Chapter 9

Lecture 30

Performance analysis V – Manoeuvres – 3

Topics

9.3.5 Parameters influencing turning performance of a jet airplane

The steps, described in subsection 9.3.4 , to determine r_{min}, $\dot{\psi}_{max}$, V_{rmin} and $V_{\dot{\psi}max}$ constitute a general procedure which is applicable to all types of airplane. However, the influence of the wing loading (W/S) and the thrust loading (T_a/W) can be examined by the following simplified analysis. It is based on the following two assumptions.

(a)Thrust available (T_a) is constant.

(b)The drag polar is parabolic with C_{DO} and K as constants.

The following relationships are observed in a steady, level, co-ordinated-turn.

$$T = D,\ \ L = nW\ ;\ n = \frac{1}{\cos\phi},\ \ r = \frac{V^2}{g\sqrt{n^2-1}},\ \ \dot{\psi} = \frac{g\sqrt{n^2-1}}{V}$$

$$\text{Hence,}\ \ T_a = \frac{1}{2}\rho V^2 S C_D = \frac{1}{2}\rho V^2 S\left\{C_{DO} + K C_L^2\right\}$$

$$\text{Or}\ \ T_a = \frac{1}{2}\rho V^2 S\left\{C_{DO} + K\left(\frac{2nW}{\rho S V^2}\right)^2\right\} \tag{9.15}$$

Solving Eq.(9.15) for n^2, gives :

$$n^2 = \frac{\frac{1}{2}\rho V^2}{K(W/S)}\left\{\frac{T_a}{W} - \frac{\frac{1}{2}\rho V^2 C_{DO}}{W/S}\right\} \tag{9.16}$$

Let the free stream dynamic pressure be denoted by 'q' :

Or $q = \frac{1}{2}\rho V^2$ (9.17)

Consequently, Eq.(9.16) can be rewritten as :

$$n^2 = \frac{q}{K(W/S)}\left\{\frac{T_a}{W} - \frac{qC_{DO}}{W/S}\right\} \tag{9.18}$$

From Eq.(9.11a)

$$r = \frac{V^2}{g\sqrt{n^2 - 1}}$$

$$\text{Or } r = \frac{2q}{g\rho\sqrt{n^2 - 1}} \tag{9.19}$$

From Eq.(9.19) it is observed that 'r' is a function of q and n. However, when the constraint of thrust available is taken into account, then n and q are related by Eq.(9.18).

The value of q which would give minimum radius of turn (r_{min}), can be obtained in two stages.

(a) Substitute the expression for 'n' as given by Eq.(9.18) in Eq.(9.19).

(b) Differentiate the resultant equation for 'r' obtained in step (a), with respect to 'q', and equate it to zero.However, the resulting expression is complicated. An alternate way is as follows.

(i) Differentiate Eq.(9.19) with respect to q and equate it to zero.

$$\frac{dr}{dq} = \frac{2g\rho\sqrt{n^2 - 1} - 2g\rho qn\left(n^2 - 1\right)^{-1/2}(dn/dq)}{g^2\rho^2\left(n^2 - 1\right)} = 0$$

$$\text{Or } n^2 - 1 - qn\frac{dn}{dq} = 0 \tag{9.20}$$

(ii)The quantity (dn/dq) is obtained by differentiating Eq.(9.18) with respect to q i.e.

$$n\frac{dn}{dq} = \frac{(T_a/W)}{2K(W/S)} - \frac{qC_{DO}}{K(W/S)^2} \tag{9.21}$$

(iii)Substituting for n^2 and n (dn/dq) in Eq.(9.20) yields :

$$\frac{q}{K(W/S)}\frac{T_a}{W} - \frac{q^2 C_{DO}}{K(W/S)^2} - 1 - \frac{q(T_a/W)}{2K(W/S)} + \frac{q^2 C_{DO}}{K(W/S)^2} = 0$$

Simplifying :

$$\frac{q(T_a/W)}{2K(W/S)} = 1 \tag{9.22}$$

Equation (9.22) yields the value of q which gives minimum radius of turn. This value is denoted by 'q_{rmin} ' i.e. :

$$q_{rmin} = \frac{2K(W/S)}{T_a/W} \tag{9.23}$$

Using Eq.(9.17), V_{rmin} is given as :

$$V_{rmin} = \sqrt{\frac{4K(W/S)}{\rho(T_a/W)}} \tag{9.24}$$

Substituting q_{rmin} in Eq.(9.18) gives :

$$n^2_{rmin} = \frac{2K(W/S)(T_a/W)}{(T_a/W)K(W/S)} - \frac{4K^2 (W/S)^2 C_{DO}}{(T_a/W)^2 K(W/S)^2} = 2 - \frac{4\,K C_{DO}}{(T_a/W)^2}$$

$$\text{Or } n_{rmin} = \sqrt{2 - \frac{4K C_{DO}}{(T_a/W)^2}} \tag{9.25}$$

Substituting from Eqs.(9.24) and (9.25) in Eq.(9.11a) gives :

$$r_{min} = \frac{V^2_{rmin}}{g\sqrt{n^2_{rmin} - 1}} = \frac{4K(W/S)}{\rho(T_a/W)} \frac{1}{g\sqrt{2 - \frac{4KC_{DO}}{(T_a/W)^2} - 1}}$$

$$= \frac{4K(W/S)}{g\rho(T_a/W)\sqrt{1 - 4K C_{DO}/(T_a/W)^2}} \tag{9.26}$$

Proceeding in a similar manner, the values of $V_{\dot{\psi}max}$, $n_{\dot{\psi}max}$ and $\dot{\psi}_{max}$, which take into account the constraint of thrust available, can be derived. The final expressions are given below.

$$V_{\dot{\psi}max} = \left[\frac{2(W/S)}{\rho}\right]^{1/2} \left(K/C_{DO}\right)^{1/4} \tag{9.27}$$

$$n_{\dot{\psi}max} = \left[\frac{T_a/W}{\sqrt{KC_{DO}}} - 1\right]^{1/2} \tag{9.28}$$

$$\dot{\psi}_{max} = g\left\{\frac{\rho}{(W/S)}\left[\frac{T_a/W}{2K} - \left(\frac{C_{DO}}{K}\right)^{1/2}\right]\right\}^{1/2} \tag{9.29}$$

Remarks:

(i)From Eqs.(9.26) and (9.29) it is observed that for a jet airplane to have a low value of r_{min} and a high value of $V_{\dot{\psi}max}$, the value of (T_a/W) should be high and that of (W/S) should be low. However, as stated in section 7.4.3 the wing loading (W/S) is a compromise between various considerations like range, take-off and landing. Consequently, the general practice is to select (T_a/W) to give the desired value of $\dot{\psi}_{max}$, taking into account the wing loading chosen from other considerations.

(ii) The constraints of $(n_{max})_{str}$ and C_{Lmax} have not been taken into account in the above analysis. Also the variation of thrust available with flight speed has been ignored.

Equation (9.25) shows that the load factor for minimum radius of turn (n_{rmin}) is less than $\sqrt{2}$. However, the load factor for maximum rate of turn ($n_{\dot{\psi}max}$), as given by Eq.(9.28), could be high, especially near the sea level where (T_a/W) is at its highest. In this situation the constraint of $(n_{max})_{str}$ needs to be taken into consideration.

(iii)The constraint of C_{Lmax} is likely to affect the value of r_{min}. Example 9.4 illustrates such a situation.

(iv) A simplified analysis of the turning performance of an airplane with engine propeller combination can be carried out by assuming that (a) THP in constant with flight velocity and (b) C_{DO} and K are constants. However, the resulting expression has the following form.

$$A\,V_{rmin}^4 + B\,V_{r\,min} + C = 0$$

This equation does not have an analytical solution and a graphical or numerical procedure is needed. Reference 1.12 chapter 2 can be consulted for details.

It can be inferred from the analysis of Ref.1.12, that if it is desired to increase $\dot{\psi}_{max}$ or decrease r_{min} of a given airplane, then the wing loading (W/S) should be reduced and / or the ratio (BHP/W) should be increased.

Example 9.4

Consider the airplane in example 9.3 with the simplification that the thrust remains constant with flight velocity and has the value of 21685 N. Obtain the values of V_{rmin}, $V_{\dot{\psi}max}$, n_{rmin}, $n_{\dot{\psi}max}$, r_{min} and $\dot{\psi}_{max}$ as given by the analysis in subsection 9.3.5.

Solution :

The given data are :

W = 176, 400 N, S = 45 m^2, C_{DO} = 0.017, K = 0.05, h = 8000 m or

ρ = 0.525 kg/m^3, T_a = 21685 N .

The constraints are : C_{Lmax} = 1.4, $(n_{max})_{str}$ = 3.5.

Consequently, W/S = 176400 /45 = 3920 N/m^2 &T_a/W = 21685/176400 = 0.1229.

Based on the analysis of subsection 9.3.5, which considers only the constraint of thrust available, the following expressions are obtained.

$$V_{rmin} = \sqrt{\frac{4K(W/S)}{\rho(T_a/W)}}$$

$$n_{rmin} = \sqrt{2 - \frac{4K\,C_{DO}}{(T_a/W)^2}}$$

$$r_{min} = \frac{4K(W/S)}{g\rho(T_a/W)\sqrt{1-4K\,C_{DO}/(T_a/W)^2}}$$

$$V_{\psi max} = \left[\frac{2(W/S)}{\rho}\right]^{1/2}(K/C_{DO})^{1/4}$$

$$n_{\psi max} = \left[\frac{T_a/W}{\sqrt{KC_{DO}}} - 1\right]^{1/2}$$

$$\dot{\psi}_{max} = g\left\{\frac{\rho}{(W/S)}\left[\frac{T_a/W}{2K} - \left(\frac{C_{DO}}{K}\right)^{1/2}\right]\right\}^{1/2}$$

Accordingly ,

$$V_{rmin} = \sqrt{\frac{4 \times 0.05 \times 3920}{0.525 \times 0.1229}} = 110.23 \text{ m/s}$$

$$n_{rmin} = \sqrt{2 - \frac{4 \times 0.05 \times 0.017}{0.1229^2}} = 1.332$$

$$r_{min} = \frac{V_{rmin}^2}{g\sqrt{n^2-1}} = \frac{110.23^2}{9.81\sqrt{1.332^2-1}} = 1407.6 \text{ m}$$

$$V_{\psi max} = \left[\frac{2 \times 3920}{0.525}\right]^{1/2}\left(\frac{0.05}{0.017}\right)^{1/4} = 160.04 \text{ m/s}$$

$$n_{\psi max} = \left[\frac{0.1229}{\sqrt{0.05 \times 0.017}} - 1\right]^{1/2} = 1.793$$

$$\dot{\psi}_{max} = \frac{g\sqrt{n_{\psi max}^2 - 1}}{V_{\psi max}} = \frac{9.81 \times \sqrt{1.793^2 - 1}}{160.04} = 0.0912 \text{ rad/s}$$

The values of lift coefficients corresponding to V_{rmin} and $V_{\psi max}$ are:

$$C_{Lrmin} = \frac{n_{rmin}\ W}{\frac{1}{2}\rho V_{rmin}^2\ S} = \frac{1.332 \times 176400}{0.5 \times 0.525 \times 110.23^2 \times 45} = 1.637$$

$$C_{L\psi max} = \frac{n_{\psi max}\ W}{\frac{1}{2}\rho V_{\psi max}^2\ S} = \frac{1.793 \times 176400}{\frac{1}{2} \times 0.525 \times 160.04^2 \times 45} = 1.045$$

It is observed that in case of $\dot{\psi}_{max}$ the values of n and C_L are 1.793 and 1.045.

These values are lower than the prescribed values of $(n_{max})_{str}$ and C_{Lmax}. Hence, this turn is possible and $\dot{\psi}_{max}$ of 0.0912 rad/s at V = 160.04 m/s is possible. However, the value of C_{Lrmin} is 1.637 which is higher than C_{Lmax} and this turn is not possible. In this situation, a new value of flight velocity (V) is to be obtained at which the values of load factor (n) given by the two constraints of thrust available and C_{Lmax}, are equal.

The value of n from the constraint of thrust available can be denoted by 'n_{Ta}'. It is given by Eq.(9.16):

$$n_{Ta} = \left[\frac{\frac{1}{2}\rho V^2}{K(W/S)} \left\{ \frac{T_a}{W} - \frac{1}{2}\rho V^2 \frac{C_{DO}}{W/S} \right\} \right]^{1/2} \tag{9.30}$$

The value of n from the constraint of C_{Lmax} can be denoted by 'n_{CLmax}'. It is given by :

$$L = n_{CLmax}\ W = \frac{1}{2}\rho V^2 S C_{Lmax}$$

$$\text{Or } n_{CLmax} = \frac{1}{2}\rho V^2 \frac{C_{Lmax}}{W/S} \tag{9.31}$$

Equating Eqs.(9.30) and (9.31) gives the value of 'V' which satisfies both the constraints i.e.

$$\left[\frac{\frac{1}{2}\rho V^2}{K(W/S)}\left\{\frac{T_a}{W}-\frac{1}{2}\rho V^2\frac{C_{DO}}{W/S}\right\}\right]^{1/2}=\frac{1}{2}\rho V^2\frac{C_{Lmax}}{W/S}$$

Simplifying yields :

$$\frac{T_a/W}{K(W/S)}=\left\{\frac{C_{DO}}{K(W/S)^2}+\frac{C_{Lmax}^2}{(W/S)^2}\right\}\frac{1}{2}\rho V^2$$

Substituting various values gives :

$$\frac{0.1229}{0.05\times 3920}=\left[\frac{0.017}{0.05\times 3920^2}+\frac{1.4^2}{3920^2}\right]\frac{1}{2}\times 0.525\times V^2$$

Or V = 126.32 m/s

Consequently,

$$n=\frac{\frac{1}{2}\rho V^2 C_{Lmax}}{W/S}=\frac{0.5\times 0.525\times 126.32^2\times 1.4}{3920}=1.496$$

$$r_{min}=\frac{126.32^2}{9.81\sqrt{1.496^2-1}}=1461.9\text{ m}$$

The value of V = 126.32 m/s satisfies the constraints of T_a and C_{Lmax}. The corresponding value of n = 1.496 is also less than $(n_{max})_{str}$ of 3.5. Hence, all constraints are satisfied.

Answers : Based on the simplified analysis at 8000 m altitude the following values are obtained.

V_{rmin} = 126.32 m/s, n_{rmin} = 1.496, r_{min} = 1461.9 m,

$V_{\dot{\psi}max}$ = 160.04 m/s, $n_{\dot{\psi}max}$ = 1.793, $\dot{\psi}_{max}$ = 0.0912 rad/s

Remark :

The values by exact analysis are :

V_{rmin} = 124 m/s, n_{rmin} = 1.451 , r_{min} = 1490 m ,

$V_{\dot{\psi} max} = 165$ m/s, $n_{\dot{\psi} max} = 1.824$ $\dot{\psi}_{max} = 0.0907$ rad/s.

The agreement between the two results is seen to be reasonable. The reasons are that (T_a/W) is rather low and the variation of T_a with V is not large.

9.3.6 Sustained turn rate and instantaneous turn rate

The maximum rate of turn in a steady level co-ordinated-turn is called 'Maximum sustained turn rate(MSTR)' (Ref. 1.12 chapter 2). An airplane can maintain this turn rate continuously for some time. However, as explained in subsections 9.3.3 and 9.3.4 this turn rate is generally limited by the thrust available. A rate of turn higher than MSTR can be obtained if the airplane is allowed to descend or slow down. In this manner, the loss of potential energy or kinetic energy can be utilized to increase the available energy during turn and increase the rate of turn. This rate of turn is called 'Instantenous rate of turn'. The maximum instantenous rate of turn will be limited by other two factors viz. C_{Lmax} and $(n_{max})_{str}$. See also item (iv) in subsection 9.4.3.

General Remark:

In the foregoing sections various types of flight situations of practical interest have been analyzed. To analyze any other flight situation one can begin by writing down the equations of motion along and perpendicular to the flight path. From these equations, the lift required, thrust required and accelerations in tangential and radial directions can be worked out.