- Newtonian and non-Newtonian fluid - macroscopic pressure-drop balance

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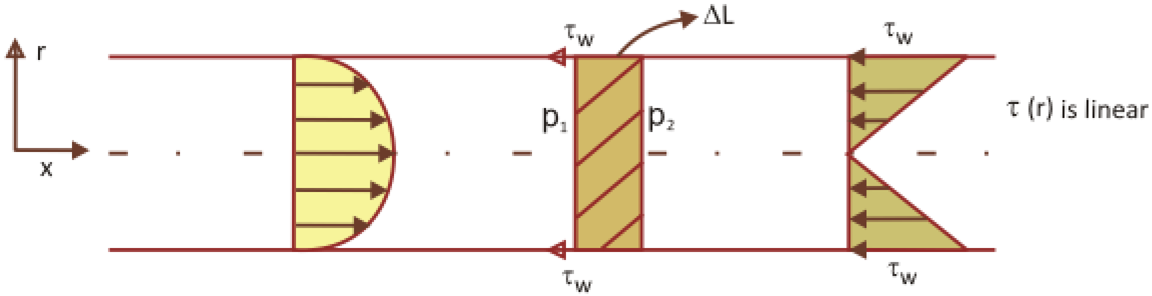
Newtonian and non-Newtonian fluid - macroscopic pressure-drop balance

In the previous lecture, we applied the NS equation to obtain an expression for the pressure–drop in a tube for the condition of the steady-state laminar flow of a Newtonian fluid. We can also derive the same expression, the Hagen- poiseuille equation, by making an integral (macroscopic) force balance.



(Fig. 18a)

If the flow is laminar (Re<2100) the velocity profile is parabolic:

$v(r) = v_{max}\,(1 - r^2/R^2)$

For the Newtonian fluid, $\tau = -\mu\left(\frac{dv}{dr}\right)$

$$\tau_w = -\mu\left(\frac{dV}{dr}\right)_w = \frac{2\mu\, V_{max}}{R}$$

and, $\tau\,(r) = \dfrac{2\mu\, V_{max}\, r}{R^2}$

Therefore, $\boxed{\tau/\tau_w = r/R}$: shear-stress varies linearly in r-direction of the tube.

Now, make a force balance over ΔL or CV of $(\Delta L.A)$

$\tau_w \times 2\,\pi R \Delta L = \Delta p \times \pi\, R^2$

$$\boxed{\frac{\Delta p}{\Delta L} = \frac{2\tau_w}{R} = \frac{4\mu v_{max}}{R^2} = \frac{8\mu\bar{V}}{R^2}}$$

which is the same as that obtained by applying differential momentum balance (NS equation) and then integrating with BCs.

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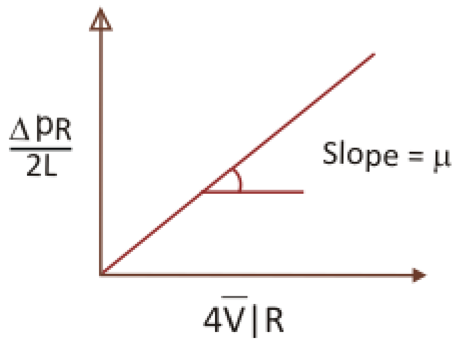
- From the above expression, one can also determine viscosity of a fluid by measuring pressure- drop across a tube for different flowrates:

$$\mu = \tau_w / \left(\frac{d}{dr}\right)_w = \frac{\Delta pR/2L}{4\,\overline{V}/R}$$



(Fig. 18b)

The above-plot is known as pseudo-shear diagram, which is generally used to determine the viscosity of a slurry-mixture. A similar approach can be followed to determine the viscosity or effective viscosity of Non-Newtonian fluids.

- Bingham plastic fluid

Such a fluid has the following shear-stress (τ) vs strain rate $\left({du}/{dr}\right)$ characteristics:

$$\boxed{\tau = \tau_o - \eta \frac{du}{dr}} \qquad \text{-----------(1)}$$

where τ_o is known as the yield stress. Fluid is supposed to be 'frozen' if

$$\tau_w < \tau_o$$

or, $\frac{\Delta p}{L} < \frac{4\tau_o}{d}$

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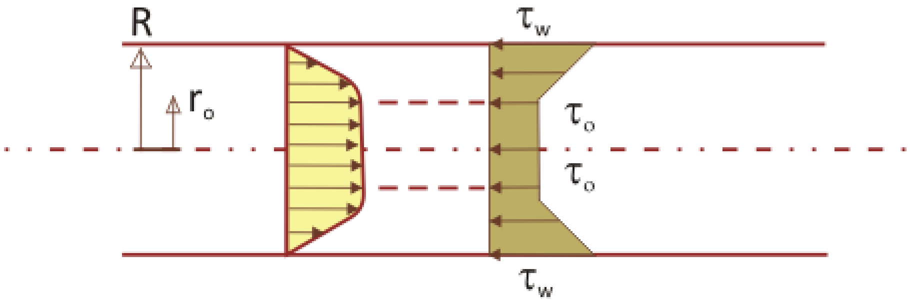
If the flow is laminar, the velocity profile is parabolic near the walls and is uniform in the central core of the tube. Shear-stress in such fluid also varies linearly with r:



(Fig. 18c)

$$\tau = \tau_o \text{ for } 0 \le r \le r_o$$

$$\frac{\tau}{\tau_o} = \frac{r}{r_o} \text{ for } r_o \le r \le R$$

Equation (1) can be integrated with $BC, V = 0$ at $r = R$ to obtain:

$$u(r) = \frac{\tau_w R}{2\eta}\left(1 - \frac{r^2}{R^2}\right) - \frac{2\tau_o R}{\eta}\left(1 - \frac{r}{R}\right)$$

$$r = r_o: \quad \tau = \tau_o = \frac{\tau_w r_o}{R}$$

Therefore, $u(r) = \frac{\tau_w R}{2\eta}\left(1 - \frac{r}{R}\right)\left(1 + \frac{r}{R} - \frac{2r_o}{R}\right)$

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The uniform velocity within the core of the tube $(r \leq r_o)$

$$= \frac{\tau_w R}{2\eta}\left(1 - \frac{r_o}{R}\right)^2$$

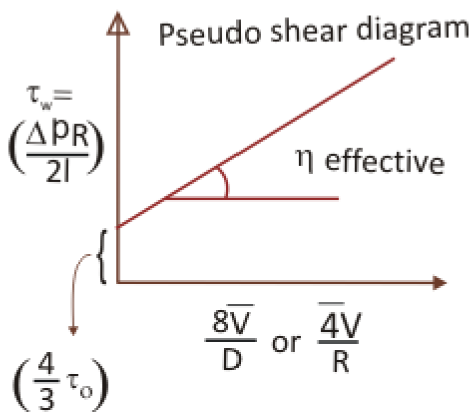
$\left(\text{Note: } \frac{dU}{dr} = 0 \text{ for } r \leq r_o\right)$

Again, $Q = \pi R^2 \overline{V} = \int_0^R 2\pi r U(r) dr$

Integrate as an exercise to obtain

$$\frac{8\overline{V}}{D} = \frac{\tau_w}{\eta}\left[1 - \frac{4}{3}\left(\frac{\tau_o}{\tau_w}\right) + \frac{1}{3}\left(\frac{\tau_o}{\tau_w}\right)^4\right]^{-1}$$

$$\simeq \frac{\tau_w}{\eta}\left[1 - \frac{4}{3}\left(\frac{\tau_o}{\tau_w}\right)\right]^{-1}$$



(Fig. 18d)

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