- Dimensional analysis and similitude–continued
- Similitude:

Previous Next

Dimensional analysis and similitude–continued

Example 2: pressure–drop in pipe–flow depends on length, inside diameter, velocity, density and viscosity of the fluid. If the roughness-effects are ignored, determine a symbolic expression for the pressure–drop using dimensional analysis.

Answer: We will apply Buckingham Pi-theorem

Variables: $\Delta p, L, D, \rho, \mu, v : 6$

Primary dimensions: $3\ (M, L, T)$

No of dimensionless (independent) group: $6 - 3 = 3$

$$\Pi_1 = f_1\,(v,\ d,\ \rho,\ \Delta p\,)$$

$$\Pi_2 = f_2\,(v,\ d,\ \rho,\ \mu)$$

$$\boxed{\begin{array}{c} v, d, \rho \text{ are repeat variables} \\ F, \mu, L \text{ are non} - \text{repeat variables} \end{array}}$$

$$\Pi_3 = f_3(v, d, \rho, L)$$

Π_1:

$$\Pi_1 = v^a\, d^b\, \rho^c\, \Delta p \qquad \left(\Delta p \equiv {}^{F}\!/\!_{A} \equiv \frac{ML}{T^2}\Big/\frac{1}{L^2} \equiv \frac{M}{LT^2}\right)$$

$$\left.\begin{array}{r} M: 0 = C + 1 \\ L: 0 = a + b - 3C - 1 \\ T: 0 = -a - 2 \end{array}\right] \begin{array}{l} c = -1 \\ a = -2 \\ b = 0 \end{array}$$

$$\Pi_1 = \frac{\Delta p}{\rho v^2}$$

$$\Pi_2 : \Pi_2 = v^a\, d^b\, \rho^c\, \mu \qquad \left(\mu \equiv \frac{M}{LT}\right)$$

$$\left.\begin{array}{r} M: 0 = C + 1 \\ L: 0 = a + b - 3C - 1 \\ T: 0 = -a - 1 \end{array}\right] \begin{array}{l} a = -1 \\ c = -1 \\ b = -1 \end{array}$$

$$\Pi_2 = \mu / vd\rho$$

$$\Pi_3 : \Pi_3 = v^a\, d^b\, \rho^c\, L$$

$$\left.\begin{array}{r} M: 0 = C \\ L: 0 = a + b - C + 1 \\ T: 0 = -a \end{array}\right] \begin{array}{l} a = 0 \\ c = 0 \\ b = -1 \end{array}$$

$$\Pi_3 = {}^{L}\!/\!_{d}$$

$$\frac{\Delta p}{\rho v^2} = f\left({}^{\mu}\!/\!_{vd\rho},\ \frac{L}{d}\right)$$

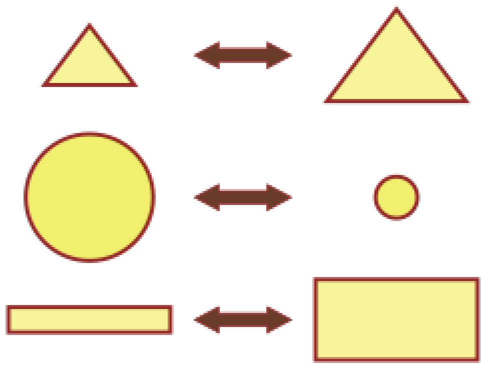
Previous Next

Module 11: Dimensional analysis and similitude

Lecture 39: Geomteric and dynamic similarities, examples

Similitude: To scale–up or down a model to the prototype, two types of similarities are required from the perspective of fluid dynamics: (1) geometrical similarity (2) dynamic similarity

1. Geometric similarity: The model and the prototype must be similar in shape.



(Fig. 39a)

This is essential because one can use a constant scale factor to relate the dimensions of model and prototype.

2. Dynamic similarity: The flow conditions in two cases are such that all forces (pressure viscous, surface tension, etc) must be parallel and may also be scaled by a constant scaled factor at all corresponding points. Such requirement is restrictive and may be difficult to implement under certain experiential conditions. Dimensional analysis can be used to identify the dimensional groups to achieve dynamic similarity between geometrically similar flows.

For example, in the flow past a sphere, drag on a model can be related to the prototype by a scale–factor if Reynolds numbers are matched. In other words,

$$\left(\frac{\rho\, ud}{\mu}\right)_{\mathrm{model}} = \left(\frac{\rho\, ud}{\mu}\right)_{\mathrm{prototype}}$$

Therefore, the types of fluid in two cases may be different:

$\left[(\rho,\mu)_{\mathrm{model-fluid}} \neq (\rho,\mu)_{\mathrm{prototype-fluid}}\right]$. Yet the drags on the two objects may be scaled as long as $(\mathrm{Re})_{\mathrm{model}} = (\mathrm{Re})_{\mathrm{prototype}}$.

Previous Next

Example: A sphere of diameter 1 ft is to be dragged at a speed of 8.45 ft/s in seawater at $5°C$. To predict the drag, we want to conduct an experiment on a 6" diameter sphere in air flowing past the model sphere. The drag on the model is to be kept at 6 lb-f. Determine the drag on the prototype.

Answer:

Ignoring any cavitation or compressibility effects, we have expression for drag:

$$F/(\rho v^2 d_p{}^2) = f\left(\frac{\rho v\, d_p}{\mu}\right)$$

Therefore, to predict the drag on the prototype one has to first match Reynolds number $\left(\frac{\rho v\, d_p}{\mu}\right)$ of two cases:

$$\left(\frac{\rho v\, d_p}{\mu}\right)_{model} = \left(\frac{\rho v\, d_p}{\mu}\right)_{prototype}$$

$d_p(\text{model}) = 6"$ and $d_p(\text{prototype}) = 1\text{ft}$

$\nu(\text{air}) = 1.5 \times 10^{-4}\ \text{ft}^2/s$ and $\nu\ (\text{sea} - \text{water}) = 1.69 \times 10^{-5}\ \text{ft}^2/s$

$v_{air} = ?$ if $v_{sea-water} = 8.45\,\text{ft/s}$

Therefore, $v_{air} = \left(\frac{1 \times 8.45}{1.69 \times 10^{-5}}\right) \times \frac{1.57 \times 10^{-4}}{0.5} = 157\,\text{ft/s}$

Therefore, $\left(F/\rho v^2\, d_p{}^2\right)_{model} = \left(F/\rho v^2\, d_p{}^2\right)_{prototype}$

$$F_{sea-water} = 6 \times \left(\frac{2}{.00238}\right)_{\rho} \times \left(\frac{8.45}{157}\right)_{v}^{2} \times \left(\frac{1}{0.5}\right)_{d_p}^{2} = 58.4\,\text{lb} - \text{f}$$

Previous Next