- Energy Conservation Equation

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Energy Conservation Equation

Similar to the derivation of continuity and momentum conservation equations, one can also apply Reynolds Transport equation to obtain the energy-conservation equation. In this course, we will, however, not derive the expression and restrict our discussion to the salient features of the equation and its application to the fluid-flow.

For a flow-system, η (specific property) for energy consists of three terms :

$$\eta = e = \left(\frac{V^2}{2} + gZ + U\right): \quad {}^{J}/_{kg}$$

where, N = total property $= \iiint \rho\eta \, d\forall$

Re-call, $\eta = 1$ for N= mass (kg)

$= \vec{V}$ for N = momentum $(kg - m/s)$

$= \left(\underset{\substack{\downarrow \\ KE}}{\frac{v^2}{2}} + \underset{\substack{\downarrow \\ PE}}{gz} + \underset{\substack{\downarrow \\ IE}}{U}\right)$ for N= energy (J)

The three terms may be recognized as kinetic energy, potential energy, and internal energy, respectively.

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Reynolds transport equation:

$$\frac{DN}{Dt} = \frac{\partial}{\partial t}\iiint_{CV} e\rho \, d\forall + \iint_{CS} e(\rho\vec{V}.dA)$$

$$\Downarrow \qquad \searrow \qquad \searrow$$

$$\begin{cases} \text{rate of energy} - \text{change in} \\ \text{system whose boundaries} \\ \text{coincide with CV at time t} \end{cases} \quad \begin{matrix} \text{local rate of} \\ \text{change} \\ \text{in energy} \end{matrix} \quad \begin{matrix} \text{energy efflux} \\ \text{(in and out)} \\ \text{because of convection} \end{matrix}$$

$$\begin{matrix} = \dot{Q} \\ \text{heat conducted} \\ \text{across CS} \end{matrix} \quad - \begin{matrix} \dot{W}\ (1^{st} \text{ law of thermodyanics:} \\ \text{work done on the} \\ \text{environment by the system)} \end{matrix}$$

$\dot{W}$ has two components; (1) surface-work because of flow $(\vec{V})$ and (2) shaft-work.

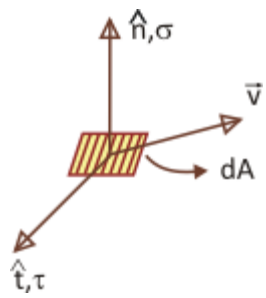
$\dot{W} = \dot{W}_f + \dot{W}_s$

- $\dot{W}_f$, surface-work also consists of two terms, or is contributed by two forces (shear and normal)

$$= -\iint \vec{V}.(\sigma\,\hat{n} + \tau\,\hat{t})dA$$

$\hat{n} = unit\ vector\ normal\ to\ the\ surface, dA$

$\hat{t} = unit\ vector\ \parallel\ to\ the\ surface\ dA, along\ the\ direction\ of\ shear\ stress$



(Fig. 19a)

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- If $\vec{V} = 0$ (stagnant fluid) there is no surface work, ie, $\dot{W}_f = 0$
- $\dot{W}_s$ is the shaft- work, for example, due to a turbine or a pump or blower, which delivers energy to the fluid or vice-versa. $\dot{W}_s = +ve$ if the machine is turbine

  $= -ve$ if the machine is pump or blower
- $\dot{W}_f$ can also be written as

  $$\dot{W}_f = \underset{\substack{\downarrow \\ \text{viscous} - \text{work}}}{\dot{W}_\tau} + \underset{\substack{\downarrow \\ \text{flow work}}}{\iint p\vec{V}.dA}$$

Note that $\dot{W}_\tau$ is zero if

- $\mu = 0$ (inviscid fluid)
- V_t (tangential velocity) = 0
- $\text{CS is perpendicular to } \vec{V}$ (choice of CS)

The general energy-balance equation can be re-arranged to obtain the following expression for a CV:

$$\frac{\partial}{\partial t}\iiint_{CV} e\rho\, d\forall = Q - \dot{W}_s - \dot{W}_\tau - \iint_{A_f} \underbrace{\left(e + \frac{p}{\rho}\right)}\rho\vec{V}.dA$$

The terms of the equation are accumulation, conduction, shaft-work, viscous work, and energy efflux across CS, including pressure- energy .

For most engineering applications of steady-state, two–port system (one inlet and one outlet), uniform flow over the ports (1-D flow), and $\vec{V}$ being perpendicular to CS (by a suitable choice), one can obtain the following expression for energy- conservation:

$$J/K_g: \left[U + \frac{V^2}{2} + g\,z + \frac{p}{\rho}\right]_{Outlet} = [\text{same terms}]_{inlet} + \underset{\substack{\uparrow \\ J/K_g(\text{conduction})}}{q} - \underset{\substack{\uparrow \\ \text{shaft} - \text{work}}}{Ws}$$

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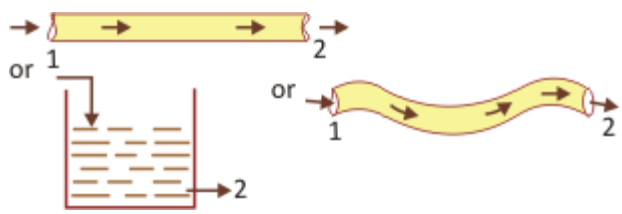
Moreover, if the flow is under isothermal condition without exchange of heat with surrounding (insulated system) $U_{inlet} = U_{outlet}$ and $\dot{q} = 0$

$$\boxed{\left[\frac{V^2}{2} + gz + \frac{p}{\rho}\right]_0 = \left[\frac{V^2}{2} + gz + \frac{p}{\rho}\right]_i - \dot{W}_s}$$

Note: unit of each term is J/kg or $W/kg - s$

If $\dot{W}_s = 0$ (no pump or turbine)

$\boxed{\frac{V^2}{2} + gz + \frac{p}{\rho} = \text{contant}}$ over two ports system, for example, in a tube or pipe or a tank:



(Fig. 19b)

The above conservation equation is also known as mechanical energy balance, consisting of KE, PE, and pressure- energy. Each term is also known as 'head'. We re-note the assumptions made in obtaining the above ME balance equation.

(1) SS (2) isothermal (3) No shaft work $(\dot{W}_S = 0)$ (4) negligible viscous work, $\dot{W}_\tau = 0$.

It is important to note that viscous work can be neglected if the fluid is inviscid or a suitable choice of CS so that $\vec{V}$ is perpendicular to CS.

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