- Momentum-Theorem (Example)

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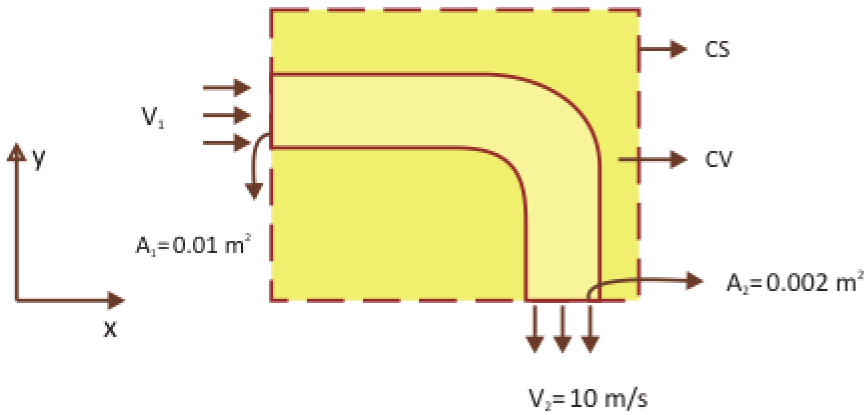
Momentum-Theorem (Example)

1. Consider the steady-state flow of water though a 90° reducing bend. The water-velocity is uniform at the inlet and outlet cross-section $(0.01\ \mathrm{m}^2 \text{ and } 0.002\ \mathrm{m}^2)$. If the water enters the bend at $200\ \mathrm{k\ Pa}$ and is discharged at $10\ \mathrm{m/s}$ to the atmosphere, determine the force required to hold the bend in place or the force acting on the bend. Neglect the weight of the bend and water in the bend. $p_{atm} = 100\ \mathrm{k\ Pa}$



(Fig. 14a)

Choose CV so that CS is perpendicular to the velocity field at the inlet and the outlet. Apply momentum theorem:

$$\vec{F}_{ext} = \frac{\partial}{\partial t}\iiint_{cv} \vec{V}\rho d\forall + \iint_{cs} \vec{V}\left(\rho\vec{V}.d\vec{A}\right)$$

Under steady-state, 1st term of the RHS is zero.

$$\vec{F}_x = \iint_{cs} \vec{V}_x\left(\rho\vec{V}.d\vec{A}\right);\ \vec{F}_y = \iint_{cs} \vec{V}_y\left(\rho\vec{V}.dA\right)$$

$$\vec{F}_x = V_1\left(-\rho V_1 A_1\right) = -\rho {V_1}^2\ A_1\ \left(\text{Note that } \vec{V_x} \text{ at the outlet} = 0\right)$$

$$\vec{F}_y = V_2\left(-\rho V_2\ A_2\right) = -\rho {V_2}^2\ A_2\ \left(\text{Note that } \vec{V_y} \text{ at the inlet} = 0\right)$$

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Note that $\rho V_1 A_1 = \rho V_2 A_2$, which follows from continuity. Apply continuity under SS: $\iint_{cs} \rho \vec{V}.d\vec{A} = 0 \Rightarrow -\rho V_1 A_1 + \rho V_2 A_2 = 0$

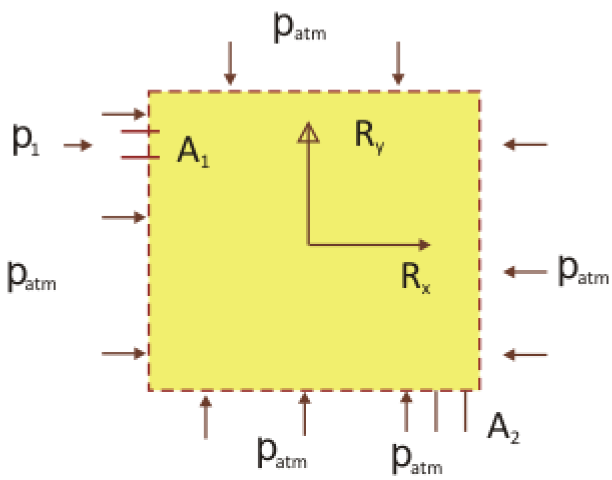
Therefore,

$$V_1 = \frac{A_2 V_2}{A_1} = \frac{0.002 \times 10}{0.01} = 2\ m/s$$

$$\therefore \vec{F}_x = -1000 \times 2^2 \times .01 = -40\ N$$

$$F_y = -1000 \times 10^2 \times .002 = -200\ N$$

Let us make free-body- diagram on CV



(Fig. 14b)

There are pressure forces acting on A_1 and A_2 by the water; reaction forces on CV, Rx and Ry (reaction forces from the support to hold bend in place). Atmospheric pressure acts uniformly over the entire CV, except on $'A_1'$ where the pressure is p_1(absolute). Therefore, add and substract p_{atm} on A_1 to make the contribution of atmosphere zero on CV. In other words, $\sum_{cs} p_{atm}\ ds = 0$.

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$F_x = -40N = R_x + (p_1 - p_{atm})\ \ A_1$

$F_y = -200N = R_y +\ \text{Wt of the bend}\ (=0) + \text{water}(=0)$

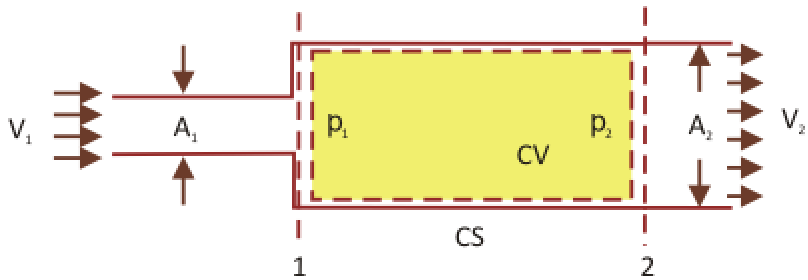
$R_x = -40 - (200 - 100) \times 0.01 \times 10^3 = -1040N$.

$R_y = 200N$

$\vec{R}$ to hold the bend in place $= (-1040i + 200j)N$

$(\text{Also, note}\ \ p_1 - p_{atm} = p_{1g})$

2. Consider the flow of an incompressible fluid under steady-state in an expander:



(Fig. 14c)

Fluid-velocity is uniform at the entrance $(\text{area} = A_1)$ and at the outlet $(\text{area} = A_2)$. Pressures are considered also uniform at sections 1 and 2 of the nozzle. Atmospheric pressure acts uniformly on the nozzle. Calculate the pressure drop $(p_1 - p_2)$.

$$\vec{F}_x = \frac{\partial}{\partial t}\iiint_{cv} \vec{V}_x \rho d\forall + \iint_{cs} \vec{V}_x\ (\rho\vec{V}.d\vec{A})$$

Choose CV as shown below:

(Fig. 14d)

$F_x = -V_1^{\,2}\ \rho A_1 + V_2^{\,2}\ \rho A_2$ (velocity everywhere in the smaller cross-section A_1 is V_1

Apply continuity over the same CV:

$\oiint \rho \vec{V}.d\vec{A} = -\rho V_1\ A_1 + \rho V_2\ A_2 = 0$

Or, $\rho V_1\ A_1 = \rho V_2\ A_2 = \dot{m}\ (\text{mass flowrate}, kg/s)$

$V_2 = \dot{m}/_{\rho A_2} = \frac{V_1 A_1}{A_2}$

Substitute, $\vec{F}_x = -V_1\ \dot{m} + V_2\ \dot{m}$

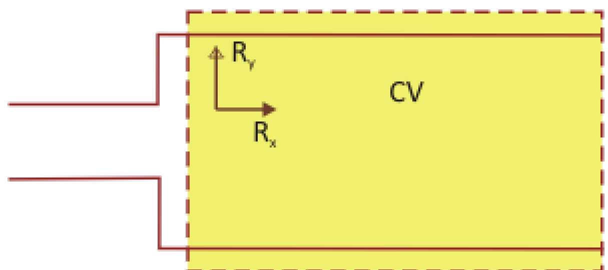
$= \dot{m}\ (V_2 - V_1) = \dot{m} V_1 \left(\frac{A_1}{A_2} - 1\right)$ neglecting viscous forces (friction on wall)

$\vec{F}_x = (p_1 A_2 - p_2 A_2) + F_v\ (=0)$, neglecting viscous forces.

$= A_2\ \ (p_1 - p_2)$

Equating, $\Delta p =\ p_1 - p_2 = \frac{\dot{m} V_1}{A_2}\ \left(\frac{A_1}{A_2} - 1\right)$ Ans.

- Choice of CV is important. If CV is chosen so that the CS cuts the nozzle, then there will be reaction forces, which will be unknown:



(Fig. 14e)

$\vec{F}_x = (p_1 A_2 - p_2 A_2) + R_x$; However there is no viscous force as in the previous case!

$\vec{F}_y =$ Weight of the nozzle and the water inside. The latter–CV may be chosen when the question posed is: determine the force to hold the nozzle in place.

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