Chapter 6

Performance analysis II – Steady climb, descent and glide

(Lectures 21,22 and 23)

Keywords: Steady climb – equations of motion, thrust and power required; maximum rate of climb; maximum angle of climb; absolute ceiling; service ceiling; glide – equations of motion, minimum angle of glide, minimum rate of sink; hodographs for climb and glide.

Topics

Chapter 6

Lecture 21

Performance analysis II – Steady climb, descent and glide – 1

Topics

6.1. Introduction

In this chapter the steady climb, descent and glide are dealt with. A glide is a descent with thrust equal to zero. The approach in this chapter is as follows.

(a) Present the forces acting on the airplane in the chosen flight,

(b) Write down equations of motion using Newton's second law,

(c) Derive expressions for performance items like rate climb, angle of climb.

(d) Obtain variation of these with flight velocity and altitude.

6.2 Equations of motion in a steady climb

During a steady climb the center of gravity of the airplane moves at a constant velocity along a straight line inclined to the horizontal at an angle γ (Fig.6.1). The forces acting on the airplane are shown in Fig.6.1.

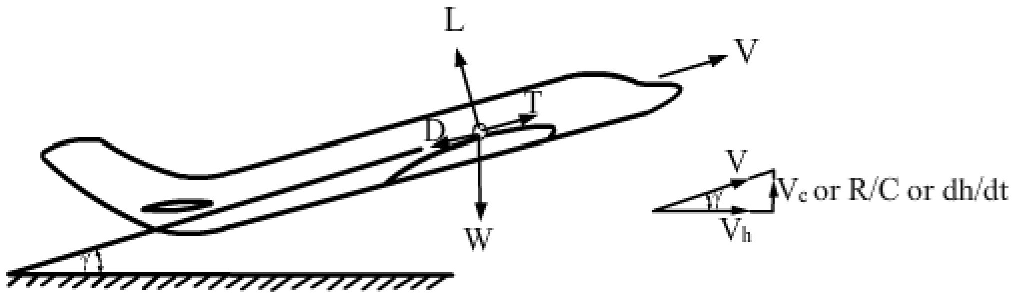

Fig.6.1 Steady climb

Since the flight is steady, the acceleration is zero and the equations of motion in climb can be obtained by resolving the forces along and perpendicular to the flight path and equating their sum to zero i.e.

$$T - D - W \sin \gamma = 0 \tag{6.1}$$

$$L - W \cos \gamma = 0 \tag{6.2}$$

Hence, $\sin \gamma = (T\text{- } D / W)$ (6.3)

From the velocity diagram in Fig.6.1, the vertical component of the flight velocity (V_C) is given by:

$$V_C = V \sin \gamma = (T\text{- } D / W)\, V \tag{6.4}$$

The vertical component of the velocity (V_C) is called rate of climb and also denoted by R/C. It is also the rate of change of height and denoted by (dh / dt). Hence,

$$V_C = R/C = dh/dt = V\sin\gamma = \frac{T\text{-}D}{W}V \tag{6.5}$$

Rate of climb is generally quoted in m/min.

Remarks :

i) Multiplying Eq.(6.1) by flight velocity V, gives:

$$T\,V = D\,V + W\,V \sin \gamma = D\,V + W\,V_C = DV + mg\frac{dh}{dt} = DV + \frac{d}{dt}(mgh) \tag{6.6}$$

In Eq.(6.6) the terms 'TV', 'DV' and '$\frac{d}{dt}(mgh)$' represent respectively, the power available, the energy dissipated in overcoming the drag and the rate of increase of potential energy. Thus, when the airplane climbs, its potential energy increases and a part of the engine output is utilized for this gain of potential energy.

Two facts may be pointed out at this juncture. (a) Energy supply to the airplane comes from the work done by the engine which is represented by the term'TV' in Eq.(6.6). (b) The drag acts in a direction opposite to that of the flight direction. Hence, energy has to be spent on overcoming the drag which is represented by the term 'DV' in Eq.(6.6). This energy (DV) is ultimately lost in the form of heat and is appropriately termed as 'Dissipated'. Continuous supply of energy is needed to overcome the drag. Thus, a climb is possible only when the engine output is more than the energy required for overcoming the drag.

It may be recalled from section 5.9 that in a level flight, at speeds equal to V_{max} and $(V_{min})_e$, the power (or thrust) available is equal to the power (or thrust) required to overcome the drag (see points D and D' in Fig.5.5 and points C and C' in Fig.5.6b). Hence, the rate of climb will be zero at these speeds. The climb is possible only at flight speeds in between these two speeds viz. V_{max} and $(V_{min})_e$. It is expected that there will be a speed at which the rate of climb is maximum. This flight speed is denoted by $V_{(R/C)max}$ and the maximum rate of climb is denoted by $(R/C)_{max}$. The flight speed at which the angle of climb (γ) is maximum is denoted by $V_{\gamma\,max}$.

ii) In a steady level flight, the lift is equal to weight but in a climb, the lift is less than weight as cos γ is less than one, when γ is not zero. Note that when an airplane climbs vertically, its attitude is as shown in Fig.6.2. It is observed that in this flight, the resolution of forces along and perpendicular flight direction gives:

$$L = 0, T = D + W$$

These expressions are consistent with Eqs.(6.1) and (6.2) when $\gamma = 90^o$ is substituted in them. Note that in this flight the thrust is more than the weight.

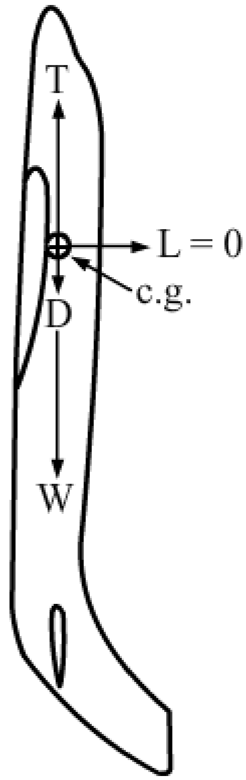

Fig.6.2 Airplane in vertical climb

6.3 Thrust and power required for a prescribed rate of climb at a given flight speed

Here it is assumed that the weight of the airplane (W), the wing area (S) and the drag polar are given. The thrust required and power required for a chosen rate of climb (V_c) at a given altitude (h) and flight speed (V) can be obtained, for a general case, by following the steps given below. It may be pointed out that the lift and drag in climb are different from those in level flight. Hence, the quantities involved in the analysis of climb performance are, hereafter indicated by the suffix 'c' i.e. lift in climb is denoted by L_c

i) Since V_c and V are prescribed, calculate the angle of climb γ from:

$$\gamma = \sin^{-1} (V_c / V)$$

ii) From Eq.(6.2) the lift required in climb (L_c) is :

$$L_c = W \cos \gamma \tag{6.7}$$

iii) Calculate the lift coefficient in climb (C_{Lc}) as:

$$C_{Lc} = \frac{L_c}{\frac{1}{2}\rho V^2 S} = \frac{W\cos\gamma}{\frac{1}{2}\rho V^2 S} \tag{6.8}$$

iv) Obtain the flight Mach number; M = V/a ; a = speed of sound at the chosen altitude.

v) Corresponding to the values of C_{Lc} and M, obtain the drag coefficient in climb (C_{DC}) from the drag polar. Hence, drag in climb (D_c) is given by:

$$D_c = (1/2\,\rho V^2\ S\ C_{DC}) \tag{6.9}$$

vi) The thrust required in climb (T_{rc}) is then given by:

$$T_{rc} = W\sin\gamma + D_c \tag{6.10}$$

and the power required in climb (P_{rc}) is :

$$P_{rc} = \frac{T_{rc}V}{1000} \text{ in kW} \tag{6.11}$$

Example 6.1

An airplane weighing 180,000N has a wing area of 45 m^2 and drag polar given by $C_D = 0.017 + 0.05\ C_L^2$. Obtain the thrust required and power required for a rate of climb of 2,000 m/min at a speed of 540 kmph at 3 km altitude.

Solution:

The given data are:

W = 180,000 N, S = 45 m^2, $C_D = 0.017 + 0.05\ C_L^2$

V_c = 2,000 m/ min = 33.33 m/s, V = 540 kmph = 150 m/s.

ρ at 3 km altitude = 0.909 kg/m^3

$\sin\gamma = V_c / V = 33.33/150 = 0.2222$ or $\gamma = 12^o$-50', $\cos\gamma = 0.975$

$L_c = W\cos\gamma = 180000 \times 0.975$

$$\text{Or } C_{Lc} = \frac{2W\cos\gamma}{\rho V^2 S} = \frac{180000 \times 0.975 \times 2}{0.909 \times 150 \times 150 \times 45} = 0.381$$

Hence, C_{DC} = 0.017 + 0.05 X0.381^2 = 0.02426

$D_c = (1/2\ \rho\ V^2\ S)\ C_{DC}$

= (1/2) X 0.909 X 150 X 150 X 45 X0.02426 = 11163 N

Hence, T_{rc} = W sin γ + D_c = 180000 X 0.2222 + 11163 = 51160 N

$P_{rc} = T_{rc}V/1000$ = 51160 X 150/1000 = 7674 kW

Answers:

Thrust required in climb (T_{rc}) = 51,160 N

Power required in climb (P_{rc}) = 7,674 kW

6.4 Climb performance with a given engine

In this case, the engine output is prescribed at a certain altitude and flight speed. This is in addition to the data on weight of the airplane (W), the wing area (S) and the drag polar.The rate of climb (V_c) and the angle of climb(γ) are required to be determined at the prescribed altitude and flight speed.

The solution to this problem is not straightforward as sin γ depends on (T- D_c) and the drag in climb (D_c) depends on the lift in climb (L_c), which in turn depends on W cos γ. Hence, the solution is obtained in an iterative manner. This is explained later in this section. However, if the drag polar is parabolic with constant coefficients, an exact solution can be obtained using Eqs. (6.1) to (6.4). The procedure is as follows.

From Eq.(6.4), sin $\gamma = V_c / V$.

Using Eq.(6.7), the lift during climb (L_c) = W cos γ = W (1-sin^2 γ)$^{1/2}$

$$= W\left[1-(V_c/V)^2\right]^{\frac{1}{2}} \qquad (6.12)$$

Hence, Lift coefficient during climb (C_{Lc}) $= \dfrac{L_c}{\frac{1}{2}\rho V^2 S} = \dfrac{W\left[1-(V_c/V)^2\right]^{\frac{1}{2}}}{\frac{1}{2}\rho V^2 S}$ (6.13)

By its definition, D = (1/2) ρ V^2 SC_D.

When the polar is parabolic, the drag in climb (D_c) can be expressed as :

$$D_c = (1/2)\,\rho\, V^2\, S\,(C_{DO} + K C_{Lc}^2) = \frac{1}{2}\rho V^2 S C_{DO} + \frac{KW^2}{\frac{1}{2}\rho V^2 S}\left[1-\left(\frac{V_c}{V}\right)^2\right] \qquad (6.14)$$

From Eq.(6.10), the thrust required in climb (T_{rc}) is given by :

$$T_{rc} = W \sin\gamma + D_c = \frac{W\,V_c}{V} + D_c$$

Substituting for D_c , yields :

$$T_{rc} = \frac{1}{2}\rho V^2 S C_{DO} + \frac{KW^2}{\frac{1}{2}\rho V^2 S}\left[1-\left(\frac{V_c}{V}\right)^2\right] + \frac{WV_c}{V} \qquad (6.15)$$

$$\text{or } A\left(\frac{V_c}{V}\right)^2 - B\left(\frac{V_c}{V}\right) + C = 0 \qquad (6.16)$$

$$\text{where, } A = \frac{KW^2}{\frac{1}{2}\rho V^2 S},\; B = W \text{ and } C = T - \frac{1}{2}\rho V^2 S C_{DO} - \frac{2KW^2}{\rho V^2 S} \qquad (6.17)$$

Equation (6.16) is a quadratic in (V_c / V), and has two solutions. The solution which is less than or equal to one, is the valid solution because V_c / V equals sin γ and sin γ cannot be more than one. Once (V_c / V) is known, the angle of climb and the rate of climb can be immediately calculated. This is illustrated in example 6.2.

Example 6.2.

For the airplane in example 6.1, obtain the angle of climb and the rate of climb at a flight speed of 400 kmph at sea level, taking the thrust available as 45,000 N.

Solution:

In this case, W = 1,80,000 N, S = 45 m^2 , $C_D = 0.017 + 0.05 C_L^2$

V = 400 kmph = 111.1 m/s, T = 45,000 N

$$\text{From Eq.(6.15), } T_{rc} = \frac{1}{2}\rho V^2 S C_{DO} + \frac{KW^2}{\frac{1}{2}\rho V^2 S}\left[1-\left(\frac{V_c}{V}\right)^2\right] + \frac{WV_c}{V}$$

Substitution of various quantities yields:

$$45000 = \frac{1}{2} \times 1.225 \times 111.1^2 \times 45 \times 0.017 + \frac{0.05 \times 180000^2}{\frac{1}{2} \times 1.225 \times 111.1^2 \times 45} \times \left(1 - \frac{V_c^2}{V^2}\right) + 180000 \times \frac{V_c}{V}$$

Simplifying, $\left(\frac{V_c}{V}\right)^2 - 37.82\left(\frac{V_c}{V}\right) + 7.24 = 0$

Solving the above quadratic gives : $(V_c / V) = 37.62, 0.192$.

Since sin γ cannot be larger than unity, the first value is not admissible.

Hence, $V_c / V = \sin\gamma = 0.192$ or $\gamma = 11^o\ 4'$

$V_c = 0.192 \times 111.1 = 21.33$ m/s = 1280 m/min.

Answers:

Angle of climb (γ) = $11^o\ 4'$; Rate of climb (V_c) = 1280 m/min

6.4.1 Iterative procedure to obtain rate of climb

When the drag polar is not given by a mathematical expression, an iterative procedure is required to obtain the rate of climb for a given thrust (T_a) or thrust horse power (THP_a). The need for an iterative solution can be explained as follows.

From Eq.(6.10), $\sin\gamma = \frac{T_a - D_c}{W}$ (6.18)

To calculate sin γ , the drag in climb (D_c) should be known. The term D_c depends on the lift in climb (L_c). In turn L_c is W cos γ , but cos γ is not known in the beginning.

To start the iterative procedure, it is assumed that the lift during climb (L_c) is approximately equal to W. Using this approximation, calculate the first estimate of the lift coefficient (C_{L1}) as :

$(C_{L1}) = W / (1/2)\rho V^2 S$

From C_{L1} and the flight Mach number obtain C_{D1} from the drag polar. Calculate the first approximation of drag (D_1) as:

$$D_1 = (1/2)\,\rho\,V^2\,S\,C_{D1}$$

Hence, the first approximation to the angle of climb (γ_1) is given by:

$$\sin\gamma_1 = \frac{T_a - D_1}{W} \qquad (6.19)$$

In the next iteration, put L = W cos γ_1 and carry out the calculations and get a second approximation to the angle of climb (γ_2). The calculations are repeated till the values of γ after consecutive iterations are almost the same. Once the angle γ is known, V_c is given as V sin γ.

It is observed that the convergence is fast and correct values of γ and V_c are obtained within two or three iterations. This is due to the following two reasons. (a)When γ is small (i.e. less than 10^o), cos γ is almost equal to one, and the approximation, L = W, is nearly valid. (b) When γ is large the lift dependent part of the drag, which is affected by the assumption of $L \approx W$, is much smaller than T_a. Consequently, the value of γ given by Eq.(6.18) is close to the final value. Example 6.3 illustrates the procedure.

Example 6.3

An airplane weighing 60,330 N has a wing area of 64 m^2 and is equipped with an engine-propeller combination which develops 500 kW of THP at 180 kmph under standard sea-level conditions. Calculate the rate of climb at this flight speed. The drag polar is given in the table below.

C_L	0.0	0.1	0.2	0.3	0.4	0.5	0.6
C_D	0.022	0.0225	0.024	0.026	0.030	0.034	0.040

C_L	0.7	0.8	0.9	1.0	1.2
C_D	0.047	0.055	0.063	0.075	0.116

Solution:

The given data are: W = 60,330 N, S = 64 m^2, V=180 kmph = 50 m/s,

THP_a = 500 kW. Hence, $T_a = (THP_a \times 1000)/V = 500 \times 1000 / 50 = 10{,}000$ N

The values of γ and V_c are obtained by the iterative procedure explained in section 6.4.1.

1st approximation: $L \approx W = \frac{1}{2}\rho V^2 S C_{L1}$

Hence, $C_{L1} = \dfrac{60330 \times 2}{1.225 \times 50 \times 50 \times 64} = 0.615$

C_{D1}: By interpolating between the values given in the above table, the value of C_{D1} is 0.041, corresponding to C_{L1} of 0.615.

Hence, $D_1 = (1/2) \times 1.225 \times 50 \times 50 \times 64 \text{ X} 0.041 = 4030$ N

From Eq.(6.19) : $\sin\gamma_1 = \dfrac{T_a - D_1}{W}$

Or $\sin\gamma_1 = \dfrac{10000-4030}{60330} = 0.099$

Or $\gamma_1 = 5^o 41'$

Hence, $V_{c1} = 50 \times 0.099 = 4.95$ m/s

$\cos\gamma_1 = 0.995$

2^{nd} approximation:

$L = W\cos\gamma_1 = 60330 \times 0.995 = 60036$ N

$C_{L2} = \dfrac{60036 \times 2}{1.225 \times 50 \times 50 \times 64} = 0.612$

From above table C_{D2} is 0.0408 corresponding to C_{L2} of 0.612.

Hence, $D_2 = (1/2) \times 1.225 \times 50 \times 50 \times 64 \times 0.0408 = 4010$ N

$\sin\gamma_2 = \dfrac{10000-4010}{60330} = 0.0993$

Hence, V_{c2} = 50 x 0.0993 = 4.965 m/s

The two approximations, V_{c1} and V_{c2} are fairly close to each other. Hence, the iteration process is stopped.

V_c = 4.965 m/s = 298 m/min.

Remark:

In the present example, γ is small ($5^0 41'$) hence 2nd iteration itself gives the correct value. For an interceptor airplane which has very high rate of climb (about 15000 m/min) few more iterations may be needed.