Module 5: Reynolds Transport Theorem

Lecture 11: Control mass, control volume, mass-, momentum-, kinetic energy balance

- Mathematical analysis of fluid motion and associated properties
- Reynolds Transport Theorem

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Mathematical analysis of fluid motion and associated properties

There are two ways to carry out the analysis:

- Control mass: In such an analysis, a fixed mass of fluid element in the flow-field is identified and conservation equations for properties such as momentum, energy or concentration are written. The identified mass moves around in the flow-field. Its property may change from one location to another; however, the property must correspond to the same contents of the identified fluid element. In general, such approach is mathematically or experimentally difficult to apply.
- Control volume: This approach is popular and widely applied in the analysis. An arbitrary fixed volume located at a certain place in the flow-field is identified and the conservation equations are written. The property under consideration or analysis may change with time.
    - If ψ is a general property associated with fluid, then $\left.\frac{D\psi}{Dt}\right|_m$ and $\left.\frac{D\psi}{Dt}\right|_{\forall}$ may be considered to describe change in the property, ψ of the fluid, following the control mass (CM) and control volume (CV)approach, respectively.
    - The surface which bounds CV is called control surface (CS). In the CV approach, the co-ordinate axis is first fixed. A ‘CV’ is then marked in the flow-field. Choice of location and shape of CV are important for mathematical formulation.

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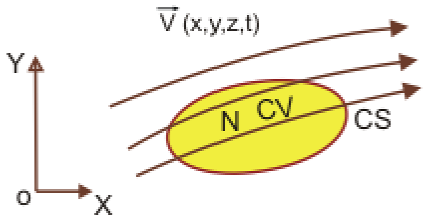
Reynolds Transport Theorem

We exclude the derivation of the theorem from the present course; however discuss the significance and the application of the same. The theorem relates the rate of change of a property associated with a CV to the material or particle rate of change of that property. Consider a CV bound by the CS through which a fluid flows, described by the flow field, $\vec{V}(x, y, z, t)$, relative to co-ordinates $x, y, \text{and } z$. As per the definition, CV is fixed in the flow field:



(Fig. 11a)

N is a property associated with flow field in CV. *N* could be either mass or momentum or energy or concentration of a species dispersed in the fluid. The theorem states:

$$\boxed{\frac{\partial N}{\partial t} = \frac{DN}{Dt} - \oint \eta \left(\rho \vec{V} . dA\right)}$$

where η is the specific property, N/M, where M = mass of fluid in volume $\forall$.

$$N = \iiint \eta \, \rho \, d\forall, \text{where } \rho \text{ is the density of the fluid } (kg/m^3)$$

or, $$\boxed{\frac{\partial}{\partial t} \iiint \eta \, \rho \, d\forall = \frac{D}{Dt} \iiint_{c\forall} \eta \, \rho \, d\forall - \oiint_{CS} \eta \left(\rho \, \vec{V} . dA\right)}$$

Note that if the property is mass, $\eta = 1$.

If the property is momentum, $\eta = \vec{V}(x, y, z, t)$.

If the property is kinetic energy, $\eta = {}^1\!/_2 \, V^2$.

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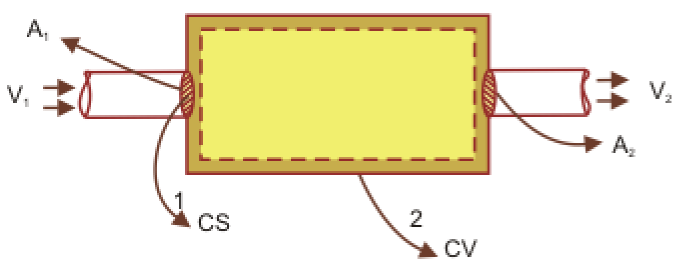
Let us give physical interpretation to the above three forms of the theorem.

- The first term is the rate of change in the property contained in the CV, which is fixed in the flow field. This term also describes if the property is under steady state or not, or if there is any accumulation of the property, N in the CV.
- The 2nd term represents the material rate of change of the property and may be seen as the rate of generation of the property. Note that if $N = m$, $\frac{Dm}{Dt} = 0$, because mass of the fluid particles initially marked in CV cannot change, nor it can be generated or destroyed, as the particles move in the flow field.
- The 3rd term represents the net rate of change in the property because of the flow of the fluid in and out of the CV through CS. $\vec{V}.d\vec{A}$ is the rate of volumetric flux in and out of dA, a differential area on CS. $\rho\vec{V}.d\vec{A}$ is the rate of mass flux. $(\rho\,\vec{V}.dA)$ is the rate of the property-flux in and out of the differential area $d\vec{A}$. $\iint \eta\,\rho\vec{V}.d\vec{A}$ is the rate of the total property over the entire CV.

Take a simple CV in the flow field, shown by the dotted line.



(Fig. 11b)

It is easier to show that the last term

$$\iint_{cs} \eta\,(\rho\vec{V}.dA) = M_1 - M_2$$
$$= \rho(Q_1 - Q_2)$$
$$= (\rho\,V_1\,A_1 - \rho\,V_2\,A_2)$$

In such case, the fluid enters and leaves the CS only through A_1 and A_2. The remaining surface of CS is impervious to the mass.

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