Module 7: Energy conservation

Lecture 24: Examples on Bernoulli equation

- Examples on Bernoulli's equation
- Apply BE

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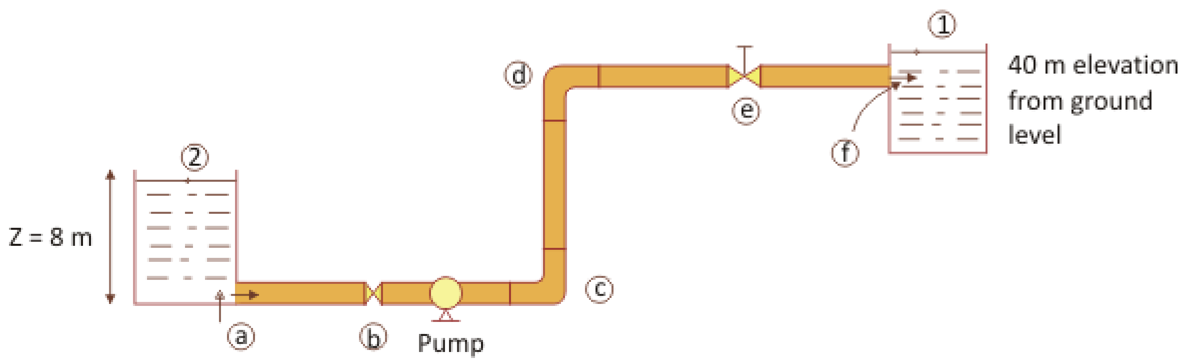
Examples on Bernoulli's equation

Example 1: A pump delivers water $(\rho = 1000 \ \mathrm{kg/m^3}, \upsilon = 10^{-6} \ \mathrm{m^2/s})$ from one reservoir to another reservoir at $5.6 \times 10^{-3} \mathrm{m^3/s}$ through 120 m of 0.05-m-diameter pipe. See the figure below for several pipe–fittings installed on the pipe-line. The surface of the pipe is rough $(\epsilon/D = .0001)$. Calculate the power of the pump required for the water–transfer.



(Fig. 24a)

Before writing energy balance equation, one should make a note that there are several minor losses between (1) and (2) and a major loss due to friction in the pipe. Let us list such minor losses:

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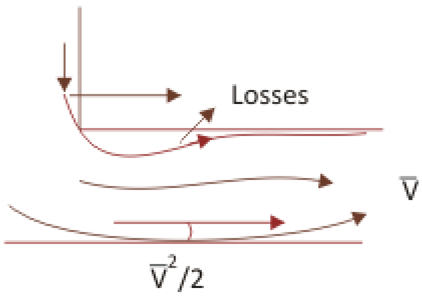
(a) Entrance loss–when the fluid enters into pipe from the tank, there is a loss.

$$w_l = K_{entrance} \overline{V}^2/2$$



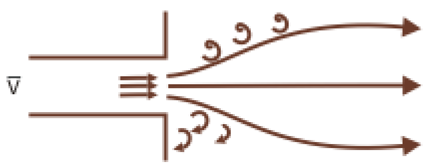
(Fig. 24b)

(b) Valve: $w_l = K_{entrance} \overline{V}^2/2$

(c) $90° - \text{bend/elbow}$ $w_l = K_{90°-bend} \overline{V}^2/2$

(d) $90° - \text{bend/elbow}$ $w_l = K_{90°-bend} \overline{V}^2/2$

(e) Exit loss: When the fluid enters into the tank, there is a frictional loss.



(Fig. 24c)

$$w_l = K_{exit} \frac{\overline{V}^2}{2}$$

All it implies is that K-factor is required to calculate minor-losses:

	K
Entrance	0.5
Exit	1.0
Gate value (half open)	2.7
90°- bend	0.95
Globe value (fully open)	6.9

Assume: valve at (b) is a gate-valve (half open) and that at (e) is a globe–valve (fully open))

Apply BE :

$$\mathrm{J/kg}: \left(\frac{P_1}{\rho} + \frac{\overline{V}_1^2}{2}\ gz_1\right) = \left(\frac{P_2}{\rho} + \frac{\overline{V}_2^2}{2}\ gz_2\right) - w_s + w_l$$

$V_1 = V_2 = 0$ (top-surface of water level; it is nearly stationary)

$P_1 = P_2 = 1\ \mathrm{atm}$

$z_1 = 8\ \mathrm{m},\ z_2 = 40\ \mathrm{m}$

w_s = shaft work $\left(\mathrm{J/kg\ or\ } {}^{W}\!/_{kg|s}\right)$

w_l = viscous – loss = Minor loss + major loss

Minor–loss = $\sum_a^f K\frac{\overline{V}^2}{2} = \ (0.5 + 1.0 + 2.7 + 0.95 + 6.9)\frac{\overline{V}^2}{2}$

(Note: $\overline{V}$ is the same for all fittings)

$$Q = \overline{V}\,A \Rightarrow \overline{V} = {}^{Q}\!/_{A} = \frac{5.6\times 10^{-3}\times 4}{\pi\times(0.05)^2} = 2.8\,\mathrm{m/s}$$

(assume that the pipe–size is the same for suction and discharge segments)

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Therefore, minor-loss = 94.272 $\mathrm{J/kg}$

Major- loss, w_l = $f\left(\frac{\bar{V}^2}{2}\right)\left(L/_D\right)$

$$R_{e,D} = \left(\frac{Vd\rho}{\mu}\right)_f = \frac{2.8 \times .050}{10-6} = 140{,}000$$

$\epsilon/_D = 0.0001$ (Flow is turbulent; $f \neq \frac{64}{R_{e,D}}$!)

From Moody's plot, f = 0.02

Therefore, major–loss $= 0.02 \times \left(\frac{2.8^2}{2}\right) \times \frac{120}{0.05}$

$$= 188.16\ \mathrm{J/k_g}$$

Substitute,

$$10 \times 8 = 10 \times 40 - w_s + (94.272 + 188.16)$$

$$w_s = 602.432\ \mathrm{J/kg}$$

$$\dot{m} = \text{moss flowrate} = Q\rho = 5.6 \times 10^{-3} \times 1000\ \mathrm{kg/_s}$$

$$= 5.6\ \mathrm{kg/_s}$$

$$\text{Pump - power} = \dot{m}\, w_s = 602.432 \times 5.6\ \mathrm{W}$$

$$= 3373.62\ \underline{\mathrm{W}}$$

$$= 4.5\ \mathrm{HP}$$

(Assume 100% efficiency of the pump)

If pump is 70% efficient, total pump–power

required$= 4.5/0.7 = \underline{6.46}\ \mathrm{HP}$

Note: very–often mechanical energy balance equation is written as

$$\left(\frac{P}{\rho} + \frac{\bar{V}^2}{2} + zg\right)_1 = (\sim)_2 + w_l - \frac{\eta w_s}{\dot{m}}$$

$$\left(w_s = \frac{\eta w_s}{\dot{m}}, \text{were } \eta \text{ in the efficiency of the pump}\right)$$

w_s = power delivered by the pump

$\dot{m}$ = flow rate of the fluid.

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