Module 8: Flow at low and high Reynolds numbers

Lecture 26: Creeping flow, Stokes-law and terminal velocity

- Creeping, potential and boundary-layer flows
- Low Reynolds-number flow

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Creeping, potential and boundary–layer flows

In the previous lectures, we took-up couple of examples on the application of the NS-equation, in particular, the flow in a circular tube. You must have realized that the NS–equation is non–linear on velocity, and the velocity and pressure–fields are coupled. In general, it is difficult to solve the NS equation, and more often numerical techniques are used to solve the equation. In this context. approximations are made to simplify the equation for solution. In this lecture, we will make use of the dimensional analysis to non–dimensionalize the NS equation, and then, explore the possibility of approximation.

$$\frac{\partial \vec{V}}{\partial t} + \vec{V}\,\nabla\,\vec{V} = \mu\,\overline{V}^2\,\vec{V} + \rho\vec{g} - \nabla p$$

x-momentum

$$\rho\left(\frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y}\right) = \mu\left(\frac{\partial^2 V_x}{\partial x2} + \frac{\partial^2 V_x}{\partial y2}\right) - \rho g_x - \frac{\partial p}{\partial x}$$

Non–dimensionalize the variables as follows:

$$X^* = \frac{X}{L}, Y^* = \frac{Y}{L}, V_X^* = \frac{V_x}{\overline{\overline{V}}}, g_X^* = \frac{g_x}{g}, p^* = \frac{p - p_2}{p_1 - p_2}, \frac{\partial}{\partial X^*} = L\frac{\partial}{\partial X},$$

The characteristic variables are generally chosen to make the dimensionless quantities vary between 0-1. Therefore, L may be the length of a tube or channel.

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$\overline{V}$ is the average velocity of the fluid, and p_1 and p_2 are the up-and downstream pressures. On substitution, the following equation is obtained:

$$V_X^* \frac{\partial V_{X^*}}{\partial X^*} = -\frac{\Delta p}{\rho \overline{V}^2} \frac{\partial p^*}{\partial X^*} + \frac{gL}{\overline{V}^2} g^* + \left(\frac{\mu}{L\rho\overline{V}}\right)\left(\frac{\partial^2 V_X^*}{\partial X^{*2}} + \frac{\partial^2 V_X^*}{\partial Y^{*2}}\right)$$

Let us define, $\begin{matrix}\text{Eu} \\ (\text{Euler \#})\end{matrix} = \frac{\rho \overline{V}^2}{\nabla p} \equiv \frac{\text{Inertial effects}}{\text{pressure effects}}$

$$\begin{matrix}\text{Fr} \\ (\text{Froude \#})\end{matrix} = \frac{\overline{V}^2}{gL} = \frac{\text{Inertial effects}}{\text{gravitational effects}}$$

$$\begin{matrix}\text{Re} \\ (\text{Reynolds \#})\end{matrix} \frac{\overline{\overline{V}}\rho L}{\mu} = \frac{\text{Inertial effects}}{\text{Viscous effects}}$$

The non–dimensionalized equation is, therefore, written as

$$\boxed{V_X^* \frac{\partial V_X^*}{\partial x^*} = -{}^{1}/_{Eu}\, \nabla^* p^* + {}^{1}/_{Fr}\, g^* + {}^{1}/_{Re}\, \nabla^{*2} p^*}$$

From the above-equation, it is clear that several approximations can be made to drop–out one or more terms from the equation, depending upon the range of Eu, Fr, and Re, the three dimensionless groups obtained for the NS-equation. We will continue our discussion in this lecture to only two approximations of common engineering application:

1. Low Re
2. High Re

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Low Reynolds-number Flow: Under this condition, the viscous effects dominate the inertial effects.

Mathematically $\mathrm{Re} \rightarrow 0$ and

$\nabla^{*^2} V^* = 0$ if $\mathrm{Eu} \simeq 1$

Such flow is called as creeping-flow

One of the most common examples of creeping flow is the flow past a spherical object at low Reynolds number.

We will not derive the expression for velocity fields in this lecture. Readers can refer to an advanced book on fluid mechanics.

However, it may be mentioned that the total force comprising of normal and shear can be combined to obtain the well–known Stoke's law for the drag (force acting along the flow–direction) on a sphere:

$$\boxed{F_D = 3\pi \mu_f d_p v_\infty}$$

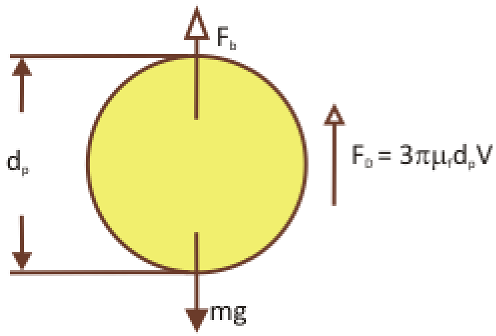
Where, $\mu_f =$ Fluid Viscosity

d_p = Diameter of the particle

v_∞ =Relative velocity of the particle with respect to the fluid–velocity

This equation is valid for $\mathrm{Re} < 1$

- One of the applications of Stoke's law is to calculate terminal velocity of a sphere in a quiescent fluid. Let us consider a sphere falling under gravity in stagnant fluid :



(Fig. 26a)

There are 3 forcess acting on the sphere

1. Gravity
2. Buoyancy
3. drag (viscous) forces

Therefore,

$$m\frac{dV}{dt} = mg - \forall\rho_f\, g - 3\pi\,\mu_f\, d_p v$$

When the particle acquires a steady-velocity or it reaches a constant (terminal) velocity

$$\frac{dV}{dt} = 0$$

$m = \forall\rho_s = {}^4/_3\,\pi\left({}^{d_p}/_2\right)^3 \rho_s$ Therefore,

$$0 = \frac{4}{3}\pi\left(\frac{d_p}{2}\right)^3 \rho_s g - \frac{4}{3}\pi\left(\frac{d_p}{2}\right)^3 \rho_f g - 3\pi\,\mu_f\, d_p\, v_\infty$$

$$\boxed{v_\infty = \frac{(\rho_s - \rho_f) g\, d_p^{\,2}}{18\ \mu_f}}$$

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