

**APPENDIX- B**

**PERFORMANCE ESTIMATION OF A TYPICAL  
SUBSONIC JET TRANSPORT AIRPLANE**

**(Lectures 38 – 40)**

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**Performance estimation of a typical subsonic  
jet transport airplane**

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**Abstract**

This report contains details of the performance estimation of a medium range jet airplane similar to B737. The following aspects are considered.

- Drag polar estimation
- Engine characteristics
- Level flight performance - stalling speed, maximum and minimum speeds
- Steady climb performance – maximum rate of climb, maximum angle of climb, service ceiling and absolute ceiling
- Range and endurance
- Steady level co-ordinated turn - minimum radius of turn, maximum rate of turn
- Take-off and landing distances

The report is intended to serve as an example of performance calculation of a typical jet airplane.

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##### **2 Estimation of drag polar**

- 2.1 Estimation of  $(C_{D0})_{WB}$
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## 1 Airplane Details

### 1.1 Overall Dimensions

Length	: 34.32 m
Wing span	: 32.22 m
Height above ground	: 11.17 m
Wheel base	: 13.2 m
Wheel track	: 5.8 m

### 1.2 Engine details

Similar to CFM 56 - 2B

Sea level static thrust	: 97.9 kN per engine
By pass ratio	: 6.5 (For which the engine characteristics are given in Ref.3*)
SFC	: $0.6 \text{ hr}^{-1}$ at $M = 0.8$ and $h = 10973 \text{ m}$ (36000 ft)

### 1.3 Weights

Gross weight	: 59175 kgf (580506.8 N)
Empty weight	: 29706 kgf (291415.9 N)
Fuel weight	: 12131 kgf (119005.1 N)
Payload	: 17338 kgf (170085.8 N)
Maximum landing weight	: 50296 kgf (493403.8 N)

### 1.4 Wing Geometry

Planform shape	: Cranked wing
Span	: 32.22 m
Area ( $S_{\text{ref}}$ )	: $111.63 \text{ m}^2$
Airfoil	: NASA - SC(2) series, $t/c = 14\%$ , $C_{\text{lopt}} = 0.5$
Root chord	: 5.59 m (Equivalent trapezoidal wing)
Tip chord	: 1.34 m (Equivalent trapezoidal wing)
Root chord of cranked wing	: 7.44 m
Portion of wing with straight trailing edge	: 11.28 m

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\* Reference numbers in this Appendix relate to those given on page 40.

Mean aerodynamic chord	: 3.9 m
Quarter chord sweep	: 27.69°
Dihedral	: 5°
Twist	: 3°
Incidence	: 1.4°
Taper ratio	: 0.24 (Equivalent trapezoidal wing)
Aspect ratio	: 9.3

### **1.5 Fuselage geometry**

Length	: 33 m
Maximum diameter	: 3.59 m

### **1.6 Nacelle geometry**

No. of nacelles	: 2
Nacelle diameter	: 1.62 m
Cross sectional area	: 2.06 m <sup>2</sup>
Length of nacelle	: 3.3 m (based on B737 Nacelle)

### **1.7 Horizontal tail geometry**

Span	: 11.98 m
Area	: 28.71 m <sup>2</sup>
Mean aerodynamic chord	: 2.67 m
Quarter chord sweep	: 32°
Root chord	: 3.80 m
Tip chord	: 0.99 m
Taper ratio	: 0.26
Aspect ratio	: 5

### **1.8 Vertical tail geometry**

Span	: 6.58 m
Area	: 25.43 m <sup>2</sup>
Root chord	: 5.90 m
Tip chord	: 1.83 m
Mean aerodynamic chord	: 4.22 m
Quarter chord sweep	: 37°

Taper ratio : 0.31  
Aspect ratio : 1.70

### 1.9 Other details

$C_{Lmax}$  without flap : 1.4  
 $C_{Lmax}$  with landing flaps : 2.7  
 $C_{Lmax}$  with T.O flaps : 2.16  
Maximum load factor ( $n_{max}$ ) : 3.5

### 1.10 Flight condition

Altitude : 10973 m (36000 ft)  
Mach number : 0.8  
Kinematic viscosity :  $3.90536 \times 10^{-5} \text{ m}^2/\text{s}$   
Density :  $0.3639 \text{ kg/m}^3$   
Speed of sound : 295.07 m/s  
Flight speed : 236.056 m/s  
Weight of the airplane : 59175 kgf (580506.8 N)

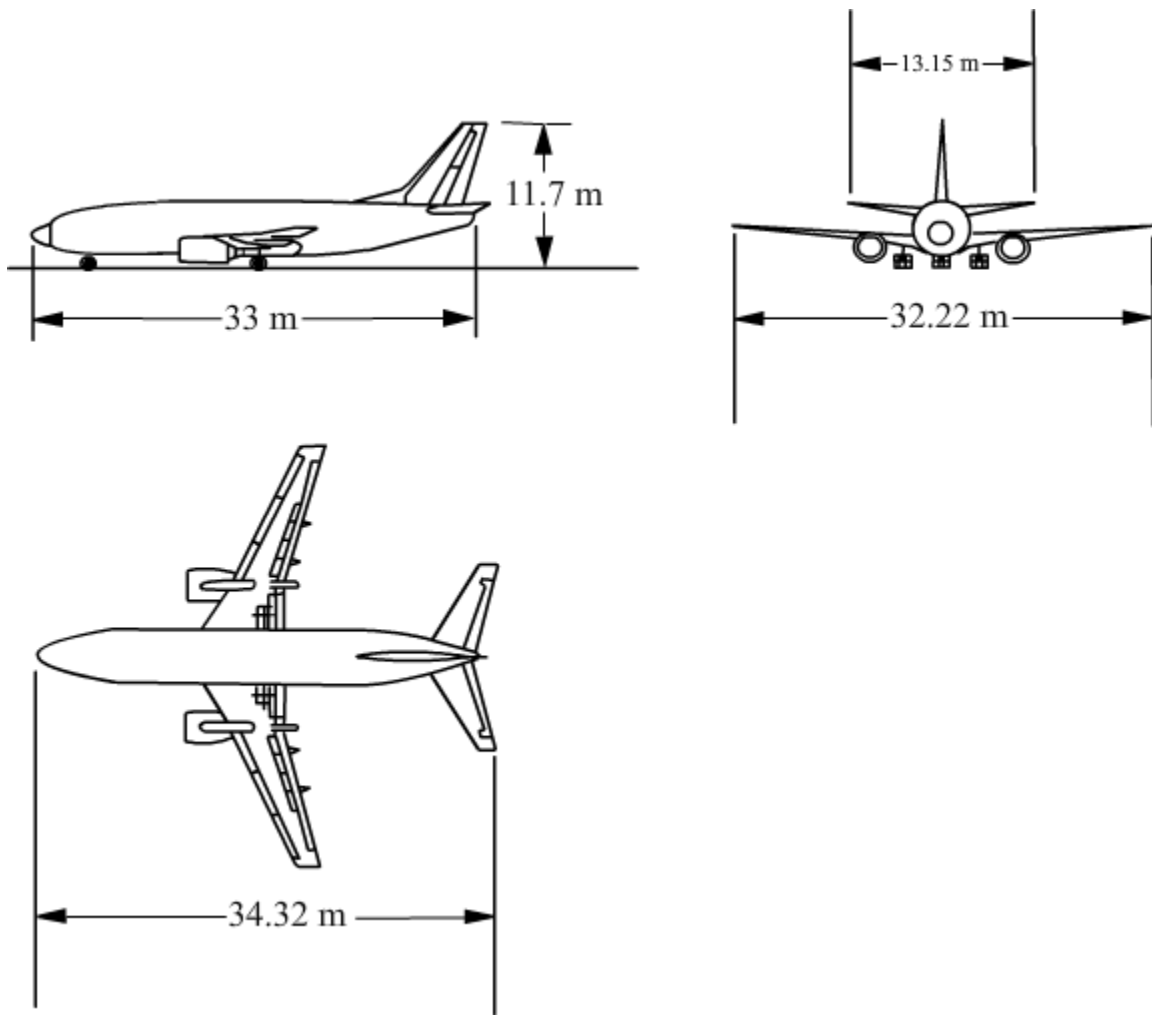


Fig.1 Three-view drawing of the airplane



## 2 Estimation of drag polar

The drag polar is assumed to be of the form:

$$C_D = C_{D0} + \frac{C_L^2}{\pi A e}$$

The quantity  $C_{D0}$  is assumed to be given by:

$$C_{D0} = (C_{D0})_{WB} + (C_{D0})_V + (C_{D0})_H + (C_{D0})_{Misc} \quad (1)$$

where suffices WB, V, H, Misc denote wing-body combination, vertical tail, horizontal tail, and miscellaneous contributions respectively.

### 2.1 Estimation of $(C_{D0})_{WB}$

Initially, the drag polar is obtained at a Mach number of 0.6 as suggested in Ref.5, section 3.1.2.  $(C_{D0})_{WB}$  is given as :

$$(C_{D0})_{WB} = (C_{D0})_w + (C_{D0})_B \frac{S_B}{S_{ref}}$$

The suffix B denotes fuselage and  $S_B$  is the maximum frontal area of fuselage.

$(C_{D0})_w$  is given as :

$$(C_{D0})_w = C_{fw} \left[ 1 + L \left( \frac{t}{c} \right) \right] \left( \frac{S_{wet}}{S_{ref}} \right)_{wing}$$

where,  $C_{fw}$  is the turbulent flat plate skin friction coefficient. The Reynolds number used to determine it ( $C_{fw}$ ) is lower of the two Reynolds numbers viz. Reynolds number based on the mean aerodynamic chord of the exposed wing ( $R_e$ ) and  $R_{ecutoff}$  based on surface roughness. Further,  $(S_{wet})_e$  is the wetted area of the exposed wing.

In the present case,  $c_r = 5.59m$ ,  $c_t = 1.34m$ ,  $b/2 = 16.11m$  and  $d_{fus} = 3.59m$ . Hence,

$$\text{Root chord of exposed wing} = c_{re} = 5.59 - \frac{5.59 - 1.34}{16.11} \times \frac{3.59}{2} = 5.116 \text{ m}$$

$$\lambda_e = \frac{1.34}{5.116} = 0.262$$

Hence, mean aerodynamic chord of exposed wing ( $\bar{c}_e$ ) is :

$$\bar{c}_e = \frac{2}{3} \left[ 5.116 \left( \frac{1 + 0.262 + 0.262^2}{1 + 0.262} \right) \right] = 3.596 \text{ m}$$

$$\text{Span of exposed wing} = (b/2)_e = 16.11 - 1.795 = 14.315m$$

Further,  $M = 0.6$ ,  $a = 295.07\text{m/s}$ . Hence,  $V = 177.12\text{m/s}$ .

Also  $v = 3.90536 \times 10^{-5} \text{ m}^2/\text{s}$ .

Hence,

$$R_e = \frac{177.12 \times 3.596}{3.90536 \times 10^{-5}} = 16.31 \times 10^6$$

The height of roughness corresponding to the standard camouflage paint, average application, is  $k = 1.015 \times 10^{-5} \text{ m}$  (Ref.5, table 3.1). Hence,  $l/k$  in this case is:

$$\frac{l}{k} = \frac{3.596}{1.015 \times 10^{-5}} = 3.543 \times 10^5$$

The  $R_{ecutoff}$  corresponding to the above  $l/k$  is  $30 \times 10^6$ . Consequently,  $C_{fw}$  corresponding to  $R_e = 16.31 \times 10^6$  is obtained from Fig.3.1 of Ref.5, as :

$$C_{fw} = 0.00265.$$

$(t/c)_{avg} = 14\%$  and  $(t/c)_{max}$  occurs at  $x/c > 0.3$  Hence,  $L = 1.2$  and

$$S_{exposedplanform} = 14.314 \left( \frac{5.116 + 1.341}{2} \right) \times 2 = 92.41 \text{ m}^2$$

$$S_{wetW} = 2 \times 92.41 (1 + 1.2 \times 0.14) = 215.8 \text{ m}^2$$

Hence,

$$(C_{Df})_w = 0.00265 (1 + 1.2 \times 0.14) \frac{215.8}{111.63} = 0.00598$$

$(C_{Do})_B$  is given as:

$$(C_{Do})_B = (C_{Df})_B + (C_{Dp})_B + C_{Db}$$

$$(C_{Do})_B = C_{Bb} \left[ 1 + \frac{60}{(l_b/d)^3} + 0.0025 \left( \frac{l_b}{d} \right) \right] \left( \frac{S_{wet}}{S_B} \right)_{fus} + C_{Db} \frac{S_{base}}{S_{ref}}$$

In the present case ,  $l_f = 33.0 \text{ m}$  ,  $d_{max} = 3.59 \text{ m}$  ,

$$R_{eb} = \frac{177.12 \times 33}{3.905 \times 10^{-5}} = 149.6 \times 10^6$$

$$\frac{l}{k} = \frac{33}{1.015 \times 10^{-5}} = 32.51 \times 10^5$$

The  $R_{ecutoff}$  corresponding to the above  $l/k$  is  $2.6 \times 10^8$ . The  $C_{fw}$  corresponds to  $R_{eb} = 149.6 \times 10^6$  measured from the graph in Ref.5, Fig.3.1 is:

$$C_{fw} = 0.0019$$

$$(S_{\text{wet}})_{\text{fus}} = 0.75 \times \pi \times 3.59 \times 33 = 279\text{m}^2$$

$$S_B = \frac{\pi}{4} \times 3.59^2 = 10.12\text{m}^2$$

Hence,

$$(C_{Df})_B = 0.0019 \times \frac{279}{10.12} = 0.0524$$

$$(C_{Dp})_B = 0.0019 \left[ \frac{60}{(33/3.59)^3} + 0.0025 \times (33/3.59) \right] \frac{279}{10.12} = 0.00524$$

Since, base area is almost zero,  $C_{Db}$  is assumed to be zero. Hence,

$$(C_{Do})_B = 0.0524 + 0.00524 + 0 = 0.0576$$

$(\Delta C_D)_{\text{canopy}}$  is taken as 0.002. Hence,  $(C_{Do})_B = 0.0596$

Finally,

$$(C_{Do})_{WB} = 0.00598 + 0.0596 \frac{10.12}{111.63} = 0.01138$$

## 2.2 Estimation of $(C_{Do})_V$ and $(C_{Do})_H$

The estimation of  $(C_{Do})_H$  and  $(C_{Do})_V$  can be done in a manner similar to that for the wing. However, the details regarding the exposed tail area etc. would be needed. In the absence of the detailed data on the shape of fuselage at rear, a simplified approach given in Ref.5, section 2.2 is adopted, wherein  $C_{Df} = 0.0025$  for both horizontal and vertical tails.

$$S_W = 2(S_h + S_v)$$

Hence,

$$(C_{Do})_{HV} = 0.0025(28.71 + 25.43) \frac{2}{111.63} = 0.0024 \quad (2)$$

## 2.3 Estimation of misc drag - nacelle

For calculating drag due to the nacelles the short cut method is used i.e.:

$$(C_{Do})_{\text{nacelle}} = 0.006 \times \frac{S_{\text{wet}}}{S_{\text{ref}}}$$

where,  $S_{\text{wet}}$  is the wetted area of nacelle. Here,  $S_{\text{wet}} = 16.79\text{m}^2$ . Since, there are two nacelles, the total drag will be twice of this. Finally,

$$(C_{Do})_{\text{nacelle}} = 0.006 \times \frac{16.79}{111.63} \times 2 = 0.0018$$

## 2.4 $C_{D0}$ of the airplane

Taking 2% for miscellaneous roughness and protuberances(Ref.5, section 3.4.6 ), the  $C_{D0}$  of the airplane is:

$$C_{D0} = 1.02 [0.01138 + 0.0024 + 0.0018] = 0.0159 \quad (3)$$

## 2.5 Induced drag

The induced drag component has the Oswald's efficiency factor  $e$  which is estimated by adding the effect of all the airplane components on induced drag (Ref.5, section 2.3).The rough estimate of  $e$  can be obtained as follows :

Figure 2.4 of Ref.5 is useful only for estimating  $e_{wing}$  of unswept wings of low speed airplanes. For the present case of swept wing, the following expression given in Ref.2 , chapter 7 is used.

$$e_{wing} = (e_{wing})_{\Lambda=0} \cos(\Lambda - 5)$$

where  $\Lambda$  is the quarter chord sweep. Ref.1, chapter 1 is used to estimate  $(e_{wing})_{\Lambda=0}$ . In the present case, with  $A = 9.3$  and  $\lambda = 0.24$ , the value of  $(e_{wing})_{\Lambda=0}$  is 0.97.

Hence,  $e_{wing} = 0.97 \times \cos(27.69 - 5) = 0.8948$ .

From Ref.5, section 2.3,  $\frac{1/e_{fus}}{(S_f/S)} = 0.8$  for a round fuselage. Hence,

$$\frac{1}{e_{fus}} = 0.8 \times \frac{10.122}{111.63} = 0.0725$$

Further, from Ref.5, section 2.3,  $\frac{1}{e_{other}} = 0.05$

Finally,

$$e = \frac{1}{0.8948^{-1} + 0.0725 + 0.05} = 0.8064$$

Hence,

$$K = \frac{1}{\pi A e} = \frac{1}{\pi \times 9.3 \times 0.8064} = 0.04244$$

### Remark:

Based on Ref.7, a detailed estimates of  $e_{wing}$  and  $e_{fuselage}$  are given in Ref.5, section 3.3.

For an untwisted wing the value of  $e_{wing}$  is given as:

$$e_{\text{wing}} = \frac{1.1(C_{L\alpha W}/A)}{R(\frac{C_{L\alpha W}}{A}) + (1-R)\pi}$$

where,

$$C_{L\alpha W} = \frac{2\pi A}{2 + \sqrt{\frac{A^2 \beta^2}{\kappa^2} \left( 1 + \frac{\tan^2 \Lambda_{1/2}}{\beta^2} \right)} + 4}$$

$C_{L\alpha W}$  = slope of lift curve of wing per radian

A = aspect ratio of wing

R = a factor which depends on (a) Reynolds number based on leading edge radius, (b) leading edge sweep ( $\Lambda_{LE}$ ), (c) Mach number (M), (d) wing aspect ratio (A) and (e) taper ratio ( $\lambda$ ).

$$\beta = \sqrt{1-M^2}$$

$\Lambda_{1/2}$  = sweep of semi-chord line

$\kappa$  = ratio of the slope of lift curve of the airfoil used on wing divided by  $2\pi$ . It is generally taken as unity.

In the present case,

M = 0.6, h = 10973 m (36000 ft), V = 177.12 m,  $\nu = 3.90536 \times 10^{-5} \text{ m}^2/\text{s}$ , S = 111.63 m<sup>2</sup>

b = 32.22 m,  $c_{re} = 5.59$  m,  $c_t = 1.34$  m,  $\Lambda_{1/4} = 27.69$  deg,

Hence, A = 9.3,  $\lambda = 0.24$ ,  $\beta = 0.8$ ,  $\tan \Lambda_{1/2} = 0.4589$ ,  $\cos \Lambda_{LE} = 0.8609$ ,

Average chord = 3.615 m

The airfoil is NASA – SC(2) with 14 % thickness. From Ref.8 the leading edge radius is 3 % of the chord.

From these data:

$$C_{L\alpha W} = 5.404$$

$R_{Ler}$  = Reynolds number based on leading edge radius =  $4.974 \times 10^4$

$$R_{Ler} \times \cot \Lambda_{LE} \times \sqrt{1-M^2 \cos^2 \Lambda_{LE}} = 7.198 \times 10^5$$

$$\frac{A\lambda}{\cos \Lambda_{LE}} = 2.592$$

Corresponding to these data,  $R = 0.943$  is obtained from Ref.5, Fig.3.14. Consequently,

$$e_{\text{wing}} = \frac{1.1 \times 5.404 / 9.3}{0.943 \times \left( \frac{5.404}{9.3} \right) + (1 - 0.943) \times \pi} = 0.8793$$

This value of  $e_{\text{wing}}$  is close to the value of 0.8948 obtained by the simpler approach. However, detailed approach is recommended for wings with sweep of above  $35^\circ$ . Reference 7, section 4.5.3 contains guidelines for estimating drag of wing-body-tail combination with allowance for trim drag.

## 2.6 Final drag polar

$$C_D = 0.0159 + 0.04244 C_L^2 \quad (4)$$

The drag polar is presented in Fig.2.

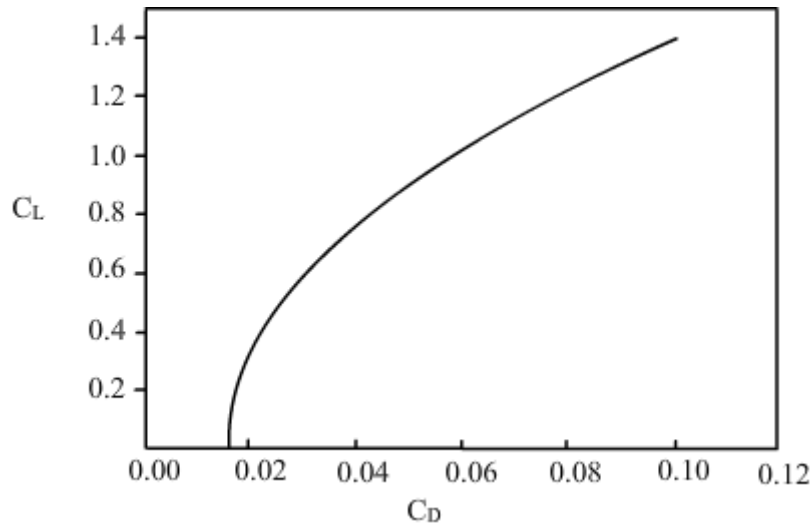


Fig.2 Drag polar at sub-critical Mach numbers

### Remarks:

- i) The polar given by Eq.(4) is valid at subcritical Mach numbers. The increase in  $C_{D0}$  and  $K$  at higher Mach numbers is discussed in section 4.2.

ii) The maximum lift to drag ratio  $((L/D)_{\max})$  is given by:

$$(L/D)_{\max} = \frac{1}{2\sqrt{C_{D0}K}}$$

Using  $C_{D0}$  and  $K$  from Eq.(4),  $(L/D)_{\max}$  is 19.25, which is typical of modern jet transport airplanes.

iii) It may be noted that the parabolic polar is an approximation and is not valid beyond  $C_{L\max}$ . It is also not accurate close to  $C_L = 0$  and  $C_L = C_{L\max}$ .