Bernoulli equation and applications

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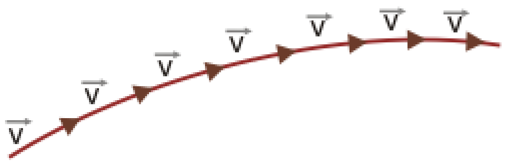
Bernoulli equation and applications

In the previous lecture, we obtained an expression for the conservation of mechanical energy in a flowing system. We will like to apply the same expression between two points or location in the flow-field. Before that, let us understand 'streamlines' and "stream tubes".

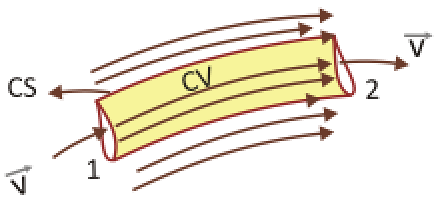
Streamline – it represents a line drawn in the flow field such that tangent drawn at every point of it is in the direction of the local velocity vector, $\vec{V}$



(Fig. 20a)

Note that streamline will change in the unsteady-state flow field. Also, note that laminar flow is represented/ characterized by streamlines (turbulent flow is characterized by eddy, irregular, unstable flow patterns). If we inject dyes or color at a certain location in the laminar flow, we can track the path of dyes and visualize streamlines. Similarly, if we inject or sprinkle several tiny (mass-less) needles in the fluid under steady- state laminar flow conditions, the needles will align themselves along the fluid- flow path. A hypothetical line connecting the head of the needles may be considered to be 'streamline.

- Two streamlines cannot cross each other, because there will be two velocities at the point of intersection, which is not possible.
- On similar note, mass cannot cross a streamline. Based on the understanding of streamlines, one can visualize a stream tube as a hypothetical 3D tube encompassing streamlines inside, whose surface also consists of streamlines.



(Fig. 20b)

Note that at the entry and exit of the stream tube (marked in bold lines), velocity is perpendicular to the CS.

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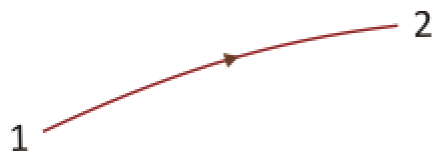
It can be said that fluid cannot cross the CS of the stream tube, and therefore, the outer boundary or CS of the tube may be replaced with a solid wall!

- We can now apply the mechanical energy balance equation to the CV of a stream tube between ports 1 and 2:
  $\frac{v^2}{2} + gz \frac{p}{\rho} =$ constant, with the assumptions

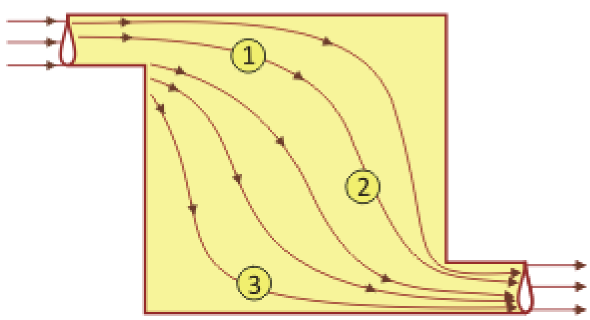
(a) SS flow (2) $\rho = C$ (3) laminar flow (4) inviscid fluid $(\mu = 0)$ or no viscous loss, and (5) isothermal.

- Under the extreme case of zero width or diameter of 3D stream tube, the equation may be applied between points 1 and 2 along a streamline



(Fig. 20c)

The same equation gets a name, the famous “Bernoulli equation” with the additional constraint (6) that this equation must be applied along a streamline. See the figure below. The BE cannot be applied between ‘1’ and ’3’, because they do not lie on a streamline, but can be applied between ‘1’ and ‘2’.

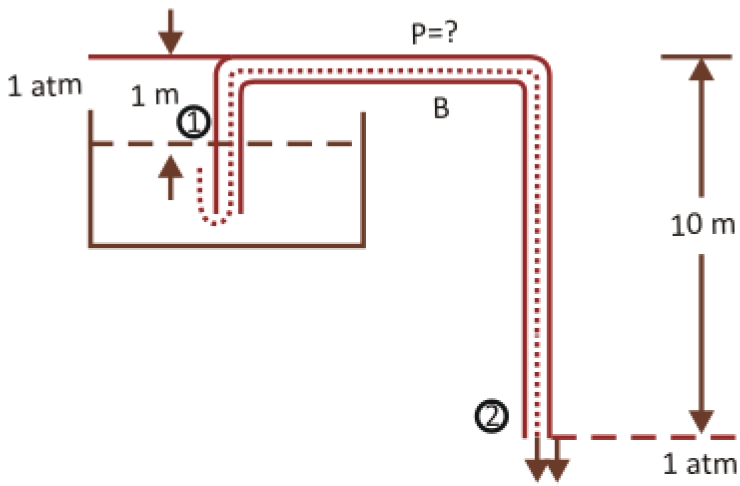


(Fig. 20d)

It is interesting to mention that if there is a pump which delivers energy to the fluid and if there are viscous losses because of friction with the walls or expansion or contraction, the BE is modified as an engineering approximation :

$$\left(\frac{V^2}{2}+gz+\frac{p}{\rho}\right)_2 = (\text{same terms})_1 + \underset{\substack{\downarrow \\ +\text{ve for pump}}}{\dot{w}_s} \quad - \quad \underset{\substack{\downarrow \\ \text{always} + \text{ve}}}{w_l} \quad : \text{J/kg or W/kg/s}$$

Example: An inverse U-tube is used to drain water from a tank or reservoir. The bend of the inverted tube is 1m above the free surface of the water in the tank and 10 m above the ground where the water is discharged. Water exits the tube at atmospheric pressure. Determine the pressure in the bend.



(Fig. 20e)

To apply BE, assume

$SS, \rho = C, \ \mu = 0$, and neglect viscous losses (losses in the bend or due to wall-friction)

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Apply BE between (1) and (B). (It is obvious that we have chosen a streamline along (1) and (B), shown as a dotted line)

$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_B}{\rho} + \frac{v_B^2}{2} + gz_B$, where

z_1 and z_B are the elevations from a reference line.

$p1 = \ 1\ \text{atm}\ ;\ v_1 \simeq 0$ (considering that the tank area is significantly larger than the tube's size, which is always true). Mathematically, one can apply continuity in such case:

$v_1\, A_1 \rho_1 = \ v_B\, A_B\, \rho_1$

$A_1 \gg A_B$ or $v_1 \ll v_B$ or $v_1 \ \simeq 0.$

Therefore, $p_B = p_1 + g(z_1 - z_B) \times \rho - \frac{v_B^2}{2} \times \rho$

$$= 10^5 + \frac{9.8(-1)}{\times 1000} - \frac{v_B^2}{2} \times 1000$$

Obtain v_B. How? Apply continuity between 'B' and '2'

$A_B\, \rho\, v_B = A_2\, \rho\, v_2 \ \Rightarrow\ v_B = v_2$

Obtain v_2. How? Apply BE between '1'and '2'

$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2$

$p_1 = p_2 = 1\ \text{atm}, v_1 \simeq 0, z_1 - z_2 = 9\ \text{m}$

Substitute to obtain $v_2 = \sqrt{9.8 \times 9 \times 2} = 13.28\ {}^{m}/_{s}$

Substitute $v_B = 13.28\ {}^{m}/_{s}$ to obtain

$$p_B = 10^5 - \left(9.8 + \frac{176.4}{2}\right) \text{x}1000$$

$$= 2000\ \text{Pa}$$

$$= 2\ \text{k Pa}$$

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