Module 2: Fluid Statics

Lecture 3: Pascal's theorem, Basic equation

- Fluid as a continuum or continuum based approach
- Fluid Statics
- The basic equation of fluid statics

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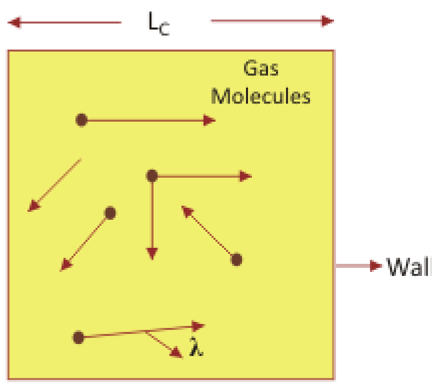
Fluid as a continuum or continuum based approach

- Fluid is made of molecules. However, for most of the engineering applications, when we speak of fluid's properties such as density, or conditions such as pressure and temperature, we do not imply such properties or conditions of individual molecules, but those of “fluid” as a whole.
- In other words, we refer to the average or macroscopic aggregate effects of the fluid-molecules, reflected in pressure, temperature, density, etc.
- Such an approach to treating a fluid is called continuum based approach. In other words, fluid is treated as continuum.
- However, there is a restriction. The continuum approach can be applied only when the mean free path of the fluid (largely, gas) is smaller (actually much smaller!!) than the physical characteristic length of the system under consideration, say, the diameter of the tube in which the gas flows, or size of a container in which gas is stored.



(Fig. 3a)

- The continuum approach is usually not valid when the gas pressure is very small (few milli-torr like in a vacuum), or the aperture size is small (like in an orifice)
- Mathematically, for the continuum approach based model to hold good, where λ is the mean free path of the gas molecule and L_C is the characteristic length of the system. Alternatively, Knudsen #defined as $\lambda / L_C << 1$.

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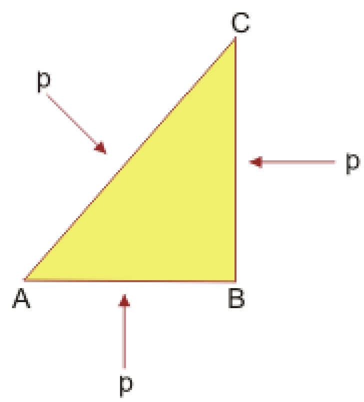
Fluid Statics

- Concerned with forces in a stationary fluid
- Recall. In a stationary fluid, shear stress $\tau = 0$. However, fluid can sustain the normal stress.

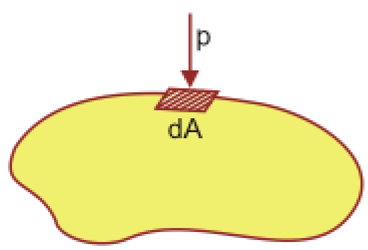
- Fluid pressure: It is a normal force per unit area of the fluid element, acting inward onto the element. Consider a fluid element, ABC:



(Fig. 3b)

Pressure acts inward on all three faces of the element.

Consider another fluid element:



(Fig. 3c)

Pressure also acts inward on the differential element dA

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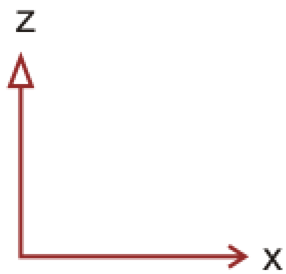
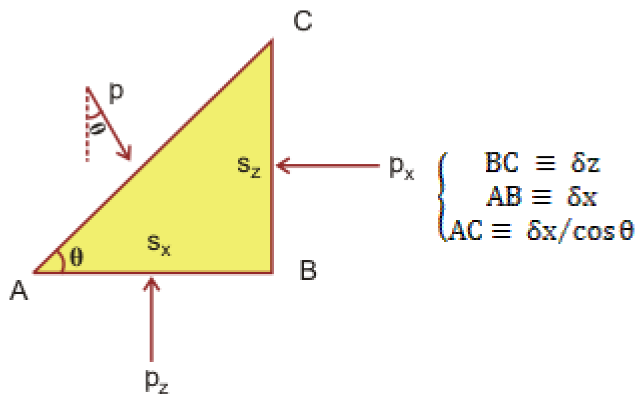
Module 2: Fluid Statics

Lecture 3: Pascal's theorem, Basic equation

- Pascal's Theorem-Pressure at any point within a static fluid is the same in all directions. Let us prove it. Consider a triangular fluid element ABC in a 2-D(x-z) scenario. Pressure acts on all 3-faces of the element inward and normal to the surface.



(Fig. 3d)



(Fig. 3e)

Force balance:

$$x-direction{:}-p_x\delta z+\left(p\frac{\delta x}{cos\theta}\right)sin\theta=0$$

$$y-direction{:}p_z\delta x+\left(p\frac{\delta x}{cos\theta}\right)sin\theta-\frac{1}{2}(\delta x\delta y.1)\rho g=0$$

Taking limit $\delta x\delta y\rightarrow 0$

$$p_z=p=p_x(Note\ \delta z=\delta x\ tan\theta)$$

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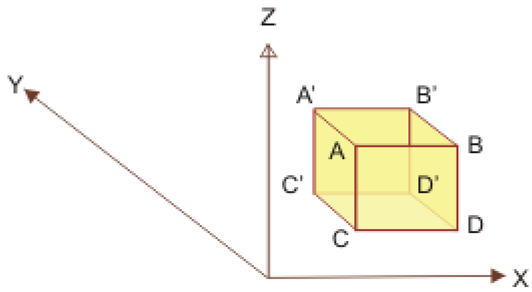
The basic equation of fluid statics

1. First, let us calculate force on a fluid element because of pressure variation. Consider an elemental fluid volume bounded by (ABCD)-(A'B'C'D') surfaces.



(Fig. 3f)

- Pressure at $c\ (x,y,z)$ is $p\ (x,y,z)$
- Volume of the fluid under consideration is $(\delta x\, \delta y\, \delta z)$
- Force acting on the face AA'C'C:

  $F_x = p\ \delta y\ \ \delta z$ (+ ve direction)

  Force acting on the face $(BB'D'D)$

  $F_x = \left(p + \frac{\partial p}{\partial x}.\delta x\right)\ \delta y\ \delta z$ (-ve direction)

  (Neglecting the higher order terms or assuming linear variation over δx)

  Differential force, $dFx = -\frac{\partial p}{\partial x}\ (\delta x\, \delta y\, \delta z)$ (acting along –ve x-direction)

  Similarly, $dFy = -\frac{\partial p}{\partial y}\ (\delta x\, \delta y\, \delta z)$

  $$dFz = -\frac{\partial p}{\partial z}\ (\delta x\, \delta y\, \delta z)$$

- Therefore, we have expression for differential pressure–force

  $\overrightarrow{dF}_{pressure} = -\left(\frac{\partial p}{\partial x}\ i + \frac{\partial p}{\partial y}\ j + \frac{\partial p}{\partial z}\ k\right)\delta x \delta y \delta z$

  or, $\boxed{\frac{\overrightarrow{d}\,F}{d\forall} = -\left(\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\right)\nabla p}$

- If there is no pressure variation or pressure is uniform around an element, the net force is zero,

  or $-\iint_s p dA = 0$

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