Chapter 7

Lecture 26

Performance analysis III – Range and endurance – 3

Topics

7.4.5 Influence of the range performance analysis on airplane design

In section 5.8 it was pointed out that the analysis of level flight performance led to improvements in design of airplanes. Similarly, the analysis of range also helped in improvements in airplane design in the following way.

The high speed airplanes are jet airplanes and for these airplanes the range (R) is proportional to :

$$\frac{1}{TSFC}\frac{1}{C_D/C_L^{1/2}} \text{ or } \frac{1}{TSFC}\frac{C_L}{C_D}\frac{1}{C_L^{1/2}}$$

Noting that $1/C_L^{1/2}$ is proportional to flight speed (V),

$$R \propto \frac{1}{TSFC}\frac{C_L}{C_D}V$$

Since, high speed airplanes fly in lower stratosphere, where speed of sound is constant,

$$R \propto \frac{1}{TSFC}\frac{C_L}{C_D}M \qquad (7.19)$$

The quantity $\frac{1}{TSFC}\frac{C_L}{C_D}M$ can be referred to as figure of merit (FM) for the following reasons.

(a) A low value of TSFC in an indicator of high engine efficiency and (b) A high value of (C_L/C_D) is an indicator of high aerodynamic efficiency.

The figure of merit provided guidelines when the supersonic airplane Concorde was being designed in early 1960's.The subsonic jets of that period like Boeing 707 would fly around M = 0.8, have $(L / D)_{max}$ around 16 and TSFC around 0.9. These values would give the FM of 0.8x16/0.9 or 14.2. If Concorde were to compete with subsonic jets, it needed to have a similar value of FM. The fighter airplanes of that period flying at Mach number of two had TSFC of 1.5 and $(L/D)_{max}$ of 5. This would give FM of (2 x 5) /1.5 = 6.66 which was far too low as compared to that for subsonic airplanes. Hence the targets for Concorde, which was being designed for a Mach number of 2.2, were fixed at $(L/D)_{max}$ of 7.5 and TSFC of 1.2. This would give FM of 2.2(1/1.2) x 7.5 = 13.75, which was comparable to the FM of subsonic airplanes. However, to achieve a TSFC of 1.2 at M =2.2, a large amount of research was carried out and the Olympus engine used on Concorde was developed jointly by Rolls-Royce of U.K. and SNECMA of France. Similarly, to achieve an $(L/D)_{max}$ of 7.5 at M = 2.2 needed a large amount of computational and experimental effort. A picture of Concorde, a technological marvel, is shown in Fig.7.3.

It may be added that for Concorde the Mach number was limited to 2.2 as the designers had chosen to use aluminum as structural material. At M = 3 the FM could be greater than that of subsonic airplanes but the aerodynamic heating would cause surface temperatures of around 300^{o}C at which the strength and modulus of elasticity of aluminum will be significantly reduced.

Fig.7.3 Concorde

(Source: www.airplane-pictures.net)

The B787 (Fig.7.4) being brought out by Boeing and called 'Dream liner' has M = 0.85, $(L/D)_{max}$ of 22 and TSFC of 0.54 hr^{-1}. These values of $(L/D)_{max}$ and TSFC indicate steady improvements in aerodynamics and engine performance over the last five decades .

Fig.7.4 Boeing 787 Dream liner

(Source: www.lotz.com)

7.5 Range in constant velocity - constant altitude flight ($R_{h,V}$)

The assumption of constant C_L during cruise gives the longest range(R). However, it is more convenient for the pilot to fly the airplane at a constant speed or Mach number. He just needs to keep an eye on the airspeed indicator or Machmeter and adjust other parameters like the angle of attack and engine setting.

To derive an expression for range in level flight at constant speed ($R_{h,V}$), an airplane with jet engine is considered and it is assumed that TSFC is constant. Equation (7.8a) is the basic equation for range of a jet airplane. When V is constant, the equation takes the following form.

$$R_{h,V} = \frac{3.6\ V}{TSFC}\int_{W_1}^{W_2}\frac{dW}{T_r} \qquad (7.20)$$

T_r = thrust required

Assuming a parabolic polar,

$$T_r = \frac{1}{2}\rho V^2 S\, C_{DO} + \frac{2K\ W^2}{\rho V^2 S} = q\, S\, C_{DO} + \frac{KW^2}{qS};\ q = \frac{1}{2}\rho V^2$$

Note: The dynamics pressure (q), is constant in a constant velocity and constant altitude flight.

Substituting for T_r in Eq.(7.20) gives:

$$R_{h,v} = \frac{3.6\ V}{TSFC\ q\ S\ C_{D0}} \int_{W_1}^{W_2} \frac{-dW}{1+aW^2} \qquad \text{where, } a = \frac{K}{q^2\ S^2\ C_{D0}}$$

$$\text{Or } R_{h,v} = \frac{3.6\ V}{qS\ C_{D0} TSFC\ \sqrt{a}} [\tan^{-1}\sqrt{a}\ W_1 - \tan^{-1}\sqrt{a}\ W_2] \qquad (7.20a)$$

where W_2 = weight of the airplane at the end of the flight.

Let $\zeta = \frac{W_f}{W_1}$, where W_f = weight of fuel

Hence, $W_2 = W_1(1-\zeta)$;

Further, let $E_1 = W_1/D_1$ = initial lift to drag ratio,

$$D_1 = qSC_{D1} = qS\left\{C_{DO} + \frac{KW_1^2}{q^2S^2}\right\} = qSC_{DO} + \frac{KW_1^2}{qS}$$

$$C_{L1} = C_L \text{ at start of flight } = \frac{W_1}{\frac{1}{2}\rho V^2 S} = \frac{W_1}{qS}$$

$$E_{max} = \frac{1}{2\sqrt{K\ C_{D0}}}$$

Noting that ,

$$\tan^{-1}\theta_1 - \tan^{-1}\theta_2 = \tan^{-1}\left\{\frac{\theta_1 - \theta_2}{1+\theta_1\theta_2}\right\},$$

Equation (7.20a) can be rewritten as :

$$R_{h,v} = \frac{3.6V}{TSFC\ qSC_{DO}\dfrac{\sqrt{K}}{qS\sqrt{C_{DO}}}}\left\{\tan^{-1}\left[\frac{\sqrt{a}W_1 - \sqrt{a}W_2}{1+aW_1W_2}\right]\right\}$$

$$= \frac{3.6V}{TSFC\sqrt{KC_{DO}}} \tan^{-1}\left[\frac{\sqrt{a}\,(W_1 - W_2)}{1 + \frac{K}{C_{DO}}\frac{W_1^2}{q^2S^2}(1-\zeta)}\right]$$

$$= \frac{7.2E_{max}V}{TSFC} \tan^{-1}\left[\frac{\sqrt{\frac{K}{C_{DO}}}\frac{W_1}{qS}\zeta}{1+\frac{K}{C_{DO}}\frac{W_1^2}{q^2S^2}(1-\zeta)}\right]$$

Multiplying the denominator and numerator of the terms in square brackets by qSC_{DO} gives :

$$R_{h,v} = \frac{7.2E_{max}V}{TSFC} \tan^{-1}\left[\frac{\sqrt{KC_{DO}}\,W_1\zeta}{qSC_{DO} + \frac{KW_1^2}{qS} - \frac{KW_1^2}{qS}\zeta}\right]$$

$$= \frac{7.2E_{max}V}{TSFC} \tan^{-1}\left[\frac{\sqrt{KC_{DO}}\,W_1\zeta}{D_1 - K\frac{W_1^2}{qS}\zeta}\right]$$

Dividing the numerator and denominator of the term in square brackets by D_1, gives :

$$R_{h,v} = \frac{7.2E_{max}V}{TSFC} \tan^{-1}\left[\frac{\sqrt{KC_{DO}}\frac{W_1}{D_1}\zeta}{\left(1 - K\frac{W_1}{qS}\frac{W_1}{D_1}\zeta\right)}\right]$$

$$\text{Or } R_{h,V} = \frac{7.2E_{max}V}{TSFC} \tan^{-1}\left[\frac{E_1\zeta}{2E_{max}(1-KC_{L1}E_1\zeta)}\right] \qquad (7.21)$$

For an airplane with an engine-propeller combination, the range at constant speed and constant altitude ($R_{h,V}$) is given as:

$$R_{h,V} = \int_{W_1}^{W_2} \frac{3600\,\eta_p\,dW}{BSFC\,T} \qquad (7.22)$$

Assuming BSFC and η_p to be constant and the drag polar as parabolic i.e.

$T = \frac{1}{2}\rho V^2 S\, C_{DO} + \frac{2KW^2}{\rho S V^2}$ and substituting in Eq.(7.22) gives:

$$R_{h_1,V} = \frac{7200\,\eta_p}{BSFC} E_{max} \tan^{-1}\left[\frac{E_1 \zeta}{2E_{max}(1 - KC_{L1} E_1 \zeta)}\right] \qquad (7.23)$$

Remarks:

i) Comparing the ranges in the constant velocity and constant C_L flights, Ref.1.1, chapter 9, shows that the maximum range in a constant velocity flight is only slightly lower than that in a constant C_L flight.

ii) In actual practice BSFC (or TSFC) and η_p may vary during the cruise. If detailed information about their variations is available, then better estimates of range and endurance can be obtained by numerical integration of Eqs.(7.8), (7.8a),(7.9) and (7.9a).

iii) Appendix ‘A’ section 6 considers the range and endurance performance of a piston engined airplane at an altitude of 8000 feet (2438 m) in constant velocity flights at different speeds. The variations in propeller efficiency and fuel consumption are taken into account. It is seen that the endurance is maximum around flight speed of 135 kmph. The range is maximum for flight speeds between 165 to 185 kmph.

iv) Section 6 of Appendix ‘B’ considers the range and endurance performance of a jet transport at an altitude of 36000 feet (10973 m) in constant velocity flights at different speeds. The endurance is near its maximum value in the speed range of 684 to 828 kmph. The maximum range occurs around 240 m/s (864 kmph). The corresponding Mach number is 0.82, which is slightly higher than the Mach number beyond which the C_{DO} and K begin to increase due to compressibility effects.

7.6 Cruising speed and cruising altitude

The cruising speed (V_{cr}) and the cruising altitude (h_{cr}) together constitute the combination at which the maximum range is obtained. To arrive at the values of V_{cr} and h_{cr} the range is calculated at various speeds at a number of altitudes and the plots as shown in Fig.7.5 are obtained. The dotted line in Fig.7.5 is the envelop of all the curves. The speed and altitude at which the maximum of this envelop occurs is called the most economical cruising speed and altitude. In some cases this speed is rather low and a higher cruising speed may be chosen from other considerations like, shorter flight time and speed appeal. i.e. a faster airplane may be more appealing to the passengers even if it consumes more fuel per kilometer of travel.

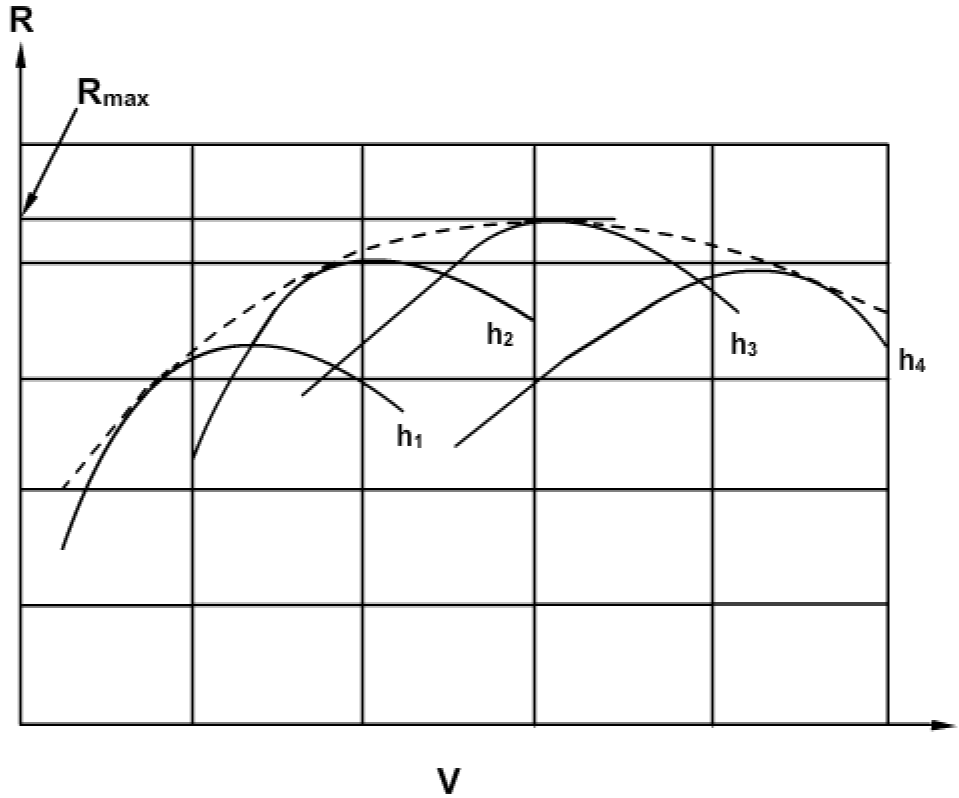

Fig.7.5 Determination of cruising speed and cruising altitude

7.7 Cruise climb

To prepare the back ground for the analysis of the cruise climb, consider Eq.(7.8a) which gives the range of a jet airplane. i.e.

$$R = \int_{W_1}^{W_2} \frac{-3.6\ V\ dW}{TSFC\ W\left(\frac{C_D}{C_L}\right)} \tag{7.24}$$

The TSFC is generally assumed to be constant during the flight. Further simplifications are needed to carry out the integration in Eq.(7.24). In the constant altitude - constant C_L flight considered in subsection 7.4.2, as the name suggests, the lift coefficient (C_L) is assumed constant during the flight. In this case, to satisfy the requirement of $L = W = \frac{1}{2}\rho V^2 S C_L$, the flight velocity is decreased as the weight of the airplane decreases due to consumption of fuel (see example 7.2). In the constant altitude – constant velocity flight, considered in section 7.5, the flight speed (V) is held constant during the flight. In this case, C_L decreases as the fuel is consumed.

Equation (7.24) suggests a third possibility, other than the above two cases, of both V and C_L being held constant during the flight.In this case, to satisfy $L = W = \frac{1}{2}\rho V^2 S C_L$, it has been suggested that the airplane be allowed to climb slowly such that the decrease of atmospheric density (ρ) with altitude compensates for the decrease of airplane weight due to consumption of fuel. With these simplifications Eq.(7.24) gives :

$$R = \frac{-3.6V}{TSFC\left(C_D/C_L\right)}\int_{W_1}^{W_2}\frac{dW}{W} = \frac{3.6V}{TSFC\left(C_D/C_L\right)}\ln\left(W_1/W_2\right) \tag{7.25}$$

The flight is called 'Cruise climb' as the altitude continuously increases during the flight.

Remarks:

(i) Exercise 7.3 would show that for a jet airplane with W_f / W_1 = 0.2 and starting the cruise climb at h = 11 km, the range would be 5141 km and the change of flight altitude between the end and the start of cruise climb would be only

1.415 km. Thus, it is observed that the change in the altitude between the start and end of cruise climb is very small as compared to the distance covered and the level flight equations (L = W and T = D) are valid.

(ii) It can be shown (Ref.1.1, chapter 5) that the range in a cruise climb is higher than that in level flight at the altitude where the cruise climb begins.

(iii) In actual practice continuous increase in altitude may not be permitted by Air Traffic Regulations. As an alternative, a stepped climb approximation may be used i.e. the flight path is divided into segments of constant altitude flights with stepped increase in altitude after certain distance.

(iv)In a cruise climb the thrust required would be

$$T = D = (1/2)\,\rho V^2 S C_D$$

Since, the flight velocity and C_L (and hence C_D) are held constant, the thrust required will be proportional to ambient density (ρ). It may be pointed out that in lower stratosphere the engine output (thrust available) is also proportional to the ambient density. Thus, in a cruise climb in lower stratosphere the thrust setting required is also constant and it becomes a very convenient flight – the pilot has just to set the Mach number and then the autopilot will take care of the flight.

7.8 Effect of wind on range and endurance

In the foregoing discussion, it was assumed that the airplane moves in a mass of air which is stationary with respect to the ground. However, in many situations the air mass has a velocity with respect to the ground and the airplane encounters head wind or tail wind. (see subsection 7.2.2 for definition of head wind and tail wind). The wind velocity is denoted by V_W . When V_W is non-zero, the velocity of the airplane with respect to the ground (V_g) and that with respect to air (V_a) are different. To analyze the effect of wind on airplane performance, it may be pointed out that the aerodynamic characteristics of the airplane (lift, and drag) and the engine characteristics depend on the velocity with respect to air (V_a), whereas the distance covered in the flight depends on the velocity with respect to the ground (V_g). In the presence of head wind the velocity of the airplane with respect to the ground will be lower than its velocity with respect to air and the range decreases. For example, in a hypothetical case of head wind being equal to the stalling speed, the airplane, in principle, can remain airborne without moving with respect to the ground. The fuel will be consumed as engine would produce thrust to overcome the drag, but no distance will be covered as the airplane is hovering! When there is tail wind the range increases.

An expression for range with effect of wind can be derived as follows.

Consider a jet airplane. Let R_g be the range in the presence of wind. Equation(7.8a) can be used to calculate R_g, but the quantity 'V' in that equation should be replaced with V_g i.e. :

$$R_g = -\int_{W_1}^{W_2} \frac{3.6\, V_g\, dW}{TSFC \times T} = \int_{W_1}^{W_2} \frac{3.6\,(V_a - V_W)\, dW}{TSFC \times T}\,,\ V_W \text{ in m/s}$$

$$R_g = -\int_{W_1}^{W_2} \frac{3.6\, V_a\, dW}{TSFC \times T} - 3.6\, V_W \int_{W_1}^{W_2} \frac{dW}{TSFC \times T} = R_a - 3.6\, V_W\, E \qquad (7.26)$$

where R_a is the range in still air = $-\int_{W_1}^{W_2} \frac{3.6\, V_a\, dW}{TSFC \times T}$

and E is the duration of flight in hours. Thus, with head wind the range decreases by $3.6\, V_W\, E$. In example 7.1 the range is 2667 km and the endurance is 3.33 hours. If a head wind of 15 m/s is encountered then the range would decrease by 15 x 3.6 x 3.33 = 180 km.

Remarks:

i) Before a flight takes- off, the information about head wind, likely to occur on the route is gathered from weather reports, and adequate amount of fuel is provided to take care of the situation.

ii) The maximum endurance (E_{max}) is not affected by the presence of wind, because E_{max} depends on airspeed only. The airspeed indicator in the cockpit, as the name suggests, indicates airspeed and the pilot only needs to fly at airspeed corresponding to E_{max}.