Chapter 3

Lecture 8

Drag polar – 3

Topics

3.2.7 Boundary layer separation, adverse pressure gradient and favourable pressure gradient

3.2.8 Boundary layer transition

3.2.9 Turbulent boundary layer over a flat plate

3.2.10 General remarks on boundary layers

3.2.7 Boundary layer separation, adverse pressure gradient and favourable pressure gradient

When the flow takes place around airfoils and curved surfaces, the velocity outside the boundary layer is not constant. From Bernoulli's equation it can be deduced that when the velocity decreases the pressure increases and vice-versa. When the velocity is decreasing i.e. dp/dx is positive, the pressure gradient is called 'Adverse pressure gradient'. When dp/dx is negative it is called 'Favourable pressure gradient'.

Figure 3.14 shows the development of a boundary layer in an external stream with adverse pressure gradient (dp/dx > 0). Such a flow may occur on the upper surface of an airfoil beyond the point of maximum thickness. Since the static pressure at a station remains almost constant across the boundary layer, the pressure inside the boundary layer at stations separated by distance Δx also increases in the downstream direction.

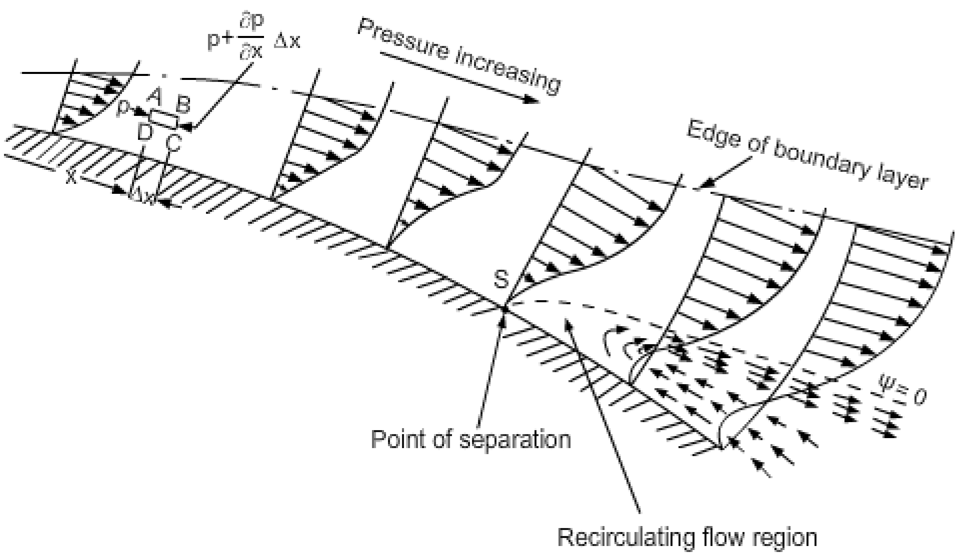

Fig.3.14 Flow in boundary layer before and after point of separation (not to scale)

Figure 3.14 also shows a small element ABCD in the boundary layer. The pressure on the face AD is p whereas that on the face BC is $\left[p+(dp/dx)\Delta x\right]$. Since dp/dx is positive in this case, the net effect causes a deceleration of the flow, in addition to that due to viscosity. The effect is more pronounced near the surface and the velocity profile changes as shown in Fig.3.14. Finally at point S the slope of the velocity profile at the wall, $(\partial U/\partial y)_{wall}$, becomes zero. Besides the change in shape, the boundary layer also thickens rapidly in the presence of adverse pressure gradient. Downstream of the point S, there is a reversal of the flow direction in the region adjacent to the wall. A line can be drawn (indicated as dotted line in Fig.3.14) in such a way that the mass flow above this line is the same as that ahead of point S. Below the dotted line, there is a region of recirculating flow and the value of the stream function (ψ) for the dotted line is zero. However, ahead of the point S, the $\psi = 0$ line is the surface of the body. Thus, after the point S, it is observed that between the main flow (i.e. region above $\psi = 0$ line) and the body surface lies a region of recirculating flow. When this happens the flow is said to be 'Separated' and S is referred to as the 'Point of separation'. Due to separation, the pressure recovery, which would have taken place in an unseparated flow, does not take place and the pressure drag of the body increases.

Remarks:

(i) If the adverse pressure gradient is very gradual then separation may not take place (Refer to Ch. 8 of Ref.3.11 for dp/dx needed for separation).

(ii) Separation does not take place when the pressure gradient is favourable.

(iii) In the two-dimensional case shown in Fig.3.14 the gradient $\partial U / \partial y$ is zero at the point of separation. Hence c_f is zero at this point. This behaviour is used in computations to determine the location of the separation point.

3.2.8 Boundary layer transition

In a laminar boundary layer, either the flow variables at a point have constant values or their values show a definite variation with time. However, as Reynolds number increases, it is found that the flow variables inside the boundary layer show chaotic variation with time. Such a boundary layer is called 'Turbulent boundary layer'. The change over from laminar to turbulent boundary layer is called 'Transition' and takes place over a distance called 'Transition length'.

Initiation of transition in a boundary layer, can be studied as an instability phenomenon. In this study, the flow is perturbed by giving a small disturbance and then examining whether the disturbance grows. Details of the analysis are available in chapter 15 of Ref.3.11 and chapter 5 of Ref.3.13. The salient features can be summarized as follows.

1. In a boundary layer on a flat plate with uniform external subsonic stream (U_e = constant), the flow becomes sensitive to some disturbances as R_x exceeds 5×10^5. This is called 'Critical Reynolds number (R_{crit})'. For boundary layers in other cases, R_{crit} depends on Mach number, surface curvature, pressure gradient in external stream and heat transfer from wall.

2. After R_{crit} is exceeded, some disturbances grow. These are called 'Tollmien – Schlichting (T-S) waves'.

3. The T-S waves lead to three-dimensional unstable waves and formation of isolated large scale vortical structures called turbulent spots.

4. The turbulent spots grow and coalesce to form fully turbulent flow.

Remarks:

(i) As R_{crit} is exceeded only some disturbances grow and hence in flows with very low free stream turbulence level, R_{crit} as high as 2.8 x 10^6 has been observed in experiments. It may be recalled from fluid mechanics that the flow in pipe can become turbulent when Reynolds number, based on pipe diameter (R_{ed}), exceeds 2000. But laminar flow has been observed, in very smooth pipes, even at R_{ed} = 40,000.

(ii) Transition process takes place over a length called transition length. Reference 3.11, chapter 15 gives some guidelines for estimating this length. Surface roughness reduces this length.

(iii) In flows with external pressure gradient, the transition is hastened by adverse pressure gradient. It is generally assumed that transition does not take place in favourable pressure gradient.

3.2.9 Turbulent boundary layer over a flat plate

When the flow is turbulent, one of its dominant features is that the velocity at a point is a random function of time(Fig.3.15). When a quantity varies in a random manner, one cannot say as to what the value would be at a chosen time, though the values may lie within certain limits. In such a situation, the flow features are described in terms of statistical averages. For example, the average $\bar{U}$ of a fluctuating quantity U is given by :

$$\bar{U}(T_0) = \frac{1}{2T}\int_{T_0-T}^{T_0+T} U\,dt \qquad (3.31)$$

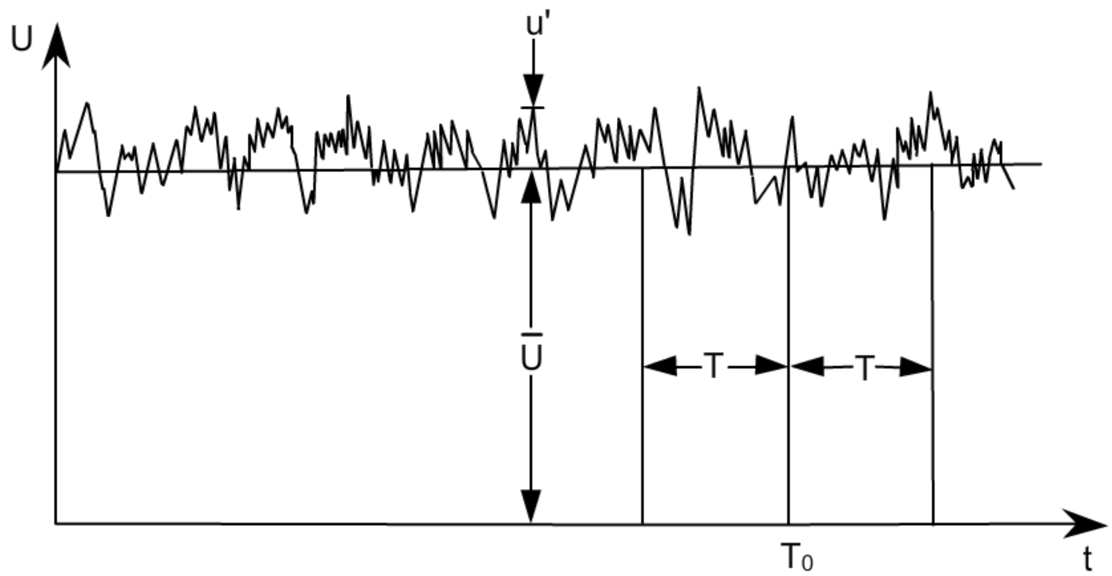

Fig.3.15 Typical turbulence signal

If the quantity $\bar{U}$ is independent of T_0 then the phenomenon is called 'Stationary random phenomenon'. The discussion here is confined to this type of flow. In such a case, the instantaneous value, $U(t)$, is expressed as :

$U(t) = \bar{U} + u'(t);\ u' = U - \bar{U}$.

By definition $\bar{u}' = 0$. Hence, to distinguish different turbulent flows the root mean square (r.m.s.) of the fluctuating quantity is used.

$$(u_{rms})^2 = \overline{u'^2} = \lim_{T \to \infty} \frac{1}{2T}\int_{-T}^{T} u'^2\, dt \tag{3.32}$$

r.m.s. value of u' is $\sqrt{\overline{u'^2}}$

Another feature of turbulent flows is that even if the mean flow is only in one direction, the fluctuations are in all three directions i.e. the instantaneous velocity vector ($\bar{\mathbf{V}}$) at a point would be

$\bar{\mathbf{V}} = (\bar{U} + u')\mathbf{i} + v'\mathbf{j} + w'\mathbf{k}$; u', v' and w' are the components of the fluctuating velocity along x, y and z directions.

Because of the random fluctuations, the transfer of heat, mass and momentum is many times faster in turbulent flows than in laminar flows. However, the part of the kinetic energy of the mean motion which gets converted into the random fluctuations is finally dissipated into heat and as such losses are higher when the flow is turbulent.

Characteristics of turbulent boundary layer:

Analysis of turbulent boundary layer is more complicated than that of laminar boundary layer. Reference 3.11, chapters 16 to 22 can be referred to for details. A few results are presented below.

Velocity profile:

Velocity profile of a turbulent boundary layer on a flat plate with zero pressure gradient is also shown in Fig.3.12. The profile can be approximated by a power law like:

$$\frac{\bar{U}}{U_e} = (y/\delta)^{1/7} ; 5\times10^5 < Re < 10^7 \quad (3.33)$$

This approximation is called '1/7th power law profile'.

Boundary layer thickness $(\delta_{0.99})$:

Though the velocity gradient $(\partial\bar{U}/\partial y)$ near the wall is much higher for turbulent boundary layer than for the laminar case, the gradient is lower away from the wall and $\delta_{0.99}$ is much higher for a turbulent boundary layer. Reference 3.13, chapter 6, gives the following expression for $\delta_{0.99}$.

$$\frac{(\delta_{0.99})_{turb}}{x} = \frac{0.16}{R_x^{1/7}} \quad (3.34)$$

Skin friction:

The value of $(\partial U/\partial y)_{wall}$ is higher for turbulent boundary layer than for laminar boundary layer (Fig.3.12). Hence, the skin friction drag for turbulent boundary layer is much higher than that for a laminar boundary layer. Reference 3.13, chapter 6 gives the following expression for C_{df}.

$$C_{df} = \frac{0.031}{R_L^{1/7}} \quad (3.35)$$

Remarks:

(i) In Eqs.(3.34) and (3.35) it is assumed that the boundary layer is turbulent from the leading edge. Corrections to these expressions can be applied by taking the start of the transition region as the origin of the turbulent boundary layer. However, at the values of R_L obtained in actual airplanes the error in C_{df}, by ignoring the laminar region is small.

(ii) In certain references following expressions are found for $\delta_{0.99}$ and C_{df}.

$$\left(\delta_{0.99} / x\right) = 0.37 / R_x^{\frac{1}{5}} \text{ and } C_{df} = 0.072 / R_L^{\frac{1}{5}}.$$

However, Ref.3.13 chapter 6 shows that Eqs. (3.34) and (3.35) are more accurate.

(iii) For the 1/7th power law profile of the turbulent boundary layer (Eq.3.33), it can be shown using Eqs.(3.26) and (3.33) that :

$$\delta_1 = \frac{\delta}{8} \quad \text{for turbulent boundary layer} \qquad (3.36)$$

Example 3.2

Consider a case with L = 0.5 m, $V_\infty = 30\,m/s$, $\upsilon = 15\times10^{-6}\,m^2/s$ which gives $R_L = 10^6$. Obtain the values of $\delta_{0.99}$, δ_1 and C_{df} in the following cases.

(i) Assume that the boundary layer is laminar throughout even at $R_L = 10^6$

(ii) Assume that the boundary layer is turbulent from landing edge of plate.

Solution:

(i) Laminar flow

$$\frac{\delta_{0.99}}{L} = \frac{5}{\sqrt{R_L}} = \frac{5}{\sqrt{10^6}} = 0.005 \text{ or } \delta_{0.99} = 2.5 \text{ mm}$$

$$\frac{\delta_1}{L} = \frac{1.721}{\sqrt{R_L}} = \frac{1.721}{\sqrt{10^6}} = 0.001721 \text{ or } \delta_1 = 0.86 \text{ mm}$$

$$C_{df} = \frac{1.328}{\sqrt{R_L}} = \frac{1.328}{\sqrt{10^6}} = 0.001328$$

(ii) Turbulent flow

$$\frac{\delta_{0.99}}{L} = \frac{0.16}{R_L^{1/7}} = 0.02223 \text{ or } \delta_{0.99} = 11.12 \text{ mm}$$

$$\delta_1 \approx \frac{\delta}{8} = 1.39 \text{ mm}$$

$$C_{df} = \frac{0.031}{R_L^{1/7}} = \frac{0.031}{\sqrt[7]{10^6}} = 0.00431$$

Remark :

The comparison of the above results points out that the values of $\delta_{0.99}$ and δ_1 are larger when the boundary layer is turbulent than when it is laminar. The value of C_{df} in the former case is nearly three times of that in the later case.

3.2.10 General remarks on boundary layers

In this subsection the following four topics are briefly touched upon.
(i) Calculation of boundary layer, (ii) Separation of turbulent boundary layer,
(iii) Laminar flow airfoil and (iv) Effect of roughness on transition and skin friction

(i) Calculation of boundary layer

To calculate the boundary layer over an airfoil the first step is to obtain the velocity distribution using potential flow theory. It may be recalled from aerodynamics, that in potential flow analysis the velocity on the surface of the body is not zero. It is assumed that this velocity distribution, given by potential flow, would roughly be the distribution of velocity outside the boundary layer (U_e). From this velocity distribution and using Bernoulli's equation, the first estimate of dp/dx is obtained. Based on this data the growth of laminar boundary layer and the location of transition point are determined. After the transition, the growth of turbulent boundary layer is calculated. After obtaining the boundary layers the displacement thickness (δ_1) is added to the airfoil shape and calculations are repeated till the displacement thickness assumed at the beginning of an iteration is almost same as that obtained after calculation of the boundary layer. Subsequently, the skin friction drag can be calculated. Section 18.4 of Ref.3.11 may be consulted for details. Presently, Computational Fluid Dynamics (CFD) is used for these calculations.

(ii) Separation of turbulent boundary layer

A turbulent boundary layer may also separate from the surface when it is subjected to adverse pressure gradient. However, due to turbulent mixing the value of $(\partial U/\partial y)_w$ for separation to take place is much higher than that in the case of laminar boundary layer. Hence, a turbulent boundary layer has a higher resistance to separation. This behaviour is used in bluff bodies to delay the separation and reduce their pressure drag. For example, in the case of a circular cylinder the laminar boundary layer separates at around 80^0 leaving a large

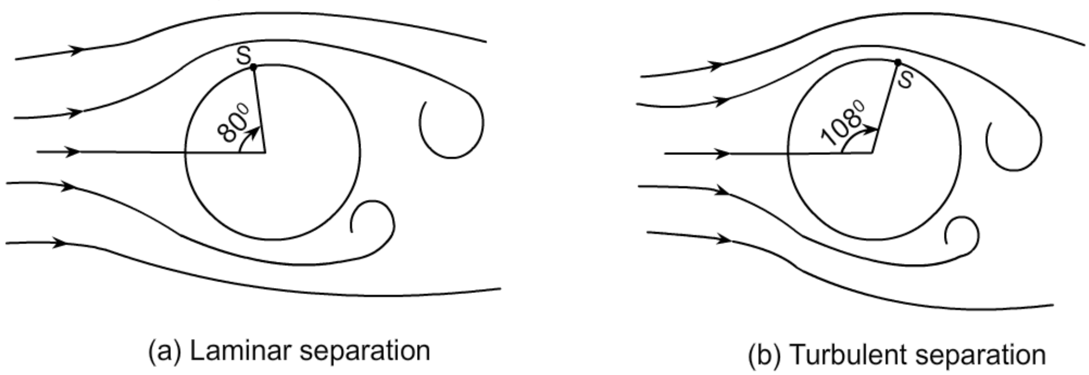

Fig.3.16 Schematic of flow past circular cylinder (a) Laminar separation (b) Turbulent separation

separated region (Fig.3.16a). However, if the transition to turbulent flow takes place before separation of laminar boundary layer, the separation is delayed. A turbulent layer separates at around 108°, giving a smaller separated region (Fig.3.16b). Since the drag of a bluff body is mainly pressure drag, the total drag decreases significantly when the flow is turbulent before separation.

For example, the drag coefficient of a circular cylinder is around 1 when the separation is laminar and it is 0.3 when the separation is turbulent (Refer chapter 1 of Ref.3.11)

(iii) **Laminar flow airfoils**

For a streamlined body, like an airfoil at low angle of attack, the drag is mainly skin friction drag. Figure 3.17 and Eqs.(3.30) and (3.35) indicate that C_{df} is much higher when boundary layer is turbulent. Hence, to reduce the drag of the airfoil, its shape is designed in such a way that the transition to turbulence is delayed and the flow remains laminar over a longer portion of the airfoil.

These airfoils are called 'Laminar flow or low drag airfoils'. Presently, efforts are in progress to delay the transition by boundary layer control (see remark in section 3.7.2).

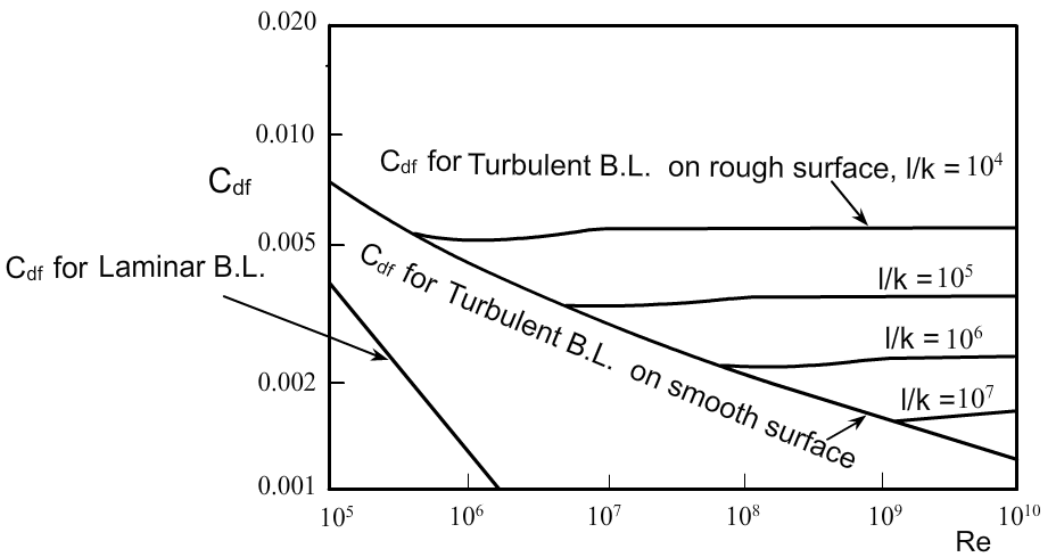

Fig.3.17 Skin friction drag coefficient at various Reynolds numbers and levels of roughness

(iv) Effect of roughness on transition

It was mentioned that the critical Reynolds number (R_{crit}) depends on factors like pressure gradient, Mach number, surface curvature and heat transfer. However, the onset of transition may be delayed when disturbance like free stream turbulence is low. However, if the surface is rough, this delay may not be observed when roughness exceeds a certain value (see chapter 15 of Ref.3.11)

(v) Effect of roughness on skin friction in turbulent boundary layer

Equation (3.35) indicates that C_{df} is proportional to $R_L^{-1/7}$ i.e. C_{df} decreases with R_L. However, when the surface is rough it is observed that the decrease in C_{df} stops after a certain Reynolds number(Fig.3.17). This Reynolds number is called 'Cut-off Reynolds number' and is denoted by $(Re)_{cut\text{-}off}$.

The roughness is quantified by the parameter (l/k), where

l = characteristic length e.g. the length (L) in case of a flat plate and chord (c) in case of an airfoil.

k = height of roughness referred to as equivalent sand roughness.

Following Ref.3.6, chapter 3, typical values of k are given in table 3.3.

Type of surface	Equivalent sand roughness (m)
Natural sheet metal	4.06×10^{-6}
Smooth paint	6.35×10^{-6}
Standard camouflage paint	1.02×10^{-5}
Mass production paint	3.048×10^{-5}

Table 3.3 Equivalent sand roughness for typical surfaces

Based on Ref.3.11, chapter 18, Fig.3.17 shows typical plots of C_{df} vs Re for turbulent boundary layer with l/k as parameter. For example, when $l/k = 10^5$, C_{df} remains almost constant at 0.0032 beyond Re = 7×10^6. Reference 3.6 section 3.1 may be seen for plot of $(Re)_{cutoff}$ vs (l/k) with Mach number as parameter. These plots are based on section 4.1.5, of Ref.3.5.