Lecture 10: Steady and unsteady state

- Steady state

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Module 4: Fluid Dynamics

Lecture 10: Steady and unsteady state

In this lecture we discuss steady state/unsteady state condition for velocity fields.

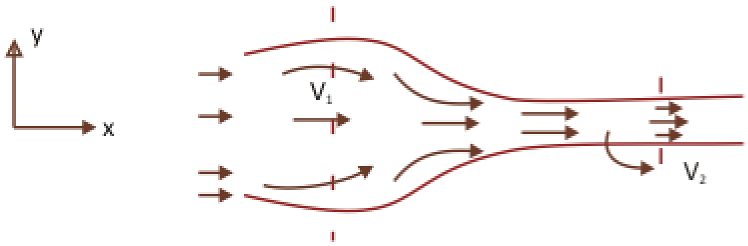
Steady state

The steady state condition for a flow-field implies that the velocity field and any property associated with the flow field remain unchanged with time.
In other words, local derivative of the velocity is zero.

Mathematically, $\frac{\partial \vec{V}}{\partial t}=0$. The concentration and temperature fields, if associated will also be under steady–state: $\frac{\partial T}{\partial t}=\frac{\partial C}{\partial t}=0$

As explained below, $\frac{D\vec{V}}{Dt}$ may or may not be zero.

Consider the flow of a fluid through a convergent nozzle. If the velocity field is steady, or the flow–field is under steady-state conditions, $\frac{\partial \vec{V}}{\partial t}=0$. However, an observer moving with the flow field will experience 'acceleration' as he moves from the larger diameter-section to the smaller diameter-section of the nozzle. This is because the velocity increases as the diameter of the nozzle decreases. Therefore, velocity remains unchanged with time anywhere in the flow- field. However, it has a spatial variation, or $\frac{D\vec{V}}{Dt}\neq 0$



(Fig. 10a)

$V_1 \neq V_2$, $\frac{\partial V_X}{\partial t}=0$, $\frac{DV_X}{Dt}\neq 0$(between 1-2)

The last expression is tantamount to saying that the velocity of a labeled or marked material-particle is not constant, as it moves from the larger section to the smaller section of the nozzle.

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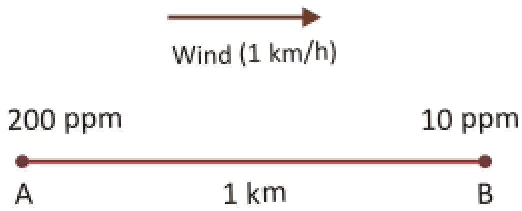
Example 1: At a certain time, the concentration of SO_2 measured at a location near a power-plant (A) is 200 ppm (parts per million). The SO_2 – concentration at 'B', 1 km downstream, is measured to be 10 ppm. At this instance, the average wind velocity is 1 km/hr. The wind is blowing in the direction of the higher (A) to lower concentration (B). If a balloon-sensor floating with the wind records a decrease of 50 ppm per hour, as it flows past 'B', determine the rate of increase of the concentration measured by a stationary observer at 'B'.



(Fig. 10b)

$$\left.\frac{DC}{Dt}\right|_B = -50; \qquad \left.\frac{\partial C}{\partial t}\right|_B = ?$$

$$\left.\frac{\partial C}{\partial t}\right|_B = \left.\frac{DC}{Dt}\right|_B - (V.\nabla)C$$

$$(V.\nabla)C = V_X \frac{\partial C}{\partial X}$$

$$\left.\frac{\partial C}{\partial t}\right|_B = -50 - 1\,\frac{10-200}{1} = 140 \text{ ppm/hr}$$

(Assume that the concentration gradient between two locations is linear)

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Example 2: A boat has a thermo-server attached to it. While floating with the fluid, it measures the temperature of the fluid. Both flow and temperature are under unsteady-state condition:

$$T(x,y,z,t) = x^2 + yz + t$$

$$\vec{V}(x,y,z,t) = 2xi + 2t^2y\hat{j} + \hat{k}$$

Determine the rate of change of the temperature recorded by the sensor at t = 1, when the boat flows past the location, whose spatial co-ordinates are $2i + j + k$.

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + (V.\nabla)T$$

$$\frac{\partial T}{\partial t} = 1$$

$$\frac{\partial T}{\partial x} = 2x; \quad \frac{\partial T}{\partial y} = z; \quad \frac{\partial T}{\partial z} = y$$

Here, $(x,y,z) = (2,1,1)$

Substituting;

$$\left.\frac{DT}{Dt}\right|_{\substack{t=1 \\ (x,y,z)=(2,1,1)}} = 1 + \underbrace{(2\times 2)}_{Vx}\underbrace{(2\times 2)}_{\frac{\partial T}{\partial x}} + \underbrace{(2\times 1^2 x\ 1)}_{Vy}\underbrace{x\,1}_{\frac{\partial T}{\partial y}} + \underbrace{1}_{V_z}\times\underbrace{1}_{\frac{\partial T}{\partial z}}$$

$$= 1 + 16 + 2 + 1 = 20$$

(Note: $V_x = 2x,\ V_y = 2t^2y,\ V_z = 1$)

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