- Mechanical energy balance: Major loss (frictional loss in pipe)

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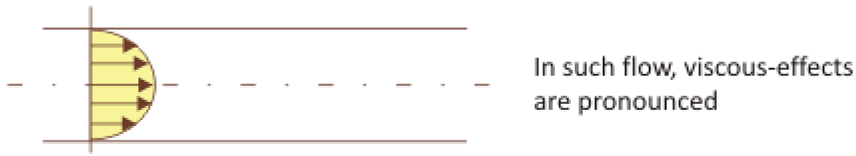
In earlier lecture, we obtained an expression for pressure-drop in a pipe or tube for the flow of a fluid under laminar conditions:

$$\frac{\Delta P}{L} = \frac{32\mu\bar{V}}{D^2}$$

The expression was obtained analytically by applying the NS equation and integrating the same with appropriate boundary conditions. The expression (also known as the Hagen–Poiseuille equation) was also derived by making the force-balance over a CV in the tube. It is important to note that such mathematical treatment can be carried out, only if the flow is laminar.

- Laminar flow refers to the flow which can be characterized by streamlines. The flow is controlled by viscous effects and the fluid velocity is relatively smaller. A parabolic-velocity profile obtained in a tube for a fluid–flow at small velocity is a good example of laminar-flow conditions.



(Fig. 23a)

Turbulent flow: Such flow occurs at relatively larger velocities and is characterized by chaotic behavior, irregular motion, large mixing, and eddies. For such flow, inertial effects are more pronounced than viscous effects. Mathematically, velocity field is represented as $V = \bar{V} + V'$, or the velocity fluctuates at small time scales around a large time-averaged velocity. Similarly, $p = \bar{p} + p', T = \bar{T} + T'$, etc

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In the latter lectures, we will see that a parameter called Reynolds number $= \left(\frac{vd\rho}{\mu}\right)_f$ is used to characterize laminar flow vis a vis turbulent flow. If $\mathrm{Re} < 2100$ for a tubular flow, the flow-characteristic is observed to be laminar and $\mathrm{Re} > 10^4$, the flow is turbulent.

Re- visiting the Hagen–Poiseuille equation for pressure- drop:

$$\frac{\Delta P}{L} = \frac{32\mu\bar{V}}{D^2}$$

or,

$$\frac{\Delta P}{\rho} = \frac{32\mu\bar{V}L}{\rho D^2}$$

It follows that, if we apply the mechanical energy balance equation between two sections of a horizontal pipe through which there is a laminar flow under steady-state conditions, we can show that

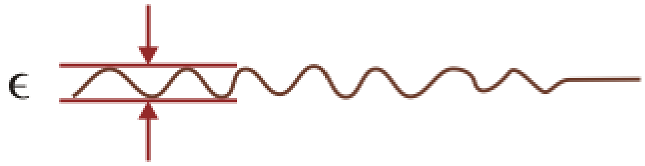
$$\frac{\Delta P}{\rho} = \frac{P_1 - P_2}{\rho} = w_l \ (\mathrm{J/kg})$$

where, $w_l = \left(\frac{32\mu\bar{V}L}{\rho D^2}\right)$

Thus, we have obtained an expression to calculate w_l, major loss in the pipe because of viscous-effect.

- Considering that, for turbulent flow the velocity or pressure–fields may not be exactly (analytically) represented, one resorts to dimensional analysis.

$$\Delta P = \Delta P(D, L, \overline{V}, \rho, \mu, \in), \text{where } \epsilon = \text{surface roughness}$$



(Fig. 23b)

$$\frac{\Delta P}{\rho\, \overline{V}^2} = f\left(\frac{\overline{V} D \rho}{\mu}, \frac{L}{D}, \frac{\in}{D}\right)$$

$$= f\left(Re, {}^{L}/_{D}, \frac{\in}{D}\right)$$

The experimental observations suggest that pressure drop in a pipe–flow under turbulent conditions depends on Reynolds number and surface roughness.

Taking an analogy from the major–loss for a laminar–flow

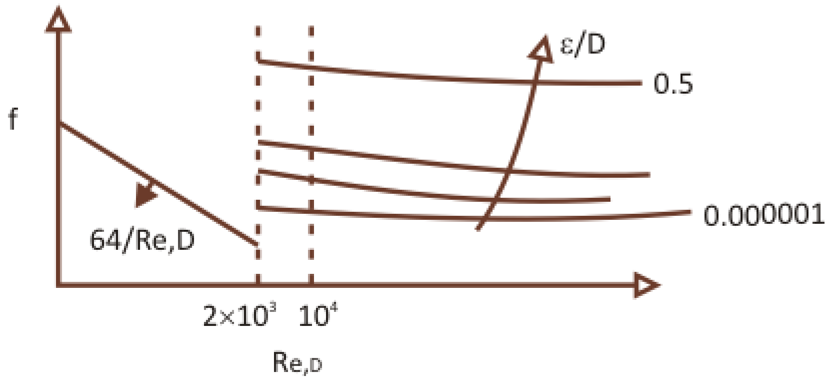
$w_l = \frac{\Delta P}{\rho} = \frac{32\mu\overline{V}L}{\rho D^2} = \frac{64}{Re}\left(\frac{\overline{V}^2}{2}\right)\left(\frac{L}{D}\right)$

At this point, an engineering parameter called friction–factor, f is defined as

$(\Delta\ p/\rho)/\left(\frac{1}{2}\,\overline{V}^2\right)$ so that

$f = \frac{64}{Re}$ for a laminar–pipe flow, and

$\boxed{\frac{\Delta P}{\rho} = f\left(\frac{\overline{V}^2}{2}\right)\left(\frac{L}{D}\right)}$ in general. This equation is known as Fanning equation. Here, *f* is experimentally shown to be dependent on the surface roughness $(\in/D\,)$ and Reynolds number, and is obtained from a plot called Moody's plot:



(Fig. 23c)

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The bottom most curve shown for $\in/D = 0.00001$ represents the friction factor for nearly hydraulically smooth tube or pipe, for which the friction factor has reached the smallest value with increasing smoothness.

Now, we have an expression to calculate energy loss (J/kg) for a pipe–flow under both laminar and turbulent conditions:

$$w_l = f\left(\frac{\overline{V}^2}{2}\right)\left(\frac{L}{D}\right)$$

where, $f = {}^{64}/_{Re}$ if $Re < 2100$

$\equiv$ to be obtained from the Moody's plot if $Re > 2100$

- A common empirical formula to calculate friction factor is proposed in literature: ${}^{1}/_{\sqrt{f}} = -1.8 \log\left[\frac{6.9}{Re{,}d} + \left(\frac{\epsilon/d}{3.7}\right)^{1.11}\right]$

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