

Chapter 5

Lecture 19

Performance analysis I – Steady level flight – 3

Topics

5.8 Influence of level flight analysis on airplane design

5.9 Steady level flight performance with a given engine

5.10 Steady level flight performance with a given engine and parabolic polar

5.10.1 Airplane with jet engine

5.8 Influence of level flight analysis on airplane design

The significant manner in which the performance analysis helped in evolution of the airplane configuration can be appreciated from the following discussion.

(a) The low speed airplanes are powered by engines delivering BHP or ESHP. In this case, the major portion of the power required is induced power, which depends on the factor K in drag polar (Eq.5.10). This factor is given as $1 / \pi A e$ where A is the aspect ratio of the wing and 'e' is the Oswald's efficiency factor (see Eq.3.46). Hence the low speed airplanes and gliders have high aspect ratio wings. It may be added that personal airplanes have aspect ratio between 6 to 8 as hanger space is also an important consideration. However, medium speed commercial airplanes have aspect ratio between 10 to 12. Gliders have aspect ratio as high as 16 to 20.

(b) For high subsonic airplanes most of the drag is parasite drag which depends on C_{DO} (see Eq.5.12). Hence, high speed airplanes have features like smooth surfaces, thin wings, streamlined fuselage, smooth fairings at wing-fuselage joint and retractable landing gear. These features reduce C_{DO} . Manufacturing techniques have also been improved to achieve smooth surface finish. High

speed airplanes also have high wing loading (W/S) to reduce the wing area. Table 3.4 may be referred to for typical values of C_{DO} , A and e of different types of airplanes. The reciprocal of (C_D / C_L) is (C_L / C_D) . It is called lift-drag ratio (L / D). The maximum value of this ratio, $(L / D)_{\max}$, is an indication of the aerodynamic efficiency of the airplane. $(L / D)_{\max}$ lies between 12 to 22 for a subsonic airplanes and between 5 to 8 for supersonic airplanes.

(c) When the weight of an airplane increases the thrust required increases in proportion to W and the power required increases in proportion to $W^{3/2}$ (Eqs.5.3 and 5.4). Hence, airplane design bureaus have a group of engineers which keeps a close watch on any increase in the weight of the airplane.

5.9 Steady level flight performance with a given engine

At the outset the following three points may be noted.

- (I) In steady level flight the thrust must be equal to drag (Eq.5.1).
- (II) The thrust is provided by the engine or the engine-propeller combination and from chapter 4, it is noted that the thrust or power output varies with engine RPM, flight speed and altitude.
- (III) For airplanes with piston engine or turboprop engine, the output is the power available at the engine shaft. Hence, to estimate the performance of such airplanes the calculations are carried-out in terms of BHP or THP. For airplanes with turbofan or turbojet engines, the output is in terms of thrust and to estimate the performance of such airplanes the calculations are carried-out in terms of thrust.

Typical variations, with altitude and flight speed, of the maximum thrust available (T_a) and the maximum thrust horse power available $(THP)_a$ are shown in Figs.5.5 and 5.6a respectively. The thrust required and power required curves are also shown in same figures.

Consider the curves of T_a and T_r corresponding to sea level conditions. It is seen that the power or thrust available is much more than the minimum power or thrust required. Hence, flights over a wide range of speeds are possible by controlling the engine output with the help of throttle and ensuring thrust equals drag.

However, as the speed increases above the speed for minimum power or thrust (V_{mp} or V_{md}), the power or thrust required increases and at a certain speed the power or thrust required is equal to the maximum available engine output (point A in Figs.5.5 & 5.6a). This speed is called the 'Maximum speed(V_{max})'. Similar intersections between power available and power required curves or thrust available and thrust required curves are seen at higher altitudes (points B, C and D in Fig.5.5, point B in Fig.5.6a and point C in Fig.5.6b).

Similarly, when the flight speed decreases below V_{mp} or V_{md} the power or thrust required increases and there is a speed at which the power or thrust required is equal to the available power or thrust - point D' in Fig.5.5 and point C' in Fig.5.6b. Figure 5.6b is drawn separately from Fig.5.6a to show the points C and C' clearly.

Thus, the minimum speed can be limited by available thrust or power output. It is denoted by $(V_{min})_e$. However, in level flight the requirement of lift equal to weight should also be satisfied(Eq.5.1). Hence, level flight is not possible below stalling speed. Thus, two factors viz. the thrust or power available and the stalling, limit the minimum flight speed of an airplane. Satisfying both these requirements, the minimum speed of the airplane at an altitude will be the higher of the two speeds viz. $(V_{min})_e$ and V_S .

Typical variations of V_{max} , $(V_{min})_e$ and V_S are shown for a jet engined airplane in Fig.5.9. The details of the calculations are given in Appendix B. Similarly, typical variations of these speeds in case of a piston engined airplane are shown in Fig.5.10 with details of calculation given in Appendix A. The following observations are made.

(i)For a jet airplane V_{max} may slightly increase initially with altitude and then decrease. However, there is an altitude at which the thrust required curve is tangential to the thrust available curve and flight is possible only at one speed. This altitude is called 'Ceiling' and denoted by h_{max} . Above h_{max} the thrust available is lower than the minimum thrust required and level flight is not possible as the requirement of $T = D$ cannot be satisfied.

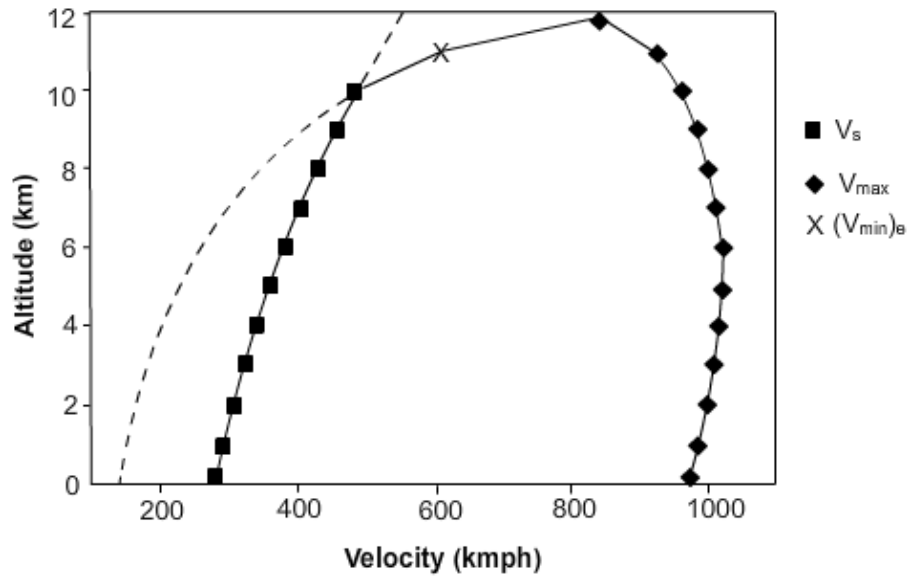


Fig.5.9 V_{max} and V_{min} for jet airplane

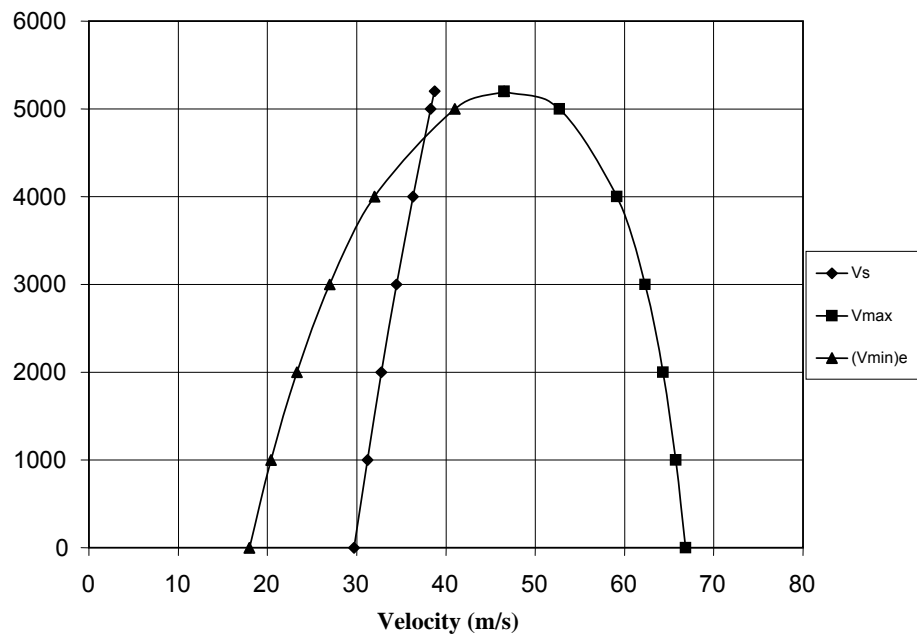


Fig.5.10 V_{max} , $(V_{min})_e$ and V_s for airplane with engine-propeller combination

(ii) The minimum speed of a jet airplane is the stalling speed (V_s) at low altitudes. However, near the ceiling, the minimum speed is that limited by the thrust available i.e. $(V_{min})_e$.

(iii) In the case of a piston engined airplane, the maximum speed seems to decrease with altitude. In this case also there is a ceiling altitude beyond which the power available is lower than the minimum power required and hence level flight is not possible. The ceiling in this case, is lower than in the case of a jet airplane because the power output of a piston engine decreases rapidly with altitude. As regards the minimum speed, it is also limited by stalling at low altitudes and by power available near the ceiling altitude.

5.10 Steady level flight with a given engine and parabolic polar

If the drag polar is parabolic and the engine output can be assumed to be constant with speed, then V_{\max} and $(V_{\min})_e$ from the engine output consideration, can be calculated analytically. i.e. by solving an equation. It may be noted from Figs.5.5 & 5.6 that the assumption of T_a or P_a as constant with V appears reasonable near the speeds where V_{\max} occurs.

5.10.1 Airplane with jet engine:

The steps to calculate V_{\max} and $(V_{\min})_e$ are as follows.

(1) Choose an altitude 'h'. Let T_a be the thrust available in the range of speeds where V_{\max} is likely to occur.

$$(2) T_r = T_a = W(C_D / C_L)$$

Hence,

$$\frac{T_a}{W} = \frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + KC_L$$

$$\text{Or } KC_L^2 - \frac{T_a}{W} C_L + C_{D0} = 0 \quad (5.25)$$

Equation (5.25) is a quadratic in C_L . Its solution gives two values of C_L at which level flight with the given thrust is possible. Let these values of C_L be denoted as C_{L1} and C_{L2} . Then, the corresponding flight speeds, V_1 and V_2 , are given as:

$$V_1 = \left(\frac{2W}{\rho S C_{L1}} \right)^{\frac{1}{2}} \text{ and } V_2 = \left(\frac{2W}{\rho S C_{L2}} \right)^{\frac{1}{2}} \quad (5.26)$$

It may be pointed out that the same results can be obtained by using Eq.(5.12), i.e.

$$T_a = T_r = \frac{1}{2} \rho V^2 S C_{DO} + K \left(\frac{2W^2}{\rho V^2 S} \right)$$

$$\text{Or } AV^4 - BV^2 + C = 0 \quad (5.27)$$

$$\text{where, } A = \frac{1}{2} \rho S C_{DO}, \quad B = T_a \quad \text{and} \quad C = \frac{2KW^2}{\rho S}$$

For given value of thrust (T_a), Eq.(5.27) also gives two solutions for level flight speeds V_1 and V_2 .

Let V_1 be the higher among V_1 and V_2 . Then, V_1 is the maximum speed and V_2 is the minimum speed, based on engine output i.e. $(V_{\min})_e$. The higher of $(V_{\min})_e$ and the stalling speed (V_S) will be the minimum speed at the chosen altitude.

The example 5.2 illustrates the procedure.

Remarks:

- i) Calculate the Mach number corresponding to V_1 . If it is more than the critical Mach number then C_{DO} and K would need correction and revised calculation, would be required.
- ii) Obtain, from the engine charts, the thrust available at V_1 . Let it be denoted by T_{a1} . If the thrust available (T_a), assumed at the start of the calculation (step 1), is significantly different from T_{a1} , then the calculations would have to be revised with new value of T_a . However, it is expected that the calculations would converge to the correct answer in a few iterations.