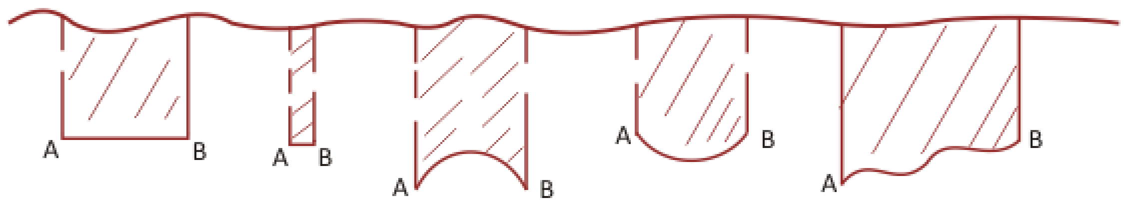
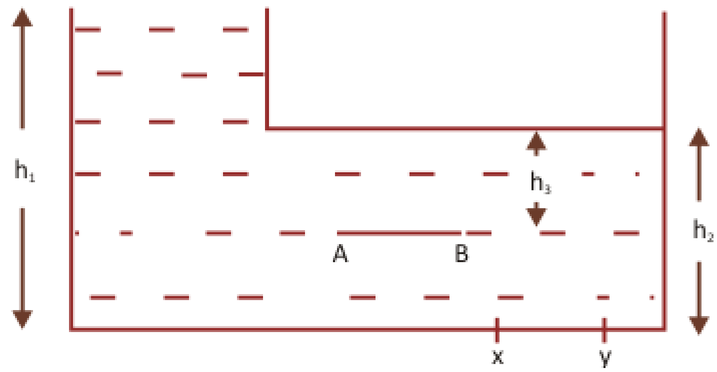


(Fig. 6f)

Special case:

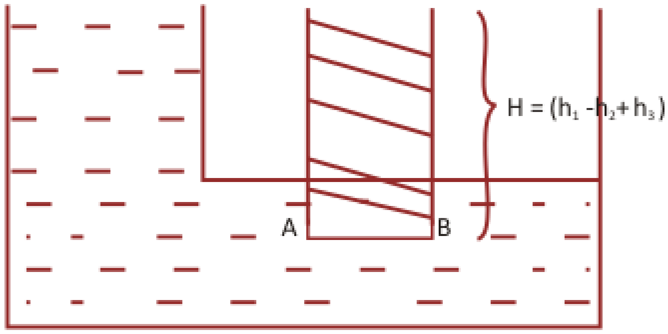


(Fig. 6g)

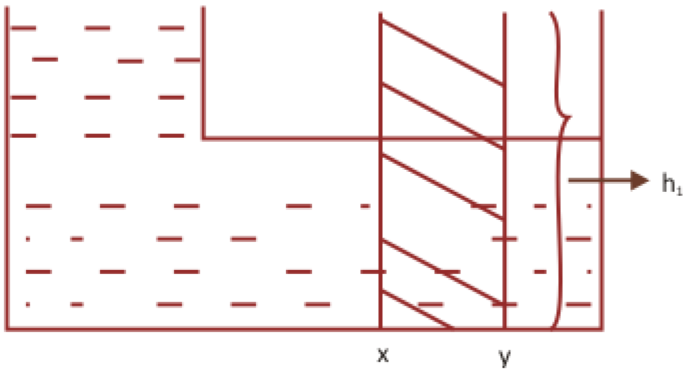
What is the hydrostatic force acting on the horizontal surface AB? $\rho g h_3$ or $\rho g\,(h_1 - h_2 + h_3)$ =?

What is the force acting or XY? $\rho g h_2$ or $\rho g h_1$?

The answer is: It is $\rho g(h_1 - h_2 + h_3)$ for the 1st case and $\rho g h_1$ for the 2nd case.



(Fig. 6h)



(Fig. 6i)

It is always the weight of the liquid column trapped between the surface and the free surface of the liquid.

Note: $\frac{dp}{dx} = 0$ along 'AB'. Therefore, the pressure on AB is the same as that on any horizontal surface at the same altitude or depth.