Chapter 5

Lecture 18

Performance analysis I – Steady level flight – 2

Topics

5.6 Thrust and power required in steady level flight when drag polar is independent of Mach number

5.7 Thrust and power required in steady level flight – consideration of parabolic polar

5.6 Thrust and power required in steady level flight when drag polar is independent of Mach number

When the Mach number is less than about 0.7, the drag polar is generally independent of Mach number. In this case, C_D / C_L and $C_D / C_L^{3/2}$ can be calculated for different values of C_L. The curves shown in Figs.5.4a and b are obtained by plotting C_D / C_L and $C_D / C_L^{3/2}$ as functions of C_L. From these curves it is observed that C_D / C_L is minimum at a certain value of C_L. This C_L is denoted by C_{Lmd} as the drag is minimum at this C_L. The power required is minimum when $C_D / C_L^{3/2}$ is minimum. The C_L at which this occurs is denoted by C_{Lmp} . Thus in steady level flight:

$$T_{rmin} = W\,(C_D / C_L)_{min} \tag{5.7}$$

$$P_{rmin} = \sqrt{\frac{2W^3}{\rho S}}\left(\frac{C_D}{C_L^{3/2}}\right)_{min} \tag{5.8}$$

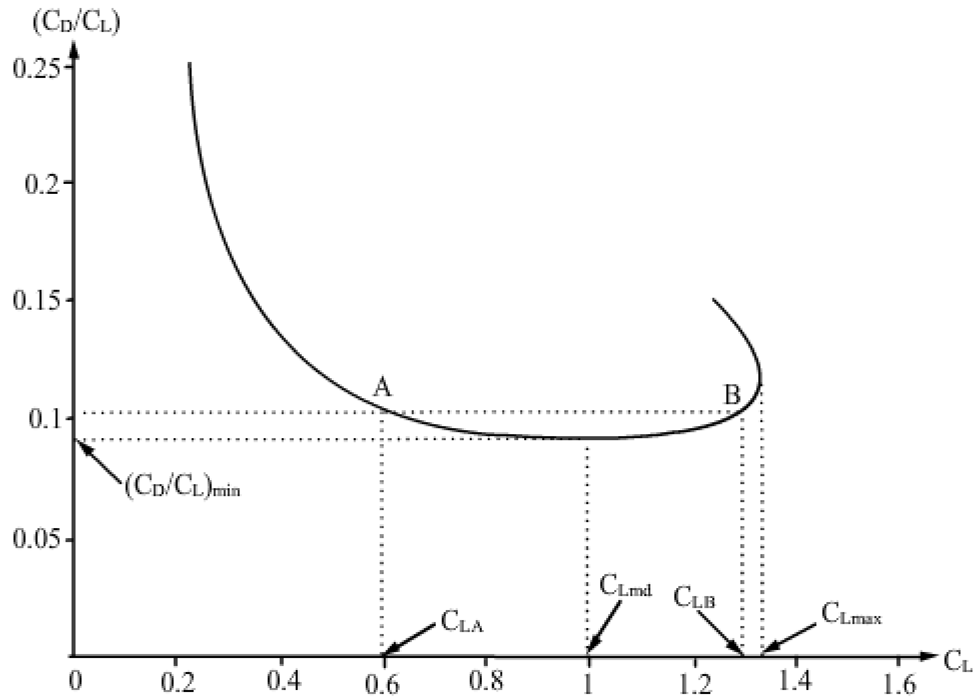

Fig.5.4a Variation of C_D / C_L with C_L

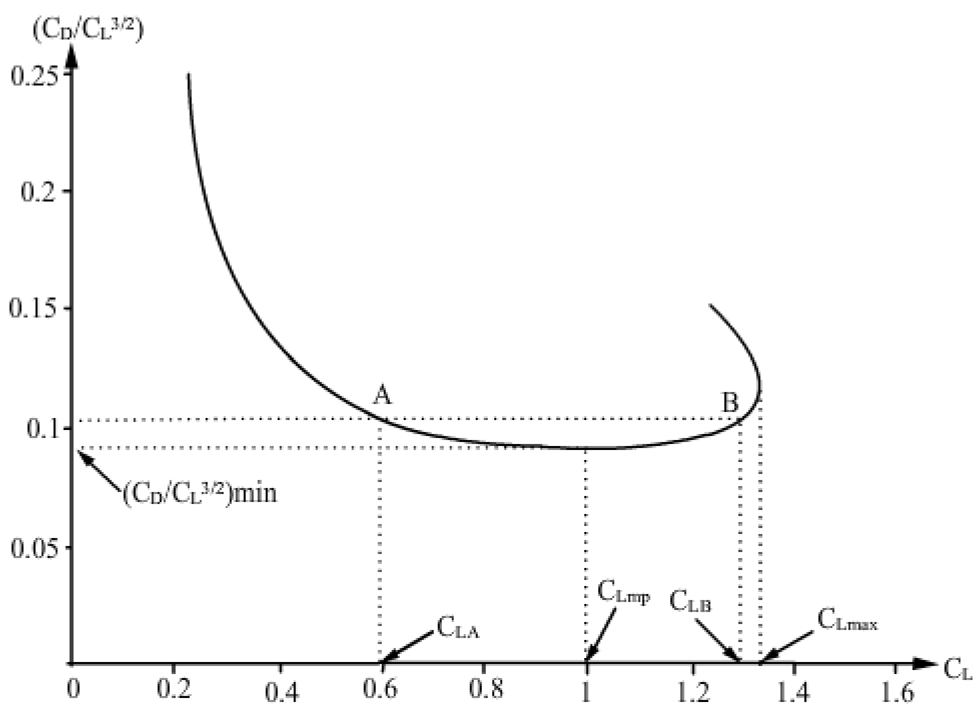

Fig.5.4b Variation of C_D / $C_L^{3/2}$ with C_L

The speeds, at which the drag and the power required are minimum, are denoted by V_{md} and V_{mp} respectively. The expressions for V_{md} and V_{mp} are:

$$V_{md} = \sqrt{\frac{2W}{\rho S C_{Lmd}}}\ ,\ V_{mp} = \sqrt{\frac{2W}{\rho S C_{Lmp}}} \qquad (5.9)$$

Note:

i) C_{Lmd} and C_{Lmp} are not equal and the corresponding speeds are different. As the density occurs in the denominator of Eq.(5.9), it implies that V_{md} and V_{mp} increase with altitude.

ii) Since for Mach number is lower than about 0.7, the drag polar is assumed to be independent of Mach number, the values of C_{Lmd}, C_{Lmp} , $(C_D / C_L)_{min}$ and $(C_D / C_L^{3/2})_{min}$ are also independent of Mach number. From Eqs.(5.7) and (5.8) it is seen that T_{rmin} is independent of altitude whereas P_{rmin} increases with altitude in proportion to $1/\sigma^{1/2}$.

iii) It is also observed in Fig.5.4a that a line drawn parallel to the X-axis cuts the curve at two points A and B. This shows that for the same value of C_D / C_L or the thrust $\{T_r = W(C_D / C_L)\}$, an airplane can have steady level flight at two values of lift coefficients viz. C_{LA} and C_{LB}. From Eq.(5.2) each value of C_L corresponds to a velocity. Hence for the same amount of thrust, in general, flight is possible at two speeds (V_A and V_B). These speeds are:

$$V_A = (2W / \rho SC_{LA})^{1/2}\ ,\ V_B = (2W / \rho SC_{LB})^{1/2} \qquad (5.9a)$$

Similarly, from Fig.5.4b it is observed that with the same power, in general, level flight is possible at two values of lift coefficient viz. C_{LA} and C_{LB} and correspondingly at two flight speeds viz. V_A and V_B .

iii) Typical variations of thrust required with flight speed and altitude are shown in Fig.5.5. Following interesting observations are made in this case where the drag polar is independent of Mach number. From Eq.(5.7) the minimum drag depends only on W and $(C_D / C_L)_{min}$ and hence is independent of altitude. However, the speed corresponding to minimum drag (V_{md}) increases with altitude (Eq.5.9). Hence, the thrust required curves at various altitudes have the same minimum thrust at all altitudes and the curves have a horizontal line, corresponding to $T = T_{rmin}$, as a common tangent (see Fig.5.5). This feature should be kept in mind when thrust required curves for subsonic airplanes are plotted.

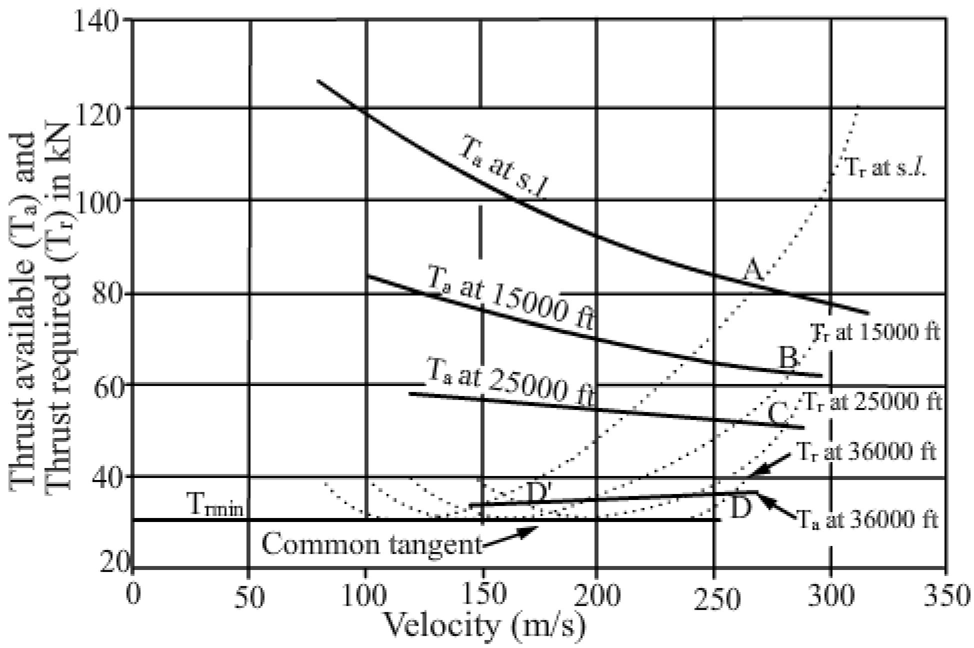

Fig.5.5 Thrust required and thrust available for subsonic jet airplane

iv) Typical variations of power required with flight speed and altitude are shown in Fig.5.6a. Interesting observations are made in this case also. From Eq.(5.8) the minimum power required (P_{rmin}) depends on $W^{3/2}$, $(C_D/C_L^{3/2})_{min}$ and $\rho^{-1/2}$. From Eq.(5.9) it is observed that V_{mp} depends on $\rho^{-1/2}$. Noting that for airplanes with piston engine or turboprop engine, the flight Mach number is less than 0.7, the drag polar is independent of Mach number. However, due to dependence on $\rho^{-1/2}$, the P_{rmin} and V_{mp} increase with altitude (Fig.5.6a) . It may be added that the slope of a line, joining a point on the P_r vs V curve and the origin, is P_r / V or T_r. However, as pointed out earlier, T_r has a minimum value (T_{rmin}) which is independent of altitude. Hence, all P_r vs. V curves have a common tangent passing through the origin. Such a tangent is shown in Fig.5.6a. This feature should be pointed out when P_r vs. V curves are plotted at different altitudes. Note that the common tangent to P_r vs. V curves does not touch at V_{mp} but at V_{md}.

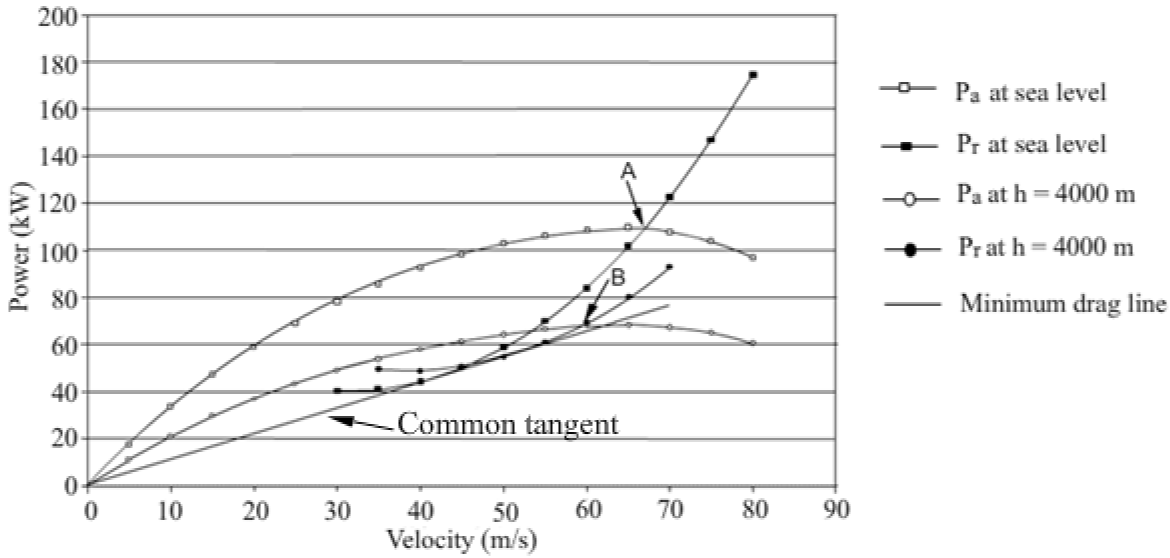

Fig.5.6a Power required and power available curves

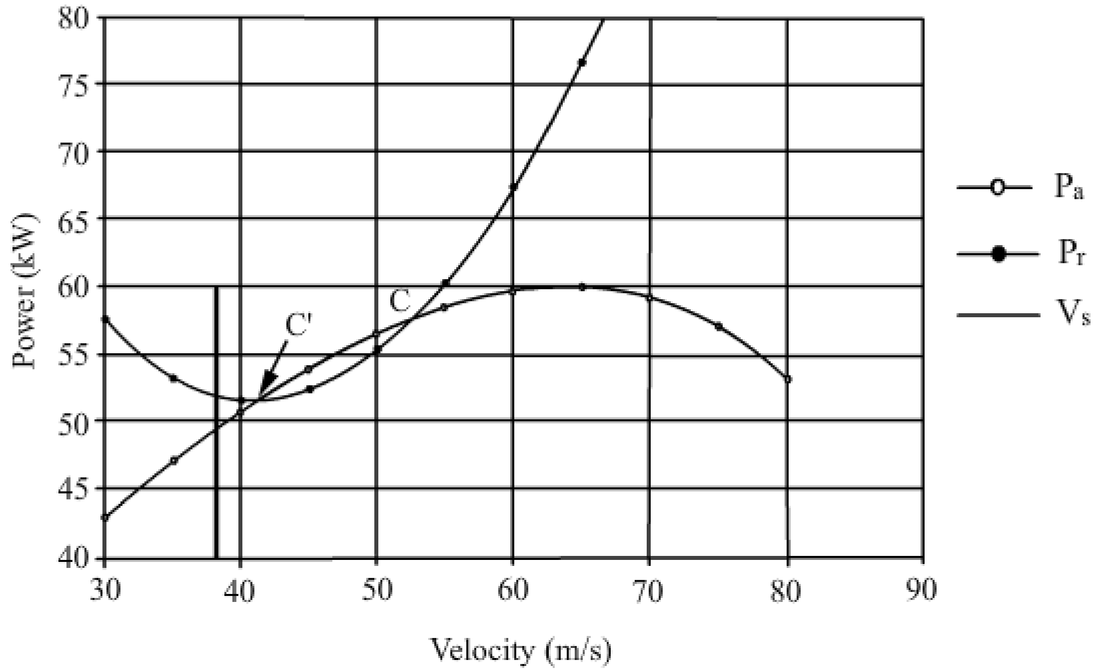

Fig.5.6b Power required and power available at an altitude near ceiling

5.7 Thrust and power required in steady level flight – consideration of parabolic drag polar

The discussion in section 5.6, was with reference to a general drag polar which may be given in tabular form or a plot. Consider the parabolic polar given by :

$$C_D = C_{D0} + KC_L^{\,2} \tag{5.10}$$

Since an equation is available for the drag polar, it is possible to obtain mathematical expressions for the power required and thrust required. In this section it is assumed that C_{DO} and K are constant with Mach number.

Substituting for C_D in expression for thrust required gives:

$$T_r = D = (1/2)\rho V^2 S C_D = (1/2)\,\rho V^2\, S\,(C_{D0} + KC_L^{\,2}) \tag{5.11}$$

Substituting for C_L as $W/\{(1/2)\rho V^2 S\}$ in Eq.(5.11) yields:

$$T_r = \frac{1}{2}\rho\, V^2\, S\left[C_{Do} + K\left(\frac{2W}{\frac{1}{2}\rho V^2 S}\right)^2\right]$$

$$\text{Or } T_r = \frac{1}{2}\rho\, V^2\, S C_{DO} + 2\,KW^2/(\rho V^2 S) \tag{5.12}$$

In Eq.(5.12) the first term (½) ρ V^2 S C_{D0} is called 'Parasite drag'. The second term 2 K W^2 / (ρV^2S) is called 'Induced drag'. Typical variations of the parasite drag, induced drag and total drag are shown in Fig.5.7.

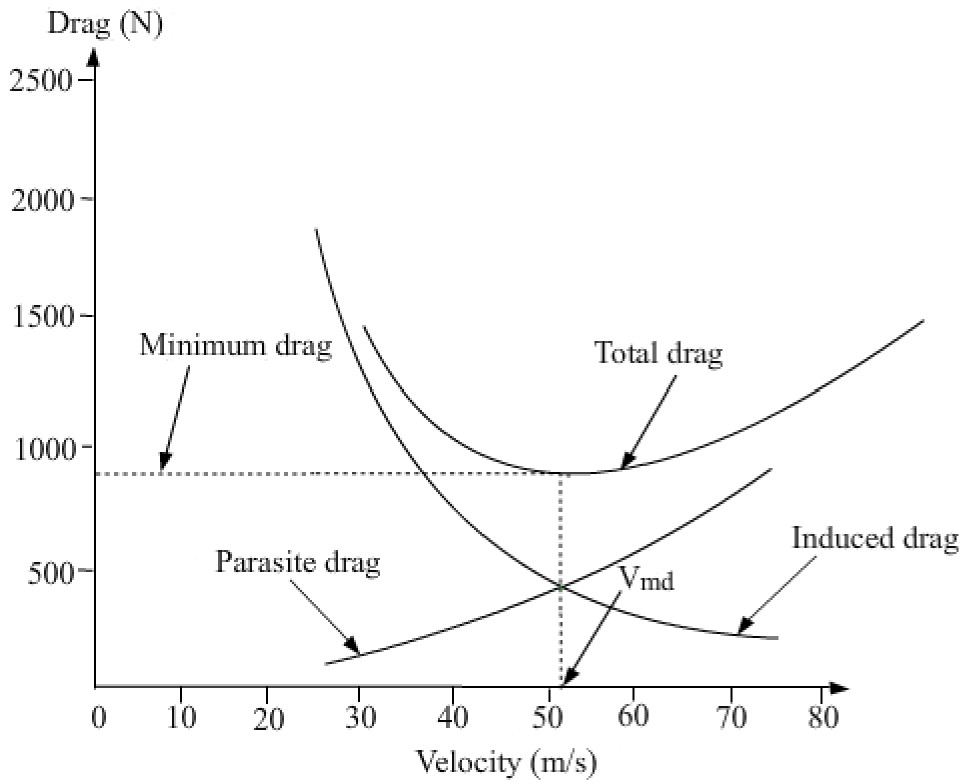

Fig.5.7 Variation of drag with flight speed

It is observed that from Fig.5.7 that the parasite drag, being proportional to V^2, increases rapidly with speed. The induced drag being proportional to $1/V^2$ is high at low speeds but decreases rapidly as speed increases. The total drag, which is the sum of the induced drag and the parasite drag, is approximately equal to induced drag at low speeds and approaches parasite drag at high speeds. It has a minimum value at a speed (V_{md}) where the parasite drag and induced drag are equal to each other (Fig.5.7). This can be verified by differentiating Eq.(5.12) with respect to V and equating it to zero i.e.

$$\frac{dT_r}{dV} = \rho\, V_{md}\, S\, C_{D0} + \frac{2KW^2}{\rho S}\,\frac{(-2)}{V_{md}^3} = 0$$

$$\text{Or}\quad V_{md} = \left(\frac{2W}{\rho S}\right)^{\frac{1}{2}} \left(\frac{K}{C_{D0}}\right)^{1/4} \qquad (5.13)$$

Substituting V_{md} in Eq.(5.12) gives minimum thrust required i.e.

$$T_{rmin} = W(C_{D0}K)^{1/2} + W(C_{D0}K)^{1/2} = 2W(C_{D0}K)^{1/2} \tag{5.14}$$

From Eq.(5.14) it is observed that when V equals V_{md}, the parasite drag and induced drag both are equal to $W(C_{D0}K)^{1/2}$. This is also shown in Fig.5.7.

Expression for power required in the present case is given by :

$$P_r = \frac{T_r V}{1000} = \frac{1}{1000}\frac{1}{2}\rho V^3 S C_D$$

Substituting for C_D from Eq.5.10 gives:

$$P_r = \frac{1}{1000}\frac{1}{2}\rho V^3 S\,[C_{D0} + KC_L^2]$$

$$\text{Or } P_r = \frac{1}{1000}\frac{1}{2}\rho V^3 S\,[C_{D0} + K(\frac{W}{\frac{1}{2}\rho V^2 S})^2]$$

$$\text{Or } P_r = \frac{1}{2000}\rho V^3 S\, C_{D0} + \frac{1}{500}\frac{KW^2}{\rho VS} \tag{5.15}$$

The first term in Eq.(5.15) is called 'Parasite power' and the second term is called 'Induced power'. The variations with flight velocity (V) of induced power, parasite power and the total power required are shown in Fig.5.8.

It is observed that the minimum power occurs at a speed, V_{mp}, at which the induced power is three times the parasite power. This can be verified by differentiating Eq.(5.15) with respect to V and equating it to zero. The verification is left as an exercise to the student.

$$V_{mp} = \left(\frac{2W}{\rho S}\right)^{1/2}\left(\frac{K}{3C_{Do}}\right)^{1/4} \tag{5.16}$$

$$P_{rmin} = \frac{1}{1000}\left(\frac{2W^3}{\rho S}\right)^{1/2}\left(\frac{256}{27}C_{Do}\,K^3\right)^{1/4} \tag{5.17}$$

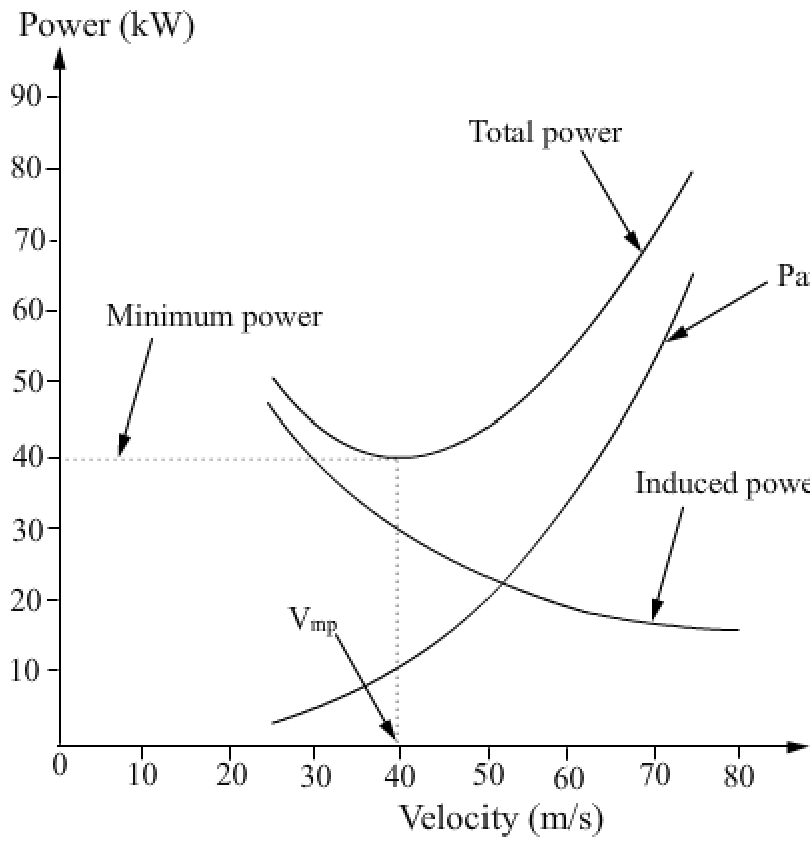

Fig.5.8 Variation of power required with flight speed

Remarks:

i) The expressions given in Eqs.(5.13) and (5.14) can be obtained in the following alternate way.

$$T_r = W\,(C_D / C_L)$$

Hence, $T_{rmin} = W\,(C_D / C_L)_{min}$ (5.18)

But, for a parabolic polar

$$\frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + K\,C_L \qquad (5.19)$$

The value of C_L at which (C_D / C_L) is minimum i.e. (C_{Lmd}) is given by :

$$\frac{d(C_D/C_L)}{dC_L} = 0 \text{ or } -\frac{C_{D0}}{C_{Lmd}^2} + K = 0$$

This gives C_{Lmd} as:

$$C_{Lmd} = (C_{Do} / K)^{1/2} \qquad (5.20)$$

The corresponding drag coefficient, C_{Dmd} is

$$C_{Dmd} = C_{DO} + \frac{KC_{DO}}{K} = 2C_{DO} \qquad (5.21)$$

Equation (5.21) shows that when T_r equals T_{min}, both parasite drag coefficient and induced drag coefficient are equal to C_{DO}. Hence under this condition, the parasite drag and induced drag both are equal to $(1/2)\rho V^2 S C_{DO}$.

Further,

$$\left(\frac{C_D}{C_L}\right)_{min} = \frac{C_{Dmd}}{C_{Lmd}} = \frac{2C_{DO}}{(C_{DO} / K)^{\frac{1}{2}}} = 2\,(C_{DO}\ K)^{\frac{1}{2}} \qquad (5.22)$$

Hence, T_{rmin} and V_{md} are:

$$T_{rmin} = 2\,W\,(C_{DO}\ K)^{1/2} \text{ and } V_{md} = \left(\frac{2W}{\rho S}\right)^{1/2}(K / C_{DO})^{1/4},$$

which are the same as Eqs.(5.14) & (5.13).

(ii) Exercise 5.4 gives expressions for T_r in terms of V/V_{md} and T_{rmin}.

(iii) Similarly, expressions given in Eqs.(5.16) and (5.17) can be obtained in the following alternate manner.

$$P_r = \frac{1}{1000}\left(\frac{2W^3}{\rho S}\right)^{1/2}\frac{C_D}{C_L^{3/2}}$$

Hence, P_{rmin} occurs when $C_D/C_L^{3/2}$ is minimum. For a parabolic polar

$$\frac{C_D}{C_L^{3/2}} = \frac{C_{DO}}{C_L^{3/2}} + K\,C_L^{1/2}$$

Therefore,

$$\frac{d\left(C_D/C_L^{3/2}\right)}{dC_L} = -\frac{3}{2}\frac{C_{DO}}{C_L^{5/2}} + \frac{1}{2}\frac{K}{C_L^{\frac{1}{2}}}$$

Equating the R.H.S. to zero, the value of C_L at which the power required is minimum (C_{Lmp}) is given as:

$$C_{Lmp} = (3C_{DO}/K)^{1/2} \tag{5.23}$$

Then the drag coefficient, corresponding to C_{Lmp} is given by:

$$C_{Dmp} = C_{DO} + \frac{3KC_{DO}}{K} = 4\,C_{DO} \tag{5.24}$$

Equation (5.24) shows that when P_r equals P_{rmin} the parasite drag coefficient is equal to C_{DO} and the induced drag coefficient is equal to $3C_{DO}$. Consequently, the parasite power is (1/2) $\rho V^3 SC_{DO}$ and induced power is 3 times of that.

Hence,

$$\left(\frac{C_D}{C_L^{3/2}}\right)_{min} = \frac{4C_{DO}}{(3C_{DO}/K)^{3/4}} = \left(\frac{256}{27}C_{DO}K^3\right)^{1/4} \tag{5.24a}$$

$$V_{mp} = \left(\frac{2W}{\rho SC_{Lmp}}\right)^{1/2} = \left(\frac{2W}{\rho S}\right)^{1/2}\left(\frac{K}{3\,C_{DO}}\right)^{1/4} = \frac{1}{3^{1/4}}V_{md} \approx 0.76\,V_{md} \tag{5.24b}$$

The above expression for V_{mp} is the same as in Eq.(5.16).

Example 5.1

An airplane weighing 100,000 N is powered by an engine producing 20,000 N of thrust under sea level standard conditions. If the wing area be 25 m^2, calculate (a) stalling speeds at sea level and at 10 km altitude,

(b) $(C_D / C_L)_{min}$, $(C_D / C_L^{3/2})_{min}$, T_{rmin}, P_{rmin}, V_{md} and V_{mp} under sea level conditions.

Assume C_{Lmax} = 1.5, $C_D = 0.016 + 0.064\ C_L^2$.

Solution:

The given data are : W = 100,000 N, T = 20,000 N, $C_D = 0.016 + 0.064\ C_L^2$,

$S = 25\ m^2$, $C_{Lmax} = 1.5$

a) $V_S = \sqrt{\dfrac{2W}{\rho S C_{Lmax}}}$,

at s.l. $\rho = 1.225\ kg/m^3$,

at 10 km $\rho = 0.413\ kg/m^3$

Hence, at sea level, $V_S = \sqrt{\dfrac{2 \times 100000}{1.225 \times 25 \times 1.5}} = 66$ m/s = 237.6 kmph

At 10 km altitude, $V_S = \sqrt{\dfrac{2 \times 100000}{0.413 \times 25 \times 1.5}} = 113.6$ m/s = 409.0 kmph.

b) $C_{Lmd} = \sqrt{\dfrac{C_{D0}}{K}} = \sqrt{0.016/0.064} = 0.5$

$C_{Dmd} = 2C_{DO} = 0.032$

Hence, $(C_D / C_L)_{min} = 0.032/0.5 = 0.064$ and

$T_{rmin} = W\ (C_D / C_L)_{min} = 100000 \times 0.064 = 6400$ N

$C_{Lmp} = \sqrt{3C_{DO}/K} = 0.866$

$C_{Dmp} = 4C_{DO} = 0.064$

$(C_D/C_L^{3/2})_{min} = 0.064/0.866^{3/2} = 0.0794$

$V_{md} = \sqrt{\dfrac{2W}{\rho\ S\ C_{Lmd}}} = \sqrt{\dfrac{2 \times 100000}{1.225 \times 25 \times 0.5}} = 114.5$ m/s = 412.2 kmph

$V_{mp} = \sqrt{\dfrac{2 \times 100000}{1.225 \times 25 \times 0.866}} = 86.30$ m/s = 310.7 kmph

Note: $V_{mp} = V_{md} / 3^{1/4}$

$$P_{rmin} = \frac{1}{1000}\sqrt{\frac{2W^3}{\rho S}}\left(C_D / C_L^{3/2}\right)_{min} = \frac{1}{1000}\sqrt{\frac{2 \times 100000^3}{1.225 \times 25}} \times 0.0794 = 641.5 \text{ kW}.$$

Answers :

a) V_S at sea level = 237.6 kmph

V_S at 10 km altitude = 409.0 kmph

b) $(C_D / C_L)_{min}$ = 0.064 ; $(C_D/C_L^{3/2})_{min}$ = 0.0794 ; T_{rmin} = 6400 N

At sea level : P_{rmin} = 641.5 kW; V_{md} = 412.2 kmph; V_{mp} = 310.7 kmph