## Module 3: Hydrostatic forces on submerged bodies

## Lecture 7: Calculation of horizontal component, buoyancy

- Forces on submerged bodies (continued)
- Buoyancy

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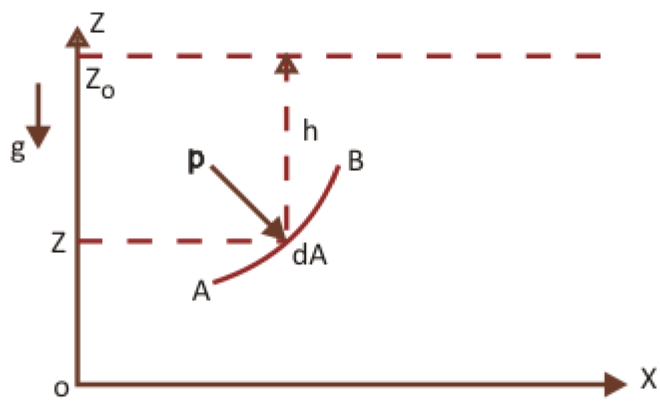
## Forces on submerged bodies (continued)

### Horizontal component

Re–consider the submerged surface 'AB' in a liquid.



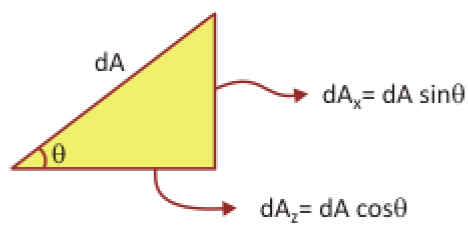

(Fig. 7a)

Pressure p acts normal to dA, the elemental surface of AB, at a distance Z from the origin or the depth 'h' from the top of the liquid surface.

$d\vec{F}_x \equiv$ differential pressure-force acting in x-direction

$$= (\rho g h)\,\hat{n}\, dA \sin\theta$$

$$|dF_x| = (\rho g h) dA_x \quad \left\{ \begin{array}{l} dA_x \text{ is the projected} \\ \text{area of the submerged} \\ \text{surface } dA \text{ on } Z-Y \text{ plane.} \end{array} \right.$$




(Fig. 7b)

$$\boxed{F_x = \int_{A_x} \rho g h \, dA_x} \qquad \text{------------(i)}$$

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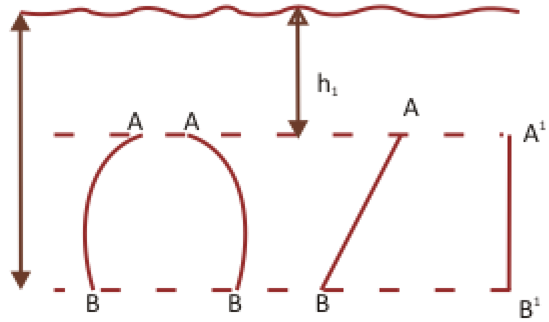
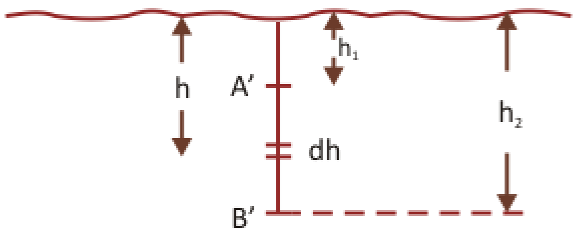
Let us calculate this integral for simple geometries.




(Fig. 7c)

See the figure presented above. All four surfaces marked as 'AB' have the same projected area, marked as A'B', on Y-Z plane because the depths (h1 and h2) of A and B are the same for all four geometries. Therefore, $F_x$ will assume identical values for all cases as follows:

$F_x = \int_{A_x} \rho g h \, dA_x = \int_{h_1}^{h_2} \rho g h \, dh.1$ (Assuming unit width of the surface)




(Fig. 7d)

$$\boxed{F_x = \rho g \frac{{h_2}^2 - {h_1}^2}{2}}$$

Let us re-arrange the right hand Side of the expression

$$= \underbrace{\rho g \left(\frac{h_2 + h_1}{2}\right)}_{\text{pressure acting on the centroid of the the A'B' surface}} \quad \underbrace{(h_2 - h_1)}_{\text{area of the projected surface A'B'}}$$

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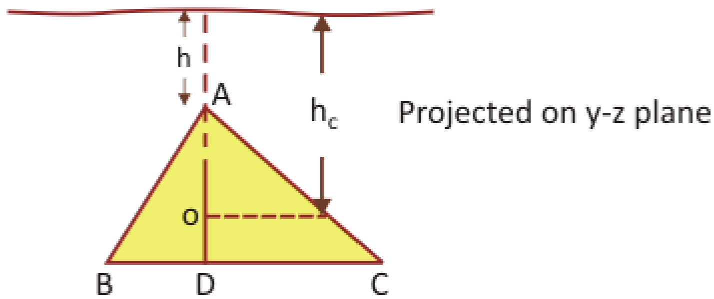
- Therefore, it may be said that the horizontal component of the hydrostatic pressure-force on a submerged surface is the hydrostatic pressure on the centroid of the projected surface on y-z plane, multiplied by the area of the projection. Alert: line of action of the force doses not pass through the centroid! One will have to determine it.
- This can be shown that this is true for all submerged surfaces. Let us consider an object whose projection on y-z plane is a triangular surface. Therefore, for such object, F_x may be calculated as follows:




(Fig. 7e)

$|F_x| = (\rho g h_c)x$ Area of triangle ABC where, $h_C$ is the depth of the centroid of $ABC = h + {}^2/_3\,AD$ , where AD is the altitude of the triangle ABC.

Area of the triangle $ABC = \frac{1}{2}\,BC\,AD$

$$\therefore |F_x| = \rho g\left(h + {}^2/_3\,AD\right)\frac{1}{2}\,BC\,AD$$

To this end, $\vec{F} = \vec{F_x} + \vec{F_z}$

Or $|F| = \sqrt{{F_x}^2 + {F_z}^2}$

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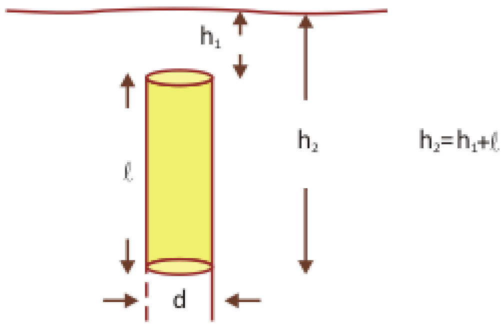
## Buoyancy

It is a net vertical force acting on the submerged body in a fluid, due to the hydrostatic pressure distribution. It is not a fundamental or physical force. It is an artifact of net pressure distribution in the vertical direction

$\boxed{\text{Buoyancy force} = \rho g \forall}$, where $\forall$ is the volume of displaced fluid. The force acts vertically upward.

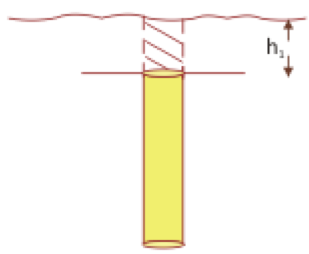
Consider a cylinder of length 'l' and diameter 'd' immersed in the fluid:




(Fig. 7f)

- Vertical force acting on the vertical side is zero.
- Vertical force acting on the top circular surface $= \left(p_{atm} + (\rho g h_1)\right) A, \text{where } A = \pi d^2/4$

Note that this is the weight of the fluid trapped between the free surface of the fluid and the top surface of the object, and acts vertically downward)




(Fig. 7g)

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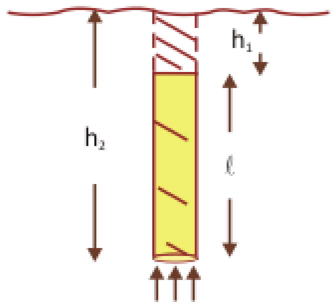
Vertical force on the bottom surface $= \left(p_{atm} + (\rho g h_2)\right)A$, which acts vertically upward. Note that this is the weight of the fluid trapped in the column of height $(h_1 + l)$over the bottom surface of the cylinder.




(Fig. 7h)

Net force (vertical) on the cylinder $= \rho g(h_2 - h_1)A \text{ or } \rho g l\, A$ ,which is the same as $\rho\ g\ \forall$, where $\forall$ is the volume of the cylinder. Thus, buoyancy is the weight of the displaced fluid by the cylinder, acting upward.

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