Chapter 7

Performance analysis III – Range and endurance

(Lectures 24-26)

Keywords: Range; endurance; safe range; gross still air range; Breguet formulae; cruising speed and altitude; cruise climb; effect of wind on range.

Topics

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Lecture 24

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Topics

7.1 Introduction

Airplane is a means of transport designed to carry men and materials safely over a specified distance. Hence, the fuel required for a trip or the distance covered with a given amount of fuel are important items of performance analysis. Similarly, airplanes used for training, patrol and reconnaissance would be required to remain in air for a certain period of time. Thus, the fuel required to remain in air for a certain length of time or the time for which an airplane can remain in air with a given amount of fuel are also important aspects of performance analysis. These two aspects viz. distance covered and the time for which an airplane can remain in air are discussed under the topic of range and endurance and are the subject matter of this chapter.

7.2 Definitions of range and endurance

Range (R) is the horizontal distance covered, with respect to a given point on the ground, with a given amount of fuel. It is measured in km. Endurance (E) is the time for which an airplane can remain in air with a given amount of fuel. It is measured in hours. The above definition of range is very general and terms like safe range and gross still air range are commonly used. These terms include details of the flight plan and are explained in the subsequent subsections.

7.2.1 Safe range

It is the maximum distance between two destinations over which an airplane can carry out a safe, reliably regular service with a given amount of fuel. This flight involves take-off, acceleration to the speed corresponding to desired rate of climb, climb to the cruising altitude, cruise according to a chosen flight plan, descent and landing. Allowance is also given for the extra fuel requirement due to factors like (i) head winds (see next subsection) normally encountered en-route (ii) possible navigational errors (iii) need to remain in air before permission to land is granted at the destination and (iv) diversion to alternate airport in case of landing being refused at the scheduled destination.

7.2.2 Head wind, tail wind, gust and cross wind

Generally the performance of an airplane is carried out assuming that the flight takes place in still air. However the air mass may move in different directions. Following three cases of air motion are especially important.

(a) Head wind and tail wind: In these two cases the direction of motion of air (V_w) is parallel to the flight direction. If V_w is opposite to that of the flight direction, it is called 'Head wind'. When V_w is in the same direction as the flight direction, it is called 'Tail wind' (Fig.7.1a). In the presence of wind, the velocitiy of the airplane with respect to air (V_a) and that with respect to ground (V_g) will be different. For the head wind case, $V_g = V_a - V_w$, and for the tail wind case, $V_g = V_a + V_w$.

(b) Gust: When the velocity of the air mass is perpendicular to flight path and along the vertical direction, it is called gust. Here the velocity of gust is denoted by V_{gu} (Fig.7.1b). This type of air movement would result in a change of the angle of attack of the airplane.

(c) Cross wind: When the velocity of the air mass is perpendicular to flight path and parallel to the sideward direction, it is called ‘Cross wind’. Here it is denoted by ‘v’ (Fig.7.1c).

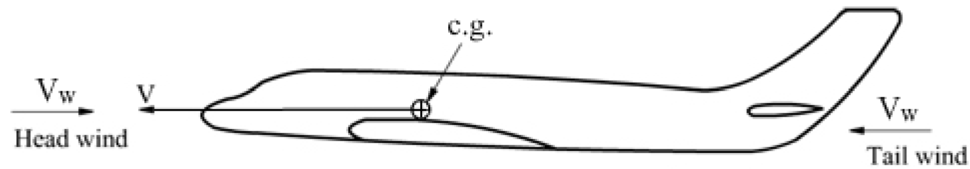

Fig.7.1a Head wind and tail wind

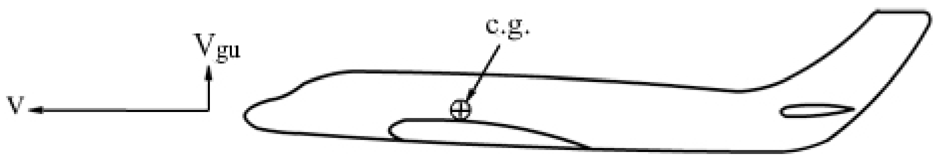

Fig.7.1b Gust

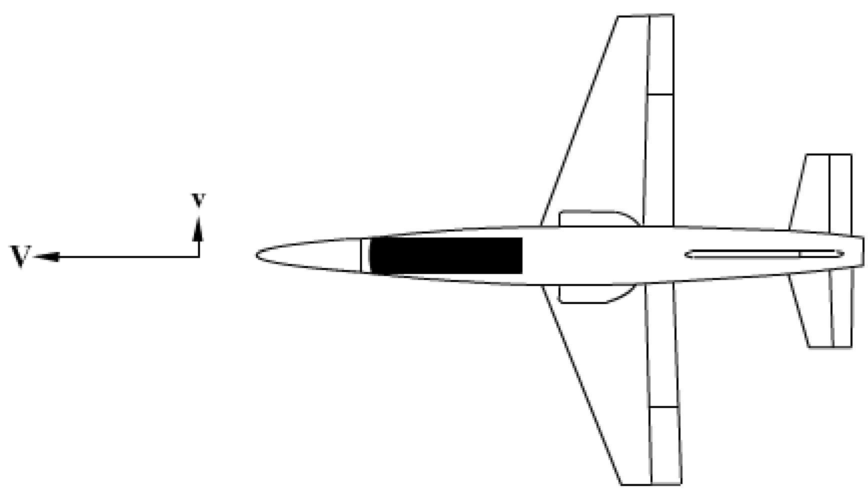

Fig.7.1c Side wind

7.2.3 Gross still air range (G.S.A.R.)

The calculation of safe range depends on the route on which the flight takes place and other practical aspects. It is not a suitable parameter for use during the preliminary design phase of airplane design. For this purpose, gross still air range (G.S.A.R.) is used. In this case, it is assumed that the airplane is already at the cruising speed and cruising altitude with desired amount of fuel and then it carries out a chosen flight plan in still air, till the fuel is exhausted. The horizontal distance covered in this flight is called ‘Gross still air range’. In the subsequent discussion the range will mean gross still air range.

Remark:

As a guideline G.S.A.R. is roughly equal to one and a half times the safe range.

7.3 Rough estimates of range and endurance

If the weight of the fuel available (W_f in N) and the average rate of fuel consumption during the flight are known, then the rough estimates of range (R) and endurance (E) are given as follows.

$$R = W_f \times (\text{km / N of fuel})_{\text{average}} \tag{7.1}$$

$$E = W_f \times (\text{hrs / N of fuel})_{\text{average}} \tag{7.2}$$

The estimation procedure is illustrated with the help of example 7.1.

Example 7.1

An airplane has a weight of 180,000 N at the beginning of the flight and 20% of this is the weight of the fuel. In a flight at a speed of 800 kmph the lift to drag ratio (L/D) is 12 and the TSFC of the engine is 0.8. Obtain rough estimates of the range and endurance.

Solution:

W_1 = Weight at the start of the flight = 180,000N

W_f = Weight of the fuel = 0.2x180,000 = 36,000N

W_2 = Weight of the airplane at the end of the flight

= 180,000 - 36,000= 144,000N.

Hence, the average weight of the airplane during the flight is :

$$W_a = \frac{180000 + 144000}{2} = 162000\text{N}$$

Consequently, the average thrust (T_{avg}) required during the flight is:

$T_{avg} = W_a \,/\, (L / D) = 162000/12 = 13500$ N

The average fuel consumed per hour is:

T_{avg} x TSFC = 13500 x 0.8 = 10800 N

Since the average speed is 800 kmph, the distance covered in 1 hr is 800 km. Noting that the fuel consumed in 1 hr is 10,800 N, gives:

$(\text{km / N of fuel})_{\text{average}}$ = 800/10800.

Consequently, $R = 36000 \times \frac{800}{10800} = 2667\,\text{km}$ and

the endurance $E = 36000 \times \frac{1}{10800} = 3.33$ hrs.

7.4 Accurate estimates of range and endurance

For accurate estimates of range and endurance, the continuous variation of the weight of the airplane during the flight and consequent changes in the following quantities are considered.

(a) The thrust required (or power required),

(b) TSFC (or BSFC) and

(c) Flight velocity and lift coefficient.

It may be recalled from subsection 4.2.4, that is the specific fuel consumption (SFC) of an engine delivering shaft horse power to a propeller is denoted by BSFC and the SFC of a jet engine is denoted by TSFC. The units of BSFC and TSFC are respectively N/kW-hr and N/N-hr (or hr^{-1}).

The steps to accurately estimate the range and endurance are as follows.

Let W be the weight of the airplane at a given instant of time and W_{fi} be the weight of the fuel consumed from the beginning of the flight up to the instant under consideration.

Then, $W = W_1 - W_{fi}$ (7.3)

where, W_1 = weight of the airplane at the start of the flight.

Let dR and dE be the distance covered in km and the time interval in hours respectively, during which a small quantity of fuel dW_f is consumed. Then,

$$dR = dW_f \times (\text{km/N of fuel}) \tag{7.4}$$

and
$$dE = dW_f \times (\text{hrs/ N of fuel}) \tag{7.5}$$

Following Ref.1.5, chapter 4, the Eqs.(7.4) and (7.5) are rewritten as:

$$dR = dW_f \left(\frac{\text{km/hr}}{\text{N of fuel /hr}}\right) \tag{7.6}$$

and $$dE = dW_f \times \{1 / (\text{N of fuel / hr})\} \tag{7.7}$$

It may be pointed out that (a) km/hr = 3.6 x V, where V is the flight speed in m/s. (b) the fuel / hr in Newtons is equal to BSFC x BHP for an airplane with engine-propeller combination and equal to TSFC x T for a jet airplane.

Hereafter, the airplane with engine-propeller combination is referred to as "E.P.C" and the jet airplane as "J.A" Note that in the case of an engine-propeller combination, the engine could be a piston engine or a turboprop engine and in the case of a jet airplane the engine could be a turbofan or a turbojet engine.

Equations (7.6) and (7.7) can be rewritten as :

$$dR = dW_f \frac{3.6\,V}{BSFC \times BHP} \quad \text{For E.P.C.} \tag{7.8}$$

$$\text{and } dR = dW_f \frac{3.6\,V}{TSFC \times T} \quad \text{For J.A.} \tag{7.8a}$$

$$dE = \frac{dW_f}{BSFC \times BHP} \quad \text{For E.P.C.} \tag{7.9}$$

$$\text{and } dE = \frac{dW_f}{TSFC \times T} \quad \text{For J.A.} \tag{7.9a}$$

Recall from section 5.2 that in a level flight,

$$T = D = W\frac{C_D}{C_L}\ ,\ L = W = \frac{1}{2}\rho V^2 S C_L\ ,\ V = \left(\frac{2W}{\rho S\, C_L}\right)^{\frac{1}{2}} = \left(\frac{2W}{\sigma\,\rho_0\, S\, C_L}\right)^{\frac{1}{2}} \tag{7.10}$$

Using, $\rho_0 = 1.225\ \text{kg/m}^3$ yields:

$$V = 1.278\left(\frac{W}{\sigma\, SC_L}\right)^{1/2} . \tag{7.11}$$

Substituting for T and V from Eqs.(7.10) and (7.11), the expression for BHP is:

$$BHP = \frac{1}{\eta_p}\frac{TV}{1000} = \frac{1}{782.6} W^{3/2} / \left[\eta_p\, (\sigma\, S)^{\frac{1}{2}} \left(C_D / C_L^{3/2}\right)\right] \qquad (7.12)$$

where η_p is the propeller efficiency.

During the analysis of range, the rate of change of weight of the airplane is only due to the consumption of fuel. Hence,

$$dW_f = - dW$$

Substituting for V, BHP, T and dW_f in Eqs. (7.8),(7.8a),(7.9) and (7.9a) gives:

$$dR = \frac{-3600\ \eta_p\ dW}{BSFC \times W\ (C_D/C_L)} \quad \text{For E.P.C.} \qquad (7.13)$$

$$\text{and } dR = \frac{-4.6\ dW}{TSFC\ (\sigma\ S\ W)^{1/2}\ (C_D/C_L^{1/2})} \quad \text{For J.A.} \qquad (7.13a)$$

$$dE = -\frac{782.6\ \eta_p\ (\sigma S)^{1/2} dW}{BSFC \times W^{3/2} \left(C_D/C_L^{3/2}\right)} \quad \text{For E.P.C.} \qquad (7.14)$$

$$\text{and } dE = \frac{-dW}{TSFC \times W\ (C_D/C_L)} \quad \text{For J.A} \qquad (7.14a)$$

Let W_2 be the weight of the airplane at the end of the flight. Integrating Eqs.(7.13), (7.13a), (7.14) and (7.14a), the range and endurance are given as:

$$R = \int_{W_1}^{W_2} dR = \int_{W_1}^{W_2} - \frac{3600\ \eta_p\ dW}{BSFC \times W\ (C_D/C_L)} \quad \text{For E.P.C.} \qquad (7.15)$$

$$\text{and } R = \int_{W_1}^{W_2} \frac{-4.6\ dW}{TSFC\ (\sigma\ S\ W)^{\frac{1}{2}}\ (C_D/C_L^{\frac{1}{2}})} \quad \text{For J.A} \qquad (7.15a)$$

$$E = \int_{W_1}^{W_2} dE = \int_{W_1}^{W_2} - \frac{782.8\ \eta_p\ (\sigma S)^{1/2}\ dW}{BSFC \times W^{3/2}\ (C_D/C_L^{3/2})} \quad \text{For E.P.C.} \qquad (7.16)$$

$$\text{and } E = \int_{W_1}^{W_2} \frac{-dW}{TSFC \times W\ (C_D/C_L)} \quad \text{For J.A.} \qquad (7.16a)$$

7.4.1 Dependence of range and endurance on flight plan and remark on optimum path

The set of Eqs.(7.15),(7.15a),(7.16) and (7.16a) or (7.8), (7.8a), (7.9) and (7.9a) when integrated, give the range and endurance. However, while doing this, it should be noted that the weight of the aircraft decreases continuously as the fuel is consumed. Further, the flight is treated as steady level flight and hence, T = D and L= W must be satisfied at each instant of time. Consequently, the thrust and power required and the flight speed may change continuously. Hence, it is necessary to prescribe the flight plan i.e., the manner in which the velocity changes with time during the flight. The following three types of flight plans can be cited as examples.

(a) Level flight at a constant velocity. In this flight, the lift coefficient decreases gradually as the weight of the airplane decreases (Eq.7.10). Simultaneously, the thrust required also decreases continuously.

(b) Level flight with constant lift coefficient (or constant angle of attack) . In this flight, in accordance with Eq.(7.10), the flight velocity and the thrust required decrease continuously as the weight of the airplane decreases.

(c) Level Flight with constant thrust. In this case, the continous decrease in the airplane weight during the flight, requires that the flight velocity and the lift coefficient (C_L) be adjusted so that at each instant of time, the thrust balances the drag and the lift balances the weight.

As mentioned earlier, the airplanes are commercial means to transport men and materials. Hence, maximization of range and endurance are important requirements. However, the right hand sides of Eqs.(7.15),(7.15a),(7.16) and (7.16a) involve integrals. The optimization of an integral is different from the optimization of an expression. The latter is done by taking the derivative of the expression and equating it to zero. Whereas, in the case of an integral, it is to be noted that the value of the integral depends on how the integrand varies with the independent variable. This variation, in mathematical terms, is called a path. For example, as mentioned above, the range will depend on the flight plan viz. constant angle of attack flight or constant velocity flight or constant thrust flight.

The problem of optimization is to find out the path that will maximize the integral. The branch of Mathematics which deals with optimization of integrals is called 'Calculus of variation'. This topic is outside the scope of the present introductory course. Interested reader may refer, chapter 20 of Ref.7.1.

Remark:

It can be shown, using calculus of variation, that if the specific fuel consumption, propeller efficiency and altitude are assumed constant, then the maximum range is obtained in a flight with constant lift coefficient. With these assumptions Eqs.(7.15),(7.15a),(7.16) and (7.16a) become easy to integrate. The expressions for range and endurance, obtained with these assumptions, are called 'Breguet formulae'. These are derived in the next subsection. It may be pointed out that Breguet was a French pioneer in aeronautical engineering.