

APPENDIX - A

PERFORMANCE ANALYSIS OF A PISTON ENGINED AIRPLANE – PIPER CHEROKEE PA-28-180 (Lectures 35 - 37)

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Performance Analysis of a piston engined airplane – Piper Cherokee PA-28-180

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ABSTRACT

The report is intended to serve as an example of performance calculation of a typical piston engined airplane.

Problem statement: Obtain the following for the prescribed airplane:

- Information about the airplane.
- Drag Polar at cruising speed and during take-off condition.
- Engine Characteristics.
- Variation of stalling speed with altitude for flaps up and flaps down conditions.
- Variations of the maximum speed (V_{\max}) and minimum speed (V_{\min}) with altitude.
- Variations of maximum rate of climb $(R/C)_{\max}$ and maximum angle of climb (γ_{\max}) with speed and altitude. Variation of $V_{R/C\max}$ and $V_{\gamma\max}$ with altitude. Values of absolute ceiling and service ceiling.
- Variations of range and endurance with flight speed in constant velocity flights at cruising altitude. Speeds corresponding to R_{\max} and E_{\max} .
- Variation of minimum radius of turn (r_{\min}) and maximum rate of turn $(\dot{\psi}_{\max})$ at selected altitudes and variations of $(V_{r\min})$ and $(V_{\dot{\psi}\max})$ with altitude.
- Take-off and landing distances.

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Lecture 35

Performance analysis of a piston engined airplane –1

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1. Information about the airplane

Airframe: **Piper Cherokee PA-28-180**

Type: Piston-engined propeller driven low speed recreational airplane.

Manufacturer and country of origin: The Piper Airplane Corporation, USA.

1.1 Overall dimensions*

Length	: 7.148 m
Wing span	: 9.144 m
Height above ground	: 2.217 m
Wheel base	: 1.897 m
Wheel track	: 3.048 m

1.2 Power plant

Name	: Lycoming O-360-A3A
Rating	: 180BHP (135 kW) at 2700 RPM
Weight	: 129 kgf (1265.5 N)
Number	: 1
Propeller	: 1.88 m diameter, fixed pitch.

1.3 Weights

Maximum take-off weight	: 1088 kgf (10673.28 N)
Empty weight	: 558 kgf (5473.98 N)
Fuel capacity	: 50 US gallons (189 litres) usable 178.63 litres
Payload	: 468.1 kgf (4592.06 N)
Maximum wing loading	: 73.2 kgf/m ² (718.1 N/m ²)
Maximum power loading (P/W)	: 0.1241 kW/kgf (0.01265 kW/N)

1.4 Wing geometry

Planform shape	: Trapezoidal near root, rectangular afterwards and elliptical fillets at the tip.
Span (b)	: 9.144 m
Reference area (S or S _{Ref})	: 14.864 m ²

* The dimensions / areas are based on Fig.1 and the additional details given in Ref.2.

Flap area	: 1.384 m ²
Aileron area	: 1.003 m ²
Airfoil	: NACA 65 ₂ -415, t/c = 15 %, C _{lopt} = 0.4
Root chord	: 2.123 m
Tip chord	: 1.600 m
Quarter chord Sweep	: 1.48°
Dihedral	: 6°
Twist	: -2°
Incidence	: 4.62° at root, 2.62° at tip
High lift devices	: Simple flaps having 3 different settings : 10°, 25° and 40°

Derived parameters of wing:

(i) Aspect ratio (A) :

$$A = b^2 / S = 9.144^2 / (14.864) = 5.625$$

(ii) Root chord of equivalent trapezoidal wing (c_{req}) :

$$S = \frac{b}{2} (c_{req} + c_t)$$

$$\text{Or } 14.864 = \frac{9.144}{2} (c_{req} + 1.60)$$

$$c_{req} = 1.651 \text{ m}$$

(iii) Root chord of exposed wing (c_{re}):

From Fig.1, the maximum fuselage width is 1.168 m. Hence semi span of the exposed wing (b_e / 2) is:

$$\frac{b_e}{2} = \frac{1}{2} (9.144 - 1.168) = 3.988 \text{ m}$$

(iv) The root chord of exposed equivalent wing (c_{re}) is obtained as follows.

An expression for the chord of the equivalent wing is

$$c = 1.651 - \frac{y}{b/2} (1.651 - 1.600)$$

Hence,

$$c_{re} = 1.651 - \frac{0.584}{9.144/2} (1.651 - 1.600) = 1.644 \text{ m}$$

(v) Taper ratio of the exposed wing (λ_e) is:

$$\lambda_e = 1.6 / 1.644 = 0.9732$$

(vi) Mean aerodynamic chord of the exposed wing (\bar{c}_e)

$$\bar{c}_e = \frac{2}{3} c_{re} \frac{(1 + \lambda_e + \lambda_e^2)}{1 + \lambda_e} = \frac{2}{3} \times 1.644 \left\{ \frac{(1 + 0.9732 + 0.9732^2)}{1 + 0.9732} \right\} = 1.622 \text{ m}$$

(vii) Planform area of the exposed wing (S_e) is:

$$S_e = 3.988 (1.644 + 1.6) = 12.937 \text{ m}^2$$

(viii) Wetted area of exposed wing ($(S_{wet})_e$) is :

$$(S_{wet})_e = 2 S_e \{ 1 + 1.2 \times (t/c) \} = 2 \times 12.937 \{ 1 + 1.2 \times 0.15 \} = 30.53 \text{ m}^2$$

1.5 Fuselage geometry

Length (l_b) : 6.547 m (measured from Fig.1)

Frontal area (S_b) : 1.412 m² (Ref.2 p.179)

Maximum width : 1.168 m

Derived parameters for fuselage:

(i) Equivalent diameter (d_e) of fuselage :

$$\frac{\pi}{4} d_e^2 = 1.412 \text{ or } d_e = 1.341 \text{ m}$$

(ii) Height of maximum cross section (h_{max})

$$h_{max} = 1.412 / 1.168 = 1.209 \text{ m.}$$

(iii) Rough estimate of wetted area of fuselage ($(S_s)_e$) is :

$$(S_s)_e = 0.75 \times (\text{perimeter of the maximum cross section}) \times l_b$$

$$= 0.75 (1.209 + 1.168) \times 2 \times 6.547 = 23.34 \text{ m}^2.$$

(iv) Fineness ratio of fuselage (A_f) :

$$A_f = l_b / d_e = 6.547 / 1.341 = 4.882$$

1.6 Horizontal tail geometry

Plan-form shape : Rectangular with elliptical fillets at tips.

Span : 3.048 m

Area : 2.267 m²

Root chord and tip chord : 0.762 m

Airfoil : NACA 0012.

Derived parameters of horizontal tail:

(i) Aspect ratio = $A_t = 3.048^2 / 2.267 = 4.098$

(ii) Exposed area of horizontal tail = area of h.tail – area inside fuselage $\approx 2.15 \text{ m}^2$
Hence wetted area of h.tail $(S_{\text{wet}})_h$ is :

$$(S_{\text{wet}})_h = 2 \times 2.15 [1 + 1.2 \times 0.12] = 4.919 \text{ m}^2$$

1.7 Vertical tail geometry

Span	: 1.219 m
Area	: 1.059 m^2
Root chord	: 1.182 m
Tip chord	: 0.517 m
Quarter chord sweep	: 21.8°
Airfoil	: NACA 0010.

Derived parameters of vertical tail:

(i) Taper ratio : 0.4374

(ii) Aspect ratio : 1.403

(iii) Exposed area of vertical tail : same as area of v.tail = 1.059 m^2

(iv) Wetted area of v.tail $(S_{\text{wet}})_v$ is :

$$(S_{\text{wet}})_v = 2 \times 1.059 \{ 1 + 1.2 \times 0.1 \} = 2.372 \text{ m}^2$$

(v) Mean aerodynamic chord of vertical tail is :

$$\bar{c}_{vt} = (2/3) \times 1.182 \times (1 + 0.4374 + 0.4374^2) / (1 + 0.4374) = 0.893 \text{ m}.$$

1.8 Landing gear

Type	: Non-retractable, nose wheel type with fairing.
Number of wheels	: Nose 1, main 2, all same size.
Thickness	: 0.135 m
Diameter	: 0.4547 m
Wheel base	: 1.897 m
Wheel track	: 3.048 m

1.9 Flight condition

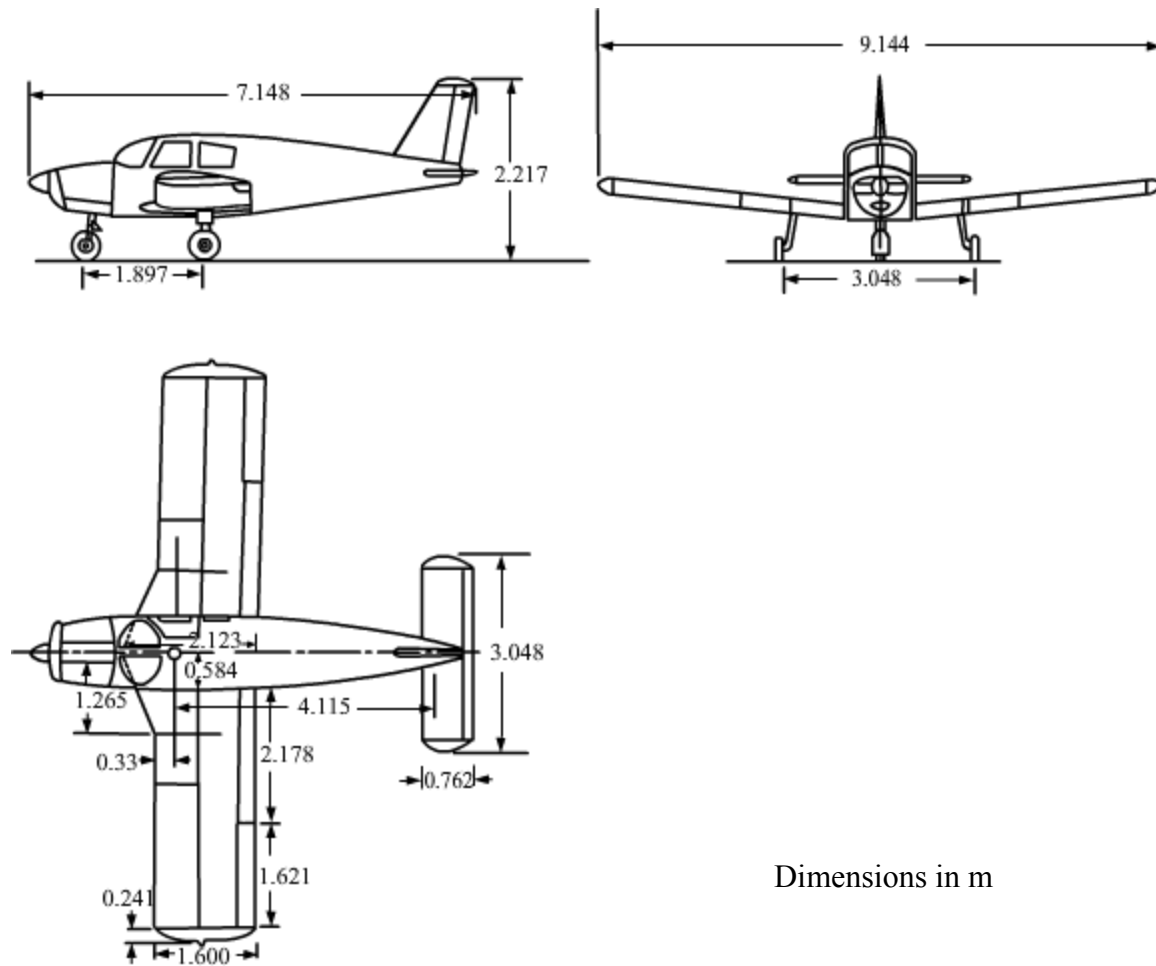
Altitude	: 2438 m (8000')
Density	: 0.9629 kg/m ³
Speed of sound	: 330.9 m/s
Kinematic viscosity (ν)	: 0.17792×10^{-4} (m ² /s)
Flight speed	: 237 km/hr (65.83 m/s)
Mach number	: 0.1992
Weight of the airplane	: 1088 kgf (10673.28 N)

1.10 Performance of PA-28-181^{\$} as given in Ref.3.

Maximum take-off weight	: 1157 kgf (2550 lbf)
Power plant rating	: 135 kW (180 BHP)
Wing loading	: 73.3kgf/ m ²
Maximum level speed	: 246 kmph
Cruising speed	: 237 kmph
Stalling speed	: 86 kmph, with flaps down condition
Maximum rate of climb	: 203 m/min at sea level
Service ceiling	: 4035 m
Take-off run	: 350 m
Take-off to 15m	: 488 m
Landing run	: 280 m
Landing distance from 15m	: 427 m

Range with allowance for taxi, take-off, climb, descent and 45 min reserves at 6000 feet (1830 m) : 924 km at 55 % power ; 875 km at 65 % power ; 820 km at 75 % power.

^{\$}**Remark:** The performance calculations are being done for PA-28-180 as a large amount of data on the airplane, the engine and the propeller are available in Ref.2. However, information on actual performance of this airplane is not given there. Ref.3 (which is easily accessible) contains information about PA-28-181 which is only slightly different from PA-28-180.



Dimensions in m

Fig.1. Three-view drawing of Piper Cherokee PA-28-180

2. Estimation of drag polar

Following Ref.1, the drag polar is assumed to be of the following form.

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} = C_{D_0} + K C_L^2 \quad (1)$$

$$C_{D_0} = C_{D_{0WB}} + C_{D_{0V}} + C_{D_{0H}} + C_{D_{0Misc}} \quad (2)$$

where suffixes WB, V, H and Misc denote wing-body combination, vertical tail, horizontal tail and miscellaneous items respectively.

2.1 Estimation of $C_{D_{0WB}}$

From Ref.1, section 3.1.1, at low subsonic Mach number, $C_{D_{0WB}}$ is given by the following expression.

$$C_{D_{0WB}} = \left\{ C_{fw} \left[1 + L \left(\frac{t}{c} \right) + 100 \left(\frac{t}{c} \right)^4 \right] R_{LS} \frac{(S_{wet})_e}{S_{Ref}} + C_{fb} \left[1 + \frac{60}{(l_f/d)^3} + 0.0025 \left(\frac{l_B}{d} \right) \right] \frac{(S_s)_e}{S_{Ref}} \right\} R_{WB} + C_{Db} \frac{S_B}{S_{Ref}} \quad (3)$$

C_{fw} = skin friction drag coefficient of wing (see below).

$L = 1.2$ when $(t/c)_{max}$ of the airfoil used on the wing is located at $(x/c) \geq 0.3$, which is the case here.

$$t/c = 0.15.$$

$R_{LS} = 1.07$ from Fig. 3.3 of Ref.1; note $M < 0.25$ and $\Lambda = 0$.

$$(S_{wet})_e = 30.53 \text{ m}^2$$

$$S_{ref} = 14.864 \text{ m}^2$$

C_{fb} = skin friction drag of fuselage (see below)

$$l_b/d_e = 4.882$$

$$(S_s)_e = 23.34 \text{ m}^2$$

R_{WB} = wing - body interference correction factor (see below)

C_{Db} = base drag coefficient. Base drag contribution is neglected as the fuselage

gradually tapers down to zero width.

Skin friction drag of wing (C_{fw}):

This quantity depends on the lower of the two Reynolds numbers viz.

(i) Reynolds number based on mean aerodynamic chord of exposed wing (\bar{c}_e) and (ii) cut-off Reynolds number ($Re_{cut-off}$) based on the roughness of the surface.

Reynolds number based on \bar{c}_e is :

$$Re = (1.622 \times 65.83) / (0.17792 \times 10^{-4}) = 6 \times 10^6$$

The roughness parameter is (l/k) where l is the reference chord, here 1.662 m.

The value of k corresponding to mass production point, from Ref.1, is :

$$3.048 \times 10^{-5} \text{ m. Hence, } l/k = 1.622 / (3.048 \times 10^{-5}) = 53215$$

Corresponding to this value of (l/k) , $Re_{cut-off}$ from Fig 3.2 of Ref.1 is 4×10^6 .

Since $Re_{cut-off}$ is lower, C_{fw} depends on it. Corresponding to Re_{cutoff} , the value of C_{fw} from Fig .3.1 of Ref.1 is 0.00348

Skin friction coefficient of fuselage (C_{fB}):

The Reynolds number based on length of the fuselage (R_{fB}) is:

$$R_{fB} = 6.547 \times 65.83 / (0.17792 \times 10^{-4}) = 24.22 \times 10^6$$

$$\text{In this case } l/k = 6.547 / (3.048 \times 10^{-5}) = 2.14 \times 10^5$$

$Re_{cut-off}$ in this case, Fig 3.2 of Ref.1, is : 18×10^6

Hence C_{fB} , from Fig 3.1 of Ref.1, is 0.00272

R_{WB} : From Fig 3.5 of Ref.1, for $M < 0.25$ and $R_{fB} = 24.22 \times 10^6$, $R_{WB} = 1.06$.

Hence,

$$\begin{aligned}
 C_{DWB} &= \{0.00348[1+1.2(0.15)+100(0.15)^4] \times 1.07 \times \frac{30.53}{14.864} \\
 &\quad + 0.00272[1 + \frac{60}{4.882^3} + 0.0025 \times 4.882] \frac{23.34}{14.864}\} \times 1.06 + 0 \\
 &= \{0.00348[1+0.18+0.051] \times 1.07 \times \frac{30.53}{14.864} + 0.00272[1+0.5156+0.0122] \times \frac{23.34}{14.864}\} \times 1.06 \\
 &= \{0.00941+0.006525\} \times 1.06 = 0.009975+0.006917 = 0.01689
 \end{aligned}$$

2.2 Estimation of C_{DoH}

The drag coefficient of horizontal tail is given by (Ref.1) as:

$$C_{DoH} = C_{fH} [1 + L(\frac{t}{c}) + 100(\frac{t}{c})^4] R_{LS} \frac{(S_{wet})_h}{S_{ref}} \quad (4)$$

The tail has NACA 0012 airfoil. Hence, $t/c = 0.12$

The wetted surface area of horizontal tail $(S_{wet})_h = 4.919 \text{ m}^2$

$$S_{ref} = 14.864 \text{ m}^2$$

The mean aerodynamic chord of exposed horizontal tail is taken equal to the root chord of the horizontal tail i.e. $\bar{c}_{et} = 0.762 \text{ m}$.

Reynolds number based on \bar{c}_{et} is :

$$0.762 \times 65.83 / (0.17792 \times 10^{-4}) = 2.52 \times 10^6$$

The value of l/k in this case is:

$$0.762 / (3.048 \times 10^{-5}) = 25000$$

$$\text{Hence, } Re_{cutoff} = 1.5 \times 10^6$$

$$\text{Consequently, } C_{fH} = 0.00414$$

$$\text{For } \Lambda = 0 \text{ and } M < 0.25, R_{LS} = 1.07$$

$$\text{Finally, } C_{DoH} = 0.00414[1+1.2 \times 0.12 + 100(0.12)^4] \times 1.07 \times 4.919 / 14.864 = 0.00171$$

2.3 Estimation of C_{Dov}

The drag coefficient C_{Dov} is given by:

$$C_{Dov} = C_{iv} \left[1 + L \left(\frac{t}{c} \right) + 100 \left(\frac{t}{c} \right)^4 \right] R_{LS} \frac{(S_{wet})_v}{S_{ref}} \quad (5)$$

The vertical tail has NACA 0010 airfoil; Hence, $t/c=0.10$

Wetted surface area of vertical tail = 2.372 m^2

$$S_{ref} = 14.564 \text{ m}^2$$

Reynolds number based on \bar{c}_{vt} is:

$$0.893 \times 65.83 / (0.17792 \times 10^{-4}) = 3.30 \times 10^6$$

The value of l/k is $0.893 / (3.048 \times 10^{-4}) = 29298$

$$Re_{cutoff} = 1.9 \times 10^6$$

Consequently, $C_{iv} = 0.00394$

Corresponding to $M < 0.25$ & $\Lambda = 21.8^\circ$, $R_{LS} = 1.07$

$$\begin{aligned} \text{Finally, } C_{Dov} &= 0.00394 [1 + 1.2 \times 0.1 + 100(0.1)^4] \times 1.07 \times 2.372 / 14.864 \\ &= 0.00076 \end{aligned}$$

2.4 Estimation of C_{DOLG} and C_{DOMisc}

The landing gear drag coefficient can be obtained from Ref.1. However, the values for Piper Cherokee given in Ref.2 are used as guidelines. Table 4.3 of Ref.2 indicates that parasite area of landing gears components would be (a) wheel strut 0.19 ft^2 , (b) wheels 0.44 ft^2 (c) wheel pants 0.40 ft^2 (see remark on p.180 of Ref.2). Thus, parasite drag area of landing gear would be:

$$0.19 + 0.44 + 0.4 = 1.03 \text{ ft}^2 = 0.0957 \text{ m}^2$$

Again from Table 4.3 of Ref.2 The sum of the parasite drag areas of miscellaneous items like beacon, antennas etc is 0.52 ft^2 or 0.0483 m^2 . Thus,

$$C_{DOLG} + C_{DOMisc} = (0.0957 + 0.0483) / 14.864 = 0.00645 + 0.00325 = 0.00970$$

Remarks:

- i) Reference 7, chapter 5 mentions that the drag of landing gear (C_{DLG}) without fairing is about 35% of the sum of the drags of major components viz. wing-body, horizontal tail and vertical tail. For landing gear with fairings, C_{DLG} would be about 25% of the aforesaid sum. In the present case :

$C_{DWB} + C_{DHT} + C_{DVT}$ is $(0.01689 + 0.00171 + 0.00076 = 0.01936)$. Thus C_{DLG} of 0.00645, estimated above appears reasonable.

- ii) The value of C_{Dmisc} of 0.00325 is about 17% of the aforesaid sum and appears reasonable.

2.5 Cooling drag and leakage drag

These drags are important for piston engined airplanes. Appendix A of Ref.7 gives some guidelines. However, Ref.2, p.179 mentions that the sum of the two drags could be approximately taken into account by multiplying the sum of all the other drags by a factor of 1.2.

2.6 Estimation of parasite drag coefficient (C_{D0})

In light of the above discussion C_{D0} can be expressed as:

$$C_{D0} = 1.2 (C_{DWB} + C_{DHT} + C_{DVT} + C_{DOLG} + C_{DOMisc}) \quad (7)$$

In the present case,

$$\begin{aligned} C_{D0} &= 1.2(0.01689+0.00171+0.00076+0.00645+0.00325) \\ &= 1.2 \times 0.02905 = 0.0349 \end{aligned} \quad (8)$$

Remark:

For comments on the above value of C_{D0} see remark at the end section 2.8

2.7 Estimation of induced drag coefficient (C_{Di})

The quantity K in Eq.(1) is given by:

$$K = \frac{1}{\pi A e}$$

where, A = Aspect ratio of the wing, e = Oswald efficiency factor

Following Ref.1 section 2.3.1 the Oswald efficiency factor is expressed as:

$$\frac{1}{e} = \frac{1}{e_{\text{wing}}} + \frac{1}{e_{\text{fuselage}}} + \frac{1}{e_{\text{other}}} \quad (9)$$

Figure 2.4 of Ref.1 presents e_{wing} for unswept wings of rectangular and tapered planforms. In the present case the taper ratio (λ) is almost unity. The value of e_{wing} for rectangular wing of

A = 5.625 is 0.845.

Further, for a fuselage of rectangular cross section and wing of aspect ratio 5.625, Fig.2.5 of Ref.1 gives:

$$\frac{1}{e_{\text{fuse}}} \left(\frac{S_b}{S_{\text{ref}}} \right) = 1.6 ; S_b = \text{frontal area of fuselage}$$

$$\text{Or } \frac{1}{e_{\text{fuse}}} = 1.6 \times \frac{1.412}{14.864} = 0.152$$

$$\text{Again from Ref 1, } \frac{1}{e_{\text{other}}} = 0.05$$

Consequently,

$$\frac{1}{e} = \frac{1}{0.845} + 0.152 + 0.05 = 1.3854$$

$$e = 0.722$$

$$\text{Hence, } K = \frac{1}{\pi \times 5.625 \times 0.722} = 0.0784$$

$$C_{Di} = 0.0784 C_L^2 \quad (10)$$

2.8 Expression for drag polar during cruise

Combining Eqs.(8) and (10) gives the drag polar in cruise condition as:

$$C_D = 0.0349 + 0.0784 C_L^2 \quad (11)$$

The value of $(L/D)_{\max}$ is given by

$$(L/D)_{\max} = \frac{1}{2\sqrt{C_{DO}K}}$$

Substituting the values of C_{DO} and K from Eq.(11) gives:

$$(L/D)_{\max} = \frac{1}{2\sqrt{0.0349 \times 0.0784}} = 9.56$$

2.8.1 Slight modification of the expression for drag polar

The value of $(L/D)_{\max}$ is an indication of the aerodynamic efficiency of the airplane. From Ref.7 chapter 3 it is observed that the value of $(L/D)_{\max}$ for Piper Cherokee is slightly more than 10. Thus, the estimated value of 9.56 is lower than that of the actual airplane and suggests need for slight modification. References 8 and 9 give the values of C_{DO} and e for similar airplanes, with non-retracted landing gear, made by manufacturers of Piper, Cessna and Beech aircraft. These values are presented in Table 1.

Airplane	A	C_{DO}	L/D	e
Piper Cherokee	6.02	0.0358	10	0.758
Piper J-3 cub	5.81	0.0373	9.6	0.75
Cessna Skyhawk	7.32	0.0317	11.6	0.747
Beechcraft D17S	6.84	0.0348	10.8	0.76

Table 1 Values of A , C_{DO} , (L/D) and e for similar airplane

From Table 1 it is seen that the estimated value of C_{DO} in the present case appears reasonable. However, the value of e should perhaps be higher, say 0.76 . With C_{DO} of 0.0349 and $e = 0.76$ the drag polar becomes :

$$C_D = 0.0349 + \frac{1}{\pi \times 5.625 \times 0.76} C_L^2$$

$$\text{Or } C_D = 0.0349 + 0.0755 C_L^2 \quad (12)$$

The expression in Eq.(12) would give $(L/D)_{\max}$ of 9.81.

Remarks:

- i) It may be added that Piper Cherokee is an airplane famous in its class but is of older design. The current trend is to have (a) smoother surfaces which would reduce C_{D0} to about 0.032 and (b) wing of larger aspect ratio of 8 and above, which would give K of around 0.053. These would give $(L/D)_{\max}$ of in excess of 12.
- ii) For subsequent calculations, the following expression for drag polar is used.

$$C_D = 0.0349 + 0.0755C_L^2$$

2.9 Expression for drag polar during take-off condition

To obtain the drag polar under take-off condition, the flight velocity is taken as $1.2 V_s$, where V_s is the stalling speed with flaps in take-off condition ($\delta_f = 10^\circ$). In the present case, $C_{L\max}$ with 10° flap deflection, from Ref.2 is 1.42. Hence,

$$V_s = \sqrt{\frac{2 \times 10673.28}{1.42 \times 1.225 \times 14.864}} = 28.73 \text{ m/s}$$

Consequently, $V_{To} = 1.2 \times 28.73 = 34.47 \text{ m/s}$

Reynolds number based on mean aerodynamic chord of the exposed wing in take-off condition is:

$$\frac{1.622 \times 34.47}{14.6 \times 10^{-6}} = 3.83 \times 10^6$$

We notice that this Reynolds number is very close to the cutoff Reynolds number for the wing (4×10^6) obtained in Section 2.1. Thus, the value of C_f and other calculations will remain almost the same. Hence, C_{D0} for the airplane in take-off condition, without the flap, can be taken as 0.0349.

Similarly K , without the flap, can be taken as 0.0755.

The correction to the drag polar for flap deflection, is carried-out using the following steps.

The flap type is plain flap.

From Fig.1, the ratio of flap chord to wing chord is 0.16 and flap deflection is 10° .

The ratio of the area of the flapped portion of the wing to the wing plan-form area is 0.4827.

The ratio of the span of the flapped portion of the wing (including the fuselage width) to the total span is 0.604.

The ratio of the fuselage width to the wing span is 0.127; the wing aspect ratio is 5.625.

Following Ref.1, section 3.4.1

$$\Delta C_{D\text{flap}} = \Delta C_{Dp} + \Delta C_{Di} + \Delta C_{Dint},$$

where, ΔC_{Dp} = increase in profile drag coefficient due to flaps,

ΔC_{Di} = increase in induced drag coefficient due to flaps and

ΔC_{Dint} = increase in interference drag due to flaps.

The increment in $C_{L\text{max}}$ due to 10° flap deflection, $\Delta C_{L\text{max}}$, as noted earlier, is 0.09.

Using these data and interpolating the curves given in Ref.1, section 3.4.1, gives ΔC_{dp} , the increment in the drag coefficient of airfoil due to flap deflection, as 0.008. Hence,

$$\begin{aligned}\Delta C_{Dp} &= \Delta C_{dp} \times (\text{area of flapped portion of the wing} / \text{wing area}) \\ &= 0.008 \times 0.4827 = 0.0038\end{aligned}$$

According to Ref.1, the increase in induced drag coefficient (ΔC_{Di}) due to flap deflection is

$$\Delta K_f^2 \times \Delta C_{L\text{max}}^2. \text{ Using Ref.1, section 3.4.1 } \Delta K_f \text{ is estimated as 0.163.}$$

$$\text{Consequently, } \Delta C_{Di} = 0.163^2 \times 0.09^2 = 0.00022$$

The interference drag due to deflection, of plain flaps is negligible.

Thus, the parasite drag coefficient in take-off condition is

$$C_{Do} = 0.0349 + 0.0038 + 0.00022 = 0.03892 \approx 0.0389$$

Hence, the drag polar in take-off condition is given by:

$$C_D = 0.0389 + 0.0755 C_L^2 \quad (13)$$

Remarks:

i) In the approach just presented, to estimate the drag polar in take-off condition, the change in the induced drag coefficient is included in the parasite drag coefficient. When the flap deflections are large, the change in the induced drag can be accounted for by reducing the value of the Oswald efficiency factor (e) by 0.05 for take-off condition and 0.1 for landing condition (Ref.4 section 3.4.1). Equations 12 and 13 are the drag polars for cruise condition and take-off condition respectively. The polars are presented in Fig.2.

ii) It may be pointed out that the parabolic drag polar is not valid beyond $C_{L\text{max}}$. It is only approximate near $C_L = 0$ and $C_L = C_{L\text{max}}$.

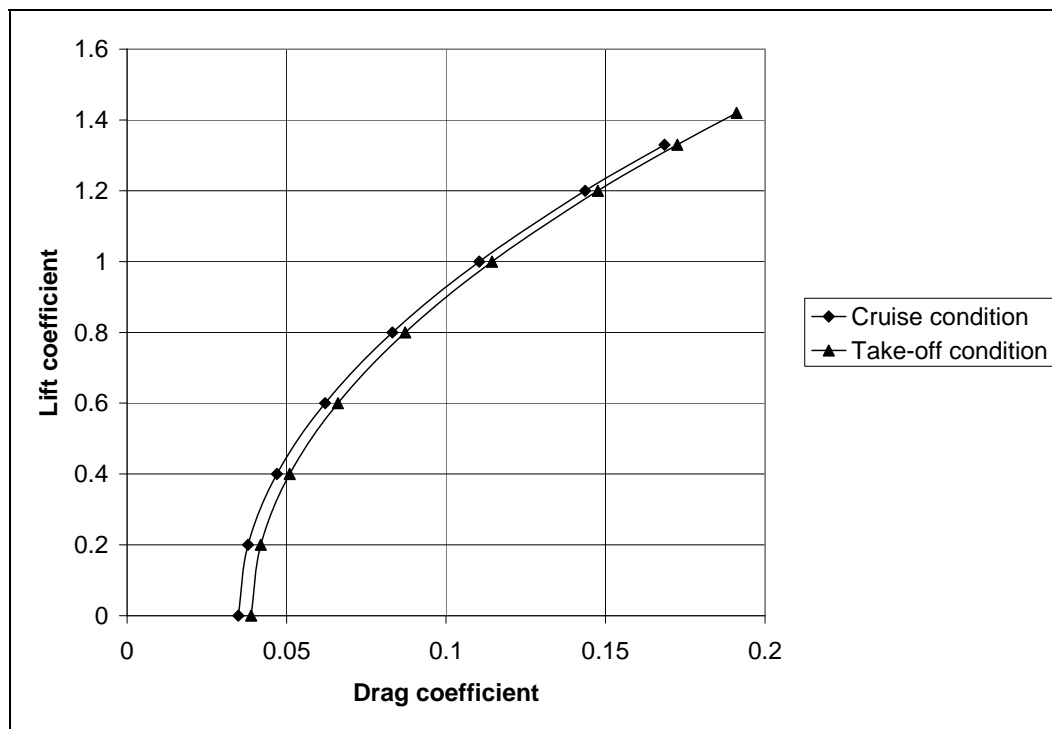


Fig.2 Drag polars at cruise and take-off conditions