Chapter 7

Lecture 25

Performance analysis III – Range and endurance – 2

Topics

7.4.2 Breguet formulae for range and endurance of airplanes with engine-propeller combination and jet engine

The derivations of these formulae are based on the assumptions that during the flight:

(i) BSFC or TSFC is constant

(ii) η_p is constant for engine propeller combination (E.P.C).

(iii) altitude is constant

(iv) C_L is constant and

(v) flight Mach number is below critical Mach number so that the drag polar is independent of Mach number.

With, these assumptions, certain terms in Eqs.(7.15) and (7.15a) can be taken outside the integral and the equations reduce to:

$$R = -\frac{3600\ \eta_p}{BSFC\ (C_D/C_L)}\int_{W_1}^{W_2}\frac{dW}{W} \quad \text{For E.P.C}$$

$$\text{and } R = \frac{-4.6}{TSFC(\sigma S)^{1/2}(C_D/C_L^{1/2})}\int_{W_1}^{W_2}\frac{dW}{W^{1/2}} \quad \text{For J.A.}$$

Hence,

$$R = \frac{8289.3\ \eta_p}{BSFC\ (C_D/C_L)} \log_{10}\left(\frac{W_1}{W_2}\right) \quad \text{For E.P.C.} \tag{7.17}$$

and $$R = \frac{9.2}{TSFC(C_D/C_L^{1/2})}\left(\frac{W_1}{\sigma S}\right)^{1/2}\left(1-\left(\frac{W_2}{W_1}\right)^{1/2}\right) \quad \text{For J.A.} \tag{7.17a}$$

Similarly, from the above assumptions, Eqs.(7.16) and (7.16a) reduce to :

$$E = -\frac{782.6\ \eta_p\ (\sigma S)^{1/2}}{BSFC \times \left(C_D/C_L^{3/2}\right)}\int_{W_1}^{W_2}\frac{dW}{W^{3/2}} \quad \text{For E.P.C}$$

and $$E = -\frac{1}{TSFC(C_D / C_L)}\int_{W_1}^{W_2}\frac{dW}{W} \quad \text{For J.A.}$$

Hence,

$$E = \frac{1565.2\ \eta_p}{BSFC \times \left(C_D/C_L^{3/2}\right)}\left[\frac{\sigma S}{W_1}\right]^{1/2}\left[W_2^{1/2} - W_1^{1/2}\right] \quad \text{For E.P.C.} \tag{7.18}$$

and $$E = \frac{2.303}{TSFC\ (C_D / C_L)}\log_{10}\left(\frac{W_1}{W_2}\right) \quad \text{For J.A.} \tag{7.18a}$$

7.4.3 Discussion of Breguet formulae – desirable values of lift coefficient and flight altitude

The following conclusions can be drawn from the above expressions for range and endurance viz. Eqs.7.17, 7.17a, 7.18 and 7.18a.

(1) For range and endurance to be high, it is evident that η_p should be high and the TSFC and BSFC should be low.

(2) Desirable values of lift coefficients for an airplane with engine-propeller combination: The endurance is maximum (Eq.7.18) when the lift coefficient is such that $C_D/C_L^{3/2}$ is minimum, i.e., $C_L = C_{Lmp}$. This can be understood from the fact, that with BSFC being assumed constant, the rate of fuel consumption per hour would be minimum, in this case, when the power required is minimum.

From Eq.(7.17), for range to be maximum, in this case, C_D/C_L should be minimum or $C_L = C_{Lmd}$. This can be understood from the fact that the range, in this case is proportional to V/THP or L / D. Hence, range is maximised when C_L corresponds to minimum drag.

(3) Desirable values of lift coefficients for a jet airplane: From Eq.(7.18a) it is observed that the endurance is maximum when C_D / C_L is minimum or $C_L = C_{Lmd}$. This can be understood from the fact that with TSFC being assumed constant, the fuel flow rate per hour would be minimum when the thrust required is minimum.

From Eq.(7.17a) the range, in this case is maximum when $C_D/C_L^{1/2}$ is minimum. This can be understood from the fact that the range, in this case, is proportional to (V / T) or $C_L^{1/2} / C_D$. The C_L corresponding to $(C_D/C_L^{1/2})_{min}$ is denoted here by C_{Lmrj}.

(4) Desirable values of flight altitude : Equation (7.17a), also shows that for a jet airplane, the range would be high, when (a) the wing loading (W/S) is high and (b) density ratio (σ) is low or the altitude is high. Hence, the jet airplanes have wing loading of the order of 4000 to 6000 N/m^2, which is much higher than that for the low speed airplanes which have a wing loading of 1000 to 2500 N/m^2. The jet airplanes also cruise at high altitude (10 to 12 km) which is not much below the ceiling altitude of 12 to 14 km for these airplanes.

From Eq.(7.18) it is observed that the endurance of an airplane with engine-propeller combination is high when (a) the wing loading is low and (b) σ is high or flight takes place near sea level.

It may be added that the final wing loading chosen for an airplane is a compromise between requirements of cruise, climb, take-off and landing. The take-off and landing distances increase in direct proportion to the wing loading (subsections 10.4.5 and 10.5.3), and hence, a high wing loading is not desirable from this point of view.

Remarks:

i) If the drag polar is parabolic, an expression for C_{Lmrj} can be derived as follows.

$$C_D = C_{DO} + K\,C_L^2$$

$$\text{Hence, } \frac{C_D}{C_L^{1/2}} = \frac{C_{DO}}{C_L^{1/2}} + K\,C_L^{3/2}$$

$$\frac{d(C_D / C_L^{1/2})}{dC_L} = \frac{C_{DO}}{2}\,C_{Lmrj}^{-3/2} + \frac{3}{2}K\,C_{Lmrj}^{1/2} = 0$$

Or $\quad C_{Lmrj} = (C_{DO} / 3K)^{1/2}$

7.4.4 Important values of lift coefficient

The points on the drag polar at which C_L is equal to C_{Lmax}, C_{Lmp}, C_{Lmd} and C_{Lmrj} are shown in Fig.7.2. The importance of these values of lift coefficient can be reemphasized as follows.

(i) The maximum lift coefficient (C_{Lmax}) decides the stalling speed which is one of the criterion for the minimum speed of the airplane. It also affects the minimum radius of turn (see subsection 9.3.3) and the take-off and landing distances (see subsections 10.4.5 and 10.5.3)

(ii) The lift coefficient corresponding to minimum power required (C_{Lmp}) influences the performance of airplanes with engine-propeller combination. It decides the flight speeds corresponding to maximum rate of climb, minimum rate of sink and maximum endurance of these airplanes.

(iii) The lift coefficient corresponding to minimum thrust required (C_{Lmd}) is also the value of C_L at which (L/D) is maximum. From Fig.7.2 it is observed that the slope of a line joining the origin to a point on the curve, is equal to (C_L/ C_D). At, $C_L = C_{Lmd}$ this line, from the origin, is tangent to the drag polar and has the maximum slope (Fig.7.2). The value of C_{Lmd} decides the flight speed for maximum range of an airplane with engine-propeller combination and the maximum endurance of a jet airplane.

(iv) The lift coefficient corresponding to $\left(C_L^{1/2}/C_D\right)_{max}$ or C_{Lmrj} decides the flight speed for maximum range of jet airplanes.

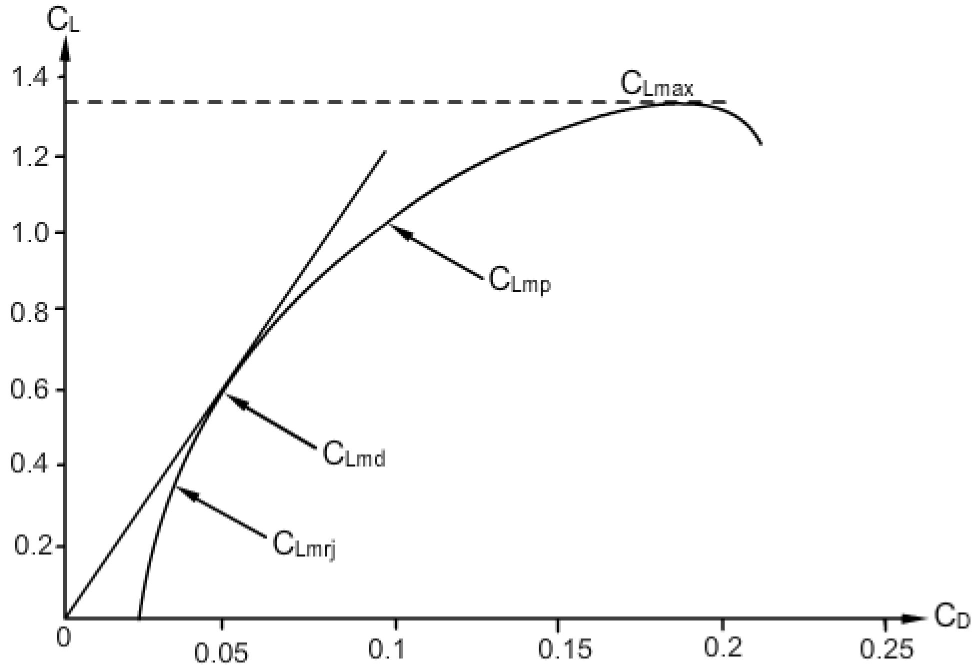

Fig.7.2 Important points on a drag polar

Example 7.2

An airplane having an engine-propeller combination weighs 88,290 N and has a wing area of 45 m^2. Its drag polar is given by: $C_D = 0.022 + 0.059C_L^2$. Obtain the maximum range and endurance at sea level in a steady level flight at a constant angle of attack from the following additional data.

Weight of fuel and oil = 15,450 N, BSFC = 2.67 N/kW-hr,

propeller efficiency (η_p) = 85%.

Note: Along with the fuel, the lubricating oil is also consumed and this fact is taken into account in this example, by specifying the weight of the oil along with the weight of fuel.

Solution:

W_1 = 88290 N, W_2 = 88290 – 15450 = 72840 N

Since BSFC, η_p and C_L are constant, the maximum range and endurance occur when C_L has the values of C_{Lmd} and C_{Lmp} respectively.

$C_{Lmd} = (C_{DO}/K)^{1/2} = (0.022/0.059)^{1/2} = 0.6106,$

$C_{Dmd} = 2\, C_{DO} = 0.044$

$C_{Lmp} = (3C_{DO}/K)^{1/2} = (3 \times 0.022/0.059)^{1/2} = 1.058,\ C_{Dmp} = 4\, C_{DO} = 0.088$

Hence, $(C_D/C_L)_{min} = 0.044/0.6106 = 0.0721$

and $(C_D/C_L^{3/2})_{min} = 0.088/(1.058)^{3/2} = 0.0808$

From Eq. (7.17):

$$R = \frac{8289.3\ \eta_p}{BSFC \times (C_D/C_L)_{min}} \log_{10}\left(\frac{W_1}{W_2}\right) = \frac{8289.3 \times 0.85}{2.67 \times 0.0721} \times \log_{10}\left(\frac{88290}{72840}\right)$$

$= 3058$ km.

Remark:

Since C_L is constant during the flight, the flight velocity and the power required change as the fuel is consumed. In the present case, the following results illustrate the changes.

Velocity at the beginning of the flight:

$V_1 = (2W_1/\rho\, S\, C_{Lmd})^{1/2} = (2 \times 88290/1.225 \times 45 \times 0.6106)^{1/2} = 72.41$ m/s.

$= 260.7$kmph.

Velocity at the end of flight:

$= (2W_2/\rho\, S\, C_{Lmd})^{1/2}$

$= (2 \times 72840/1.225 \times 45 \times 0.6106)^{1/2} = 65.8$ m/s $= 236.8$ kmph.

Power required in the beginning of the flight:

$$= \frac{T_1 \times V_1}{1000} = \frac{W_1 \times (C_D/C_L) \times V_1}{1000} = \frac{88290 \times 0.044}{1000 \times 0.6106} \times 72.41 = 460.7\,\text{kW}$$

Power required at the end of flight:

$$= \frac{T_2 \times V_2}{1000} = \frac{W_2 \times (C_D/C_L) \times V_2}{1000} = \frac{72840 \times 0.044}{1000 \times 0.6106} \times 65.8 = 345.5\,\text{kW}$$

Maximum endurance:

From Eq.(7.18) the maximum endurance is :

$$E_{max} = \frac{1565.2}{BSFC} \frac{\eta_p}{\left(C_D/C_L^{3/2}\right)_{min}} \left[\frac{\sigma S}{W_1}\right]^{1/2} \left[\left(\frac{W_1}{W_2}\right)^{1/2} - 1\right]$$

$$= \frac{1565.2 \times 0.85}{2.67 \times 0.0808} \left[\frac{1.00 \times 45}{88290}\right]^{1/2} \left[\left(\frac{88290}{72840}\right)^{1/2} - 1\right] = 14.06\text{hrs}$$

In this flight C_L equals 1.058. Proceeding in a manner similar to the remark above, it can be shown that the speeds at the beginning and end of the flight for E_{max} are 197.8 kmph and 179.7 kmph respectively. The power outputs required at the beginning and the end of this flight are 402.8 kW and 302.0 kW respectively.

Example 7.3

A jet airplane has a weight of 922,140 N and wing area of 158 m^2. The weight of the fuel and oil together is 294,300 N. The drag polar is given by:

$$C_D = 0.017 + 0.0663\, C_L^2$$

Obtain the maximum range in constant C_L flight at an altitude of 10 km assuming the TSFC to be 0.95 hr^{-1}.

Solution:

In a flight with constant C_L the maximum range occurs when

$$C_L = C_{Lmrj} = (C_{D0} / 3K)^{1/2}$$

$$C_{Lmrj} = \left(\frac{0.017}{3 \times 0.0663}\right)^{1/2} = 0.292$$

$$C_{Dmrj} = 0.017 + 0.0663 \times (0.292)^2 = 0.02265$$

$$(C_{Dmrj}/C_{Lmrj}^{1/2}) = 0.02265 / 0.292^{1/2} = 0.04192$$

σ at 10 km altitude is: 0.3369

From Eq.(7.17a):

$$R_{max} = \frac{9.2}{TSFC \times \left(C_D/C_L^{1/2}\right)}\left[\frac{W_1}{\sigma S}\right]^{1/2}\left[1-\left(\frac{W_2}{W_1}\right)^{1/2}\right]$$

$$= \frac{9.2}{0.95 \times 0.04192} \times \left(\frac{922140}{0.3369 \times 158}\right)^{1/2}\left[1-\left(\frac{922140-294300}{922140}\right)^{1/2}\right] = 5317 \text{ km}$$

Remarks:

i)The flight velocity corresponding to a C_L of 0.292 at an altitude of 10 km is equal to: $[2 \times 922140 / (158 \times 0.413 \times 0.292)]^{1/2}$ = 311.1 m/s.

The speed of sound at 10 km is 299.5 m/s. Thus the Mach number at this speed would be 311.1/299.5 = 1.04. This value is definitely higher than the critical Mach number of the airplane. Consequently, the prescribed drag polar is not valid. The C_D will actually be much higher and the range much lower.

As an alternative, let the critical Mach number be taken as 0.85 and the range be calculated in a flight at constant C_L which begins at this Mach number.

Consequently, V = 0.85 x 299.5 = 254.5 m/s.

Hence, $C_L = (2 \times 922140/0.413 \times 158 \times 254.5^2) = 0.436$

Consequently, $C_D = 0.017 + 0.0663 \times (0.436)^2 = 0.0296$

and $C_D/C_L^{1/2} = 0.0296/0.436^{1/2} = 0.0448$

The range in a constant C_L flight with C_L=0.436 would be:

$$= \frac{9.2}{0.95 \times 0.0448} \times \left(\frac{922140}{0.3369 \times 158}\right)^{1/2}\left[1-\left(\frac{922140-294300}{922140}\right)^{1/2}\right] = 4975 \text{ km}.$$

(ii) The data given in this example, roughly corresponds to that of Boeing 727, the famous jetliner of 1970's. The value of TSFC corresponds to engines of that period. The value of K equal to 0.0663 includes the change in K, when Mach number lies in the transonic range.