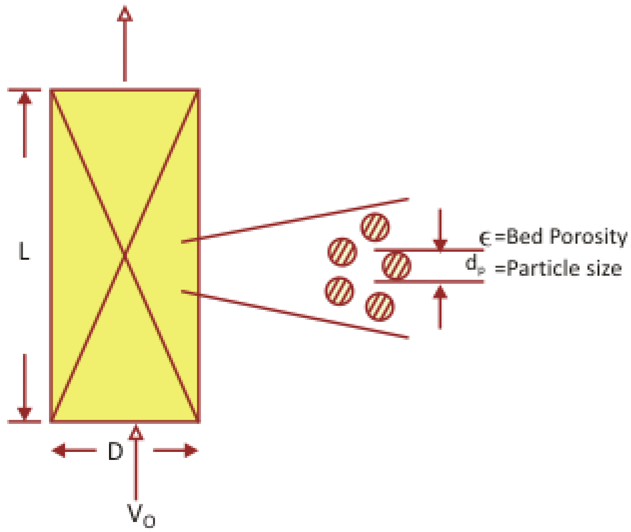
Module 9: Packed beds

Lecture 30: Pressure-drop: Ergun's equation

- Flow through packed bed (continued)

Flow through packed bed (continued)

Pressure–drop calculation:



Notes

1. Actual or real packed beds are randomly packed with irregular size particles
2. The flow–path of a fluid though the packed bed is tortuous.

- For the theoretical analysis to calculate pressure–drop, actual flow channels are replaced with parallel cylindrical conduits of constant cross–section. Particles are assumed to be of the same size and shape having constant sphericity, ϕ_s.
- Pressure–drop occurs due to inertial and viscous effects. At high Reynolds number, inertial effects prevail, whereas the viscous effects are important at low Reynolds number. In general,

$$(\Delta p)_{total} = (\Delta p)_{viscous} + (\Delta p)_{inertial}$$

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For a packed–bed

$$\frac{F_D}{As} = \frac{\text{Drag over the channel} - \text{walls consiting of packed bed particles}}{\text{Total surface area of particles}}$$

$$\boxed{= K_1\left(\frac{\mu_f V}{r_h}\right) + K_2\left(\rho_f V^2\right)} \text{ (Propose)}$$

(Re-call: wall shear-stress in tubular laminar flow,

$$\tau = \frac{4\mu_f V}{r} \text{ or } \tau \, \alpha \, \frac{\mu_f V}{r_h}$$

Similarly, pressure–drop at high Reynolds number, $\Delta p \, \alpha \, \rho_f V^2$. Therefore, Pressure-drop in packed beds is related to pressure–drop due to viscous and inertial effects via two empirical constants, K_1 and K_2.

$$r_h = \text{hydraulic radius} = \frac{\text{Total cross} - \text{section of conduits}}{\text{Wetted parameter}}$$

$$= \frac{\text{Total volume of voids}}{\text{Total surface area of particles}}$$ (multiply both numerator and denominator by L)

$$= \frac{(S_o L)\epsilon}{As}$$, S_O = cross sectional area of packed–bed

$$As = \underbrace{N_p}_{\textit{Total \# of particles}} \times \underbrace{S_p}_{\textit{Surface area of one particle}}$$

$$= \frac{S_o L(1-\epsilon)}{v_p} \times s_p, \text{ where } v_p = \text{volume of one partide}$$

$$\frac{v_p}{s_p} = \frac{6}{\phi_p \, d_p}; \, v = \frac{v_o}{\epsilon}, \text{ where } v_o = \text{superficial velocity}$$

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Therefore,

$$\frac{F_D}{As} = \frac{F_D \phi_p\, d_p}{S_o L(1-\epsilon)\times 6} = \left[K_1\, \frac{\mu_f\, v_o}{\epsilon^2\, S_o L} \times \frac{S_o L(1-\epsilon)6}{\phi_p\, d_p} + K_2 \rho_f \frac{V_o^2}{\epsilon^2}\right]$$

$$= \frac{\rho_f\, v_o^2}{\epsilon^2}\left[6K_1 \frac{\mu_f\,(1-\epsilon)}{v_o \phi_s\, d_p\;\; \rho_f} + K_2\right]$$

$$F_D = \text{drag} - \text{force} = (\Delta p)_{total} \times S_o \epsilon$$

Substituting,

$$\frac{(\Delta p)_{total}\,(S_o \epsilon)\phi_s\, d_p}{S_o L(1-\epsilon)\times 6} = \rho_f \frac{V_o^2}{\epsilon^2}\left[\frac{6K_1 \mu_f\,(1-\epsilon)}{\phi_s\, d_p v_o\, \rho_f} + K_2\right]$$

Or,

$$\frac{\Delta p}{L}\left(\frac{\epsilon^3}{1-\epsilon}\right)\left(\frac{\phi_s\, d_p}{\rho_f\, V_o^2}\right) = \frac{36\, K_1\, \mu_f\,(1-\epsilon)}{\phi_s\, d_p v_o\, \rho_f} + 6K_2$$

Setting $K_1 = \frac{150}{36}$ and $K_2 = \frac{1.75}{6}$ (based on experimental data)

We obtain

$$\boxed{\frac{\Delta p}{L}\left(\frac{\epsilon^3}{1-\epsilon}\right)\left(\frac{\phi_s\, d_p}{\rho_f\, V_o^2}\right) = \frac{150(1-\epsilon)\mu_f}{\phi_s\, d_p v_o\, \rho_f} + \; 1.75}$$

--------- Ergun's equation

One also defines f_p as the friction factor for a packed–bed

$$f_\rho = \frac{\Delta p}{L\, \rho_f\, V_o^2}\left(\frac{\phi_s\, d_p \epsilon^3}{(1-\epsilon)}\right)$$

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Therefore,

$$\boxed{f_p = \frac{150(1-\epsilon)}{\phi_s\, R_e,p} + 1.75}$$

If $R_e,p << 1$ (viscous effects), $f_p \simeq \frac{150(1-\epsilon)}{\phi_s\, ,R_e,p}$ (Kozeny – Carman Equation)

$R_e,p >> 1000$ (inertial effects) $f_p \simeq 1.75$ (Blake – Plummer equation)

It is pointed out that the Ergun's equation is applied to calculate pressure- drop across packed bed consisting of small size particles $(d_p < 25\ \mathrm{mm})$

Also, note that $\left(\Delta p/_{\rho}\right)$ is to be interpreted as energy loss due to drag or friction per unit mass of the fluid $(\mathrm{J/kg})$, so that the term can be substituted in the general mechanical energy balance equation, consisting of KE, gravitational and shaft-work heads.

$$(\mathrm{J/K_g}) : g\,\Delta Z + \Delta\left(\frac{v^2}{2}\right) + \left(\frac{\Delta p}{\rho}\right) + w_p + w_s = 0$$

where $w_s =$ frictional–pressure drop

$$= \left(\frac{\Delta p}{\rho}\right)_{\text{Friction}}$$

$$= \frac{150(1-\epsilon)^2\, \mu_f\, v_o L}{\epsilon^3\, d_p{}^2\, \phi_s{}^2\, \rho_f} + \frac{1.75\,(1-\epsilon)\, V_o^2 L}{\epsilon^3\, \phi_s\, d_p}$$