Module 2: Fluid Statics

Lecture 4: Basic equation: derivation, pressure variation in an incompressible fluid

- The basic equation of fluid statics (continued)
- Pressure variation in an incompressible fluid

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The basic equation of fluid statics (continued)

2- Now, let us calculate pressure – gradient in a static fluid.

- Identify all forces: pressure forces, and external body force which is gravity in the present case.Consider a differential fluid elemental volume, $\delta \forall$.

- Force Balance (in vector form)
  $\sum$ (pressure force + gravitational force) = 0

or, $\vec{f}_p \, \delta V + \rho g \, \delta V = 0$

or, $\bar{f}\rho = \frac{d\vec{F}}{d\forall} = -\nabla p$(form previous lecture)

or, $\boxed{-\nabla p + \rho \vec{g} = 0}$

This is the basic (vector) equation for fluid statics.

The equation in the scalar form:

$-\frac{\partial p}{\partial x} + \rho g_x = 0$: x direction

$-\frac{\partial p}{\partial y} + \rho g_y = 0$: y direction

$-\frac{\partial p}{\partial z} + \rho g_z = 0$: z direction

Where, $\vec{g} = \hat{i}g_x + \hat{j}g_y + \hat{k}g_z$

- If gravity acts in the negative z-direction $\begin{pmatrix} g_x = g_y = 0, \\ \vec{g} = -\hat{k}g \end{pmatrix}$

$\boxed{\frac{dp}{dz} = -\rho g}$ or $p = p_o - \rho g z$

where $p = p_o$ at $z = 0$

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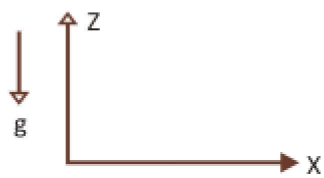
Notes:

1. Pressure gradient in a static fluid is zero if there is no gravity. Alternatively, the net force acting on a fluid volume is zero because pressure–force balances the force due to gravity. Alternatively, pressure-variation occurs in a static fluid because of gravity only.
2. If the fluid is water, static pressure is often referred as 'hydrostatic' pressure.
3. If a body is submerged in a fluid, which is in contact with (or open to) atmosphere, the atmospheric pressure acts uniformly on the body. For most of the engineering applications one is interested in calculating pressure due to the fluid only, ignoring the atmospheric pressure. In such case, the pressure is specified as a gauge pressure (above the atmosphere pressure): $p_{gauge} = \left(p - p_{atmosphere}\right)$. Therefore, the gauge pressure may be negative if the pressure in the fluid is sub or below atmospheric. Absolute pressure is always positive
4. Pressure in the atmosphere may vary, of course, over a relatively longer altitude, because the density of air is small (1 kg/m^3 in comparison to 1000 kg/m^3 for water).
   Consider an atmosphere consisting of ideal gases.
   Static pressure gradient

$$\frac{dp}{dz} = -\rho g$$

$$\left(\frac{dp}{dx} = \frac{dp}{dy} = 0\right)$$



(Fig. 4a)

And, $p = \rho RT$

Substituting, $\frac{dp}{p} = -\left({}^{g}/_{RT}\right)dz$

or, $\ln {}^{p}/_{p_o} = -{}^{g}/_{RT}\, z$

If $T \neq f(z)$ or an isothermal atmosphere

And, $p = p_0 \; at \; z = z_0$

or, $p = p_0 \exp\left(-\frac{gz}{RT}\right)$

The pressure varies (decreases) exponentially with altitude, if temperature variation is considered negligible.

Module 2: Fluid Statics

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- Engineering calculations based on the fluid static equation

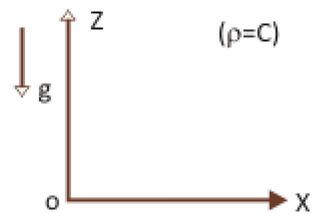
$$\boxed{-\nabla p + \rho \overrightarrow{g} = 0}$$

1. Pressure variation in an incompressible fluid

$$\frac{dp}{dz} = -\rho g$$

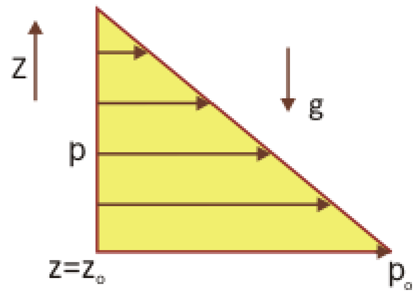
$p = p_o$ at $z = z_o$

Integrating, $p = p_o - \rho g z_o$



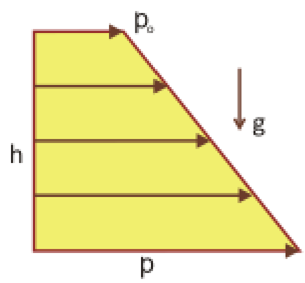
(Fig. 4b)

Therefore, pressure decreases lineally with altitude, or increases with depth in the fluid



(Fig. 4c)

or,



(Fig. 4d)

($p = p_o + \rho g h$, where h is the depth of the fluid measured from the top)

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