

## Chapter 6

### Lecture 23

## Performance analysis II – Steady climb, descent and glide – 3

### Topics

#### 6.6. Climb hydrograph

#### 6.7. Absolute ceiling and service ceiling

#### 6.8 Time to climb

#### 6.9 Steady descent

#### 6.10 Glide

6.10.1 Glide performance – minimum angle of glide, minimum rate of sink and maximum range and endurance in glide.

#### 6.11 Glide hodograph

### 6.6 Climb hodograph

From Fig.6.1 it is observed that in a climb, the vertical velocity is the rate of climb ( $V_c$ ) and the horizontal velocity is  $V_h$ . From the discussion in section 6.5 it is observed that for a chosen altitude, the vertical velocity ( $V_c$ ) and the horizontal velocity ( $V_h$ ) change with the flight speed ( $V$ ). A plot of the values of  $V_c$  and  $V_h$  at a particular altitude, in which  $V_c$  is plotted on y-axis and  $V_h$  is plotted on the x-axis is called 'Climb hodograph'. Figure 6.5 shows a hodograph, based on the sea level climb performance (Fig.6.3) of a jet airplane.

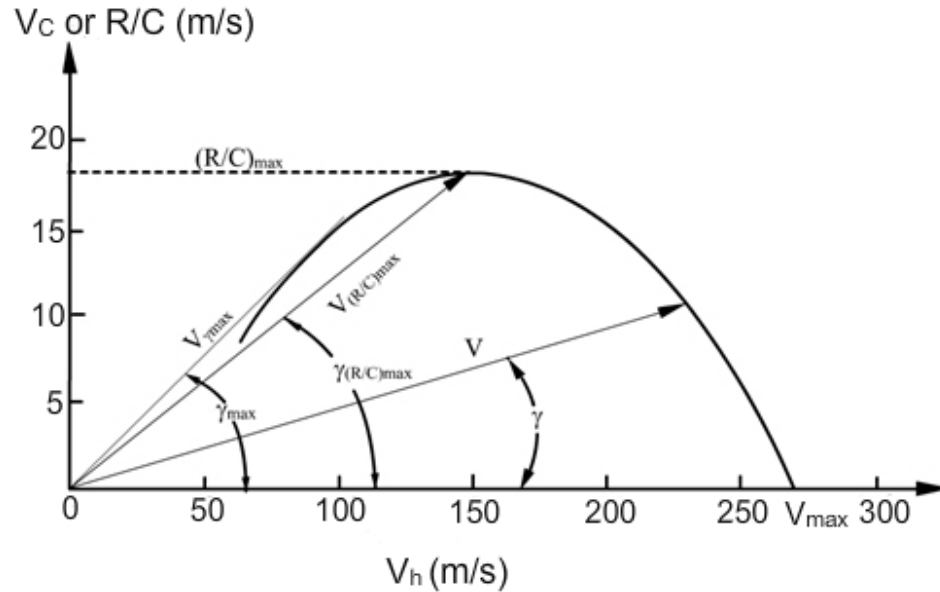


Fig.6.5 Climb hodograph

In a hodograph the line, joining the origin to a point on the curve, has the length proportional to the flight velocity ( $V$ ) and the angle this line makes to the horizontal axis ( $V_h$  - axis) is the angle of climb ( $\gamma$ ). This becomes evident when it is noted that  $V_c$  and  $V_h$  are the components of the flight velocity ( $V$ ) (see Fig.6.1).

A line from the origin which is tangent to the hodograph gives the value of  $\gamma_{\max}$  and also the velocity corresponding to it (Fig.6.5). Actually, a climb hodograph gives complete information about the climb performance at the chosen altitude especially  $\gamma_{\max}$ ,  $V_{\gamma_{\max}}$ ,  $(R/C)_{\gamma_{\max}}$ ,  $(R/C)_{\max}$ ,  $V_{(R/C)_{\max}}$ ,  $\gamma_{(R/C)_{\max}}$  and  $V_{\max}$ . These quantities are marked in Fig.6.5

### 6.7 Absolute ceiling and service ceiling

Figures 6.3e and 6.4d present the variations of maximum rate of climb,  $(R/C)_{\max}$ , with altitude. It is observed that  $(R/C)_{\max}$  decreases as the altitude increases. The altitude at which the  $(R/C)_{\max}$  is zero is called 'Absolute ceiling'. It is denoted by  $h_{\max}$ . At this altitude level flight is possible only at one speed (see sec. 5.9).

Near absolute ceiling, the rate of climb is very small and the time to climb becomes very large. It is not possible to reach the absolute ceiling (see remark (ii)

in section 6.8). Hence, for practical purposes an altitude at which the maximum rate of climb is 100 ft /min(30.5 m/min) is used as 'Service ceiling'.

To obtain the absolute ceiling and service ceiling the values of  $(R/C)_{\max}$  at different altitudes are plotted as shown in Figs.6.3e and 6.4d. Subsequently, the  $(R/C)_{\max}$  vs  $h$  curve is extrapolated till  $(R/C)_{\max} = 0$ . The altitude at which  $(R/C)_{\max}$  equals zero is the absolute ceiling. The altitude at which  $(R/C)_{\max}$  equals 100 ft/min (30.5 m/min) is the service ceiling. From Fig.6.3e and Appendix 'B' the absolute ceiling and service ceiling for the jet transport are 11.95 and 11.71 km respectively. From Fig.6.4d and Appendix A the values of these ceilings for a piston engined airplane are 5.20 and 4.61 km respectively.

### 6.8 Time to climb:

From the knowledge of the variation of rate of climb with altitude, the time required ( $t$ ) to climb from an altitude  $h_1$  to  $h_2$  can be calculated as follows.

$$V_C = \frac{dh}{dt} \text{ or } dt = \frac{dh}{V_C}$$

$$\text{Hence, } t = \int_{h_1}^{h_2} \frac{dh}{V_C} \quad (6.31)$$

The rate of climb ( $V_C$ ) in Eq.(6.31) depends on the speed and altitude at which the climb takes place. The appropriate values of  $V_C$  can be taken from plots similar to those given in Figs.6.3e or 6.4d.

#### Remarks:

i) It may be noted that in a climb which attempts to fly at  $(R/C)_{\max}$  at each altitude, the flight velocity,  $V_{(R/C)_{\max}}$ , increases with altitude (Figs.6.3c and 6.4c).

Consequently, such a flight is an accelerated climb and the values of  $V_C$  obtained using steady climb analysis will need to be appropriately corrected for the acceleration (see section 8.3.2 on accelerated climb).

ii) As an exercise the student should plot the height ( $h$ ) on y-axis and the time to climb ( $t$ ) on x-axis. It is observed that this curve reaches the absolute ceiling ( $h_{\max}$ ) in an asymptotic manner. In other words, the time taken to reach absolute

ceiling would be infinite and not practically attainable. Hence, service ceiling is used for practical purposes.

### 6.9 Steady descent

Figure 6.6 shows an airplane in a steady descent. In such a flight thrust is less than drag. The equations of motion are as follows.

$$T + W \sin \gamma - D = 0 \quad (6.32)$$

$$L - W \cos \gamma = 0 \quad (6.33)$$

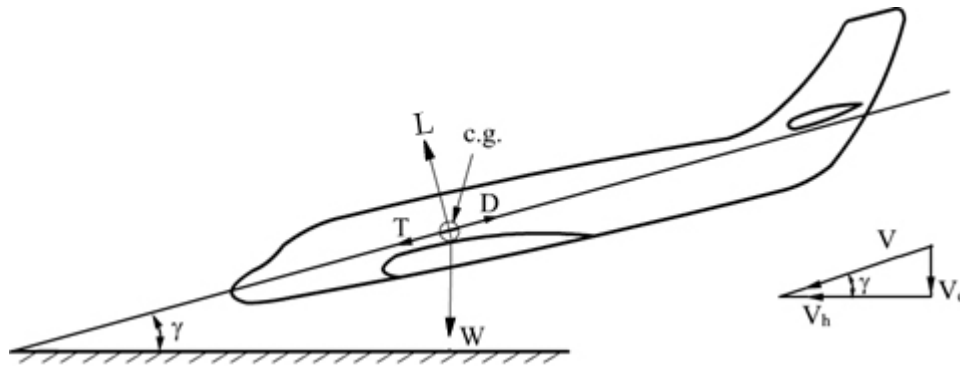


Fig.6.6 Descent or glide

$$\text{Hence, } \sin \gamma = \frac{D - T}{W} \quad (6.34)$$

$$\text{Rate of descent ( } V_d \text{ )} = \frac{D - T}{W} V \quad (6.35)$$

The rate of descent is also called rate of sink and denoted by (R/S).

### 6.10 Glide

In a glide the thrust is zero. This may happen in a powered airplane due to failure of engine while in flight. In a class of airplanes called gliders there is no engine and the thrust is always zero. With thrust equal to zero, the following equations of motion for glide, are obtained from Eqs.(6.32) and (6.33).

$$W \sin \gamma - D = 0 \quad (6.36)$$

$$L - W \cos \gamma = 0 \quad (6.37)$$

Hence,

$$\sin \gamma = D / W \text{ and} \quad (6.38)$$

$$V_d = V \sin \gamma = \frac{D}{W} V \quad (6.39)$$

The angle of glide,  $\gamma$ , is generally small. Hence,  $L \approx W$  and one can write,

$$\sin \gamma = \gamma = \frac{D}{W} \approx \frac{D}{L} \approx \frac{C_D}{C_L} \quad (6.40)$$

$$V \approx \left( \frac{2L}{\rho S C_L} \right)^{1/2} \approx \left( \frac{2W}{\rho S C_L} \right)^{1/2}$$

$$\text{and } V_d = \frac{DV}{W} \approx \frac{DV}{L} \approx \left( \frac{2W}{\rho S} \right)^{1/2} \frac{C_D}{C_L^{3/2}} \quad (6.41)$$

**Remarks:**

i) Multiplying Eq.(6.31) by V gives:

$$W V \sin \gamma - D V = 0$$

$$\text{Or } W V_d - D V = 0$$

Noting that  $V_d$  is the rate of descent and equals  $dh / dt$ ,

$$W \frac{dh}{dt} - DV = 0 \quad (6.42)$$

From Fig.6.6 it is to be noted that 'V' is along the glide path and hence in the downward direction. Consequently in Eq.(6.42)  $dh/dt$  is negative as the altitude is decreasing. As a result, the potential energy of the glider decreases with time. This loss of potential energy is utilized to provide for the energy required to overcome the drag (the second term in Eq.6.42). Hence, for a glider to stay aloft, it must be brought to a certain height and speed before it can carry out the glide. This is done by launching the glider by a winch or by towing the glider by another powered airplane.

### 6.10.1 Glide performance – minimum angle of glide and minimum rate of sink and maximum range and endurance in glide

The performance in a glide is stated in terms of the following four quantities.

(a) Minimum angle of glide ( $\gamma_{\min}$ ) (b) Minimum rate of climb ( $(R/S)_{\min}$  or  $V_{d\min}$ ).

- (c) The distance covered in glide from a certain height or 'range in glide ( $R_{\text{glide}}$ )'  
(d) The time elapsed in descending from a given height or 'Endurance in glide ( $E_{\text{glide}}$ )'.

From Eq.(6.40) the minimum angle of glide ( $\gamma_{\text{min}}$ ), occurs when  $C_D / C_L$  is minimum or at  $C_L = C_{L\text{md}}$ . From Eq.(6.41) the minimum rate of sink  $(R/S)_{\text{min}}$  or  $V_{\text{dmin}}$  occurs when  $C_D / C_L^{3/2}$  is minimum or at  $C_L = C_{L\text{mp}}$ . This can be understood from the following alternate explanation. When a glider sinks, it is expending energy to overcome the drag, which comes from the potential energy initially imparted to it. Thus, the rate of sink would be minimum when the rate of power consumption is minimum and this occurs when  $V$  equals  $V_{\text{mp}}$ .

Gliders with very low rate of sink (around 0.5 m/s) are called 'Sail planes'. From Eq.(6.41) it is observed that a low rate of sink is achieved by (a) low wing loading (b) low  $C_{D0}$  with smooth surface finish and (c) large aspect ratio (16 to 20) to reduce  $K$ . Note from Eq.(5.24a) that  $(C_D / C_L^{3/2})_{\text{min}}$  depends on  $C_{D0}^{1/4}$  and  $K^{3/4}$ .

If a glider is left at a height 'h' above the ground, then the horizontal distance covered in descending to the ground is called 'Range in glide' and denoted by  $R_{\text{glide}}$ . Assuming  $\gamma$  to be constant during the glide, the range in glide can be expressed as:

$$R_{\text{glide}} = \frac{h}{\tan \gamma} \approx \frac{h}{\gamma} = h \frac{C_L}{C_D} \quad (6.43)$$

Thus, the range in glide would be maximum when the flight is at  $C_L$  corresponding to  $C_{L\text{md}}$  or at  $V = V_{\text{md}}$ .

The time to descend from a height  $h$  is called  $E_{\text{glide}}$ . Assuming  $V_d$  to be constant,  $E_{\text{glide}}$  equals  $(h / V_d)$ . The quantity  $E_{\text{glide}}$  would be maximum when the descent takes place at  $C_L = C_{L\text{mp}}$  or  $V = V_{\text{mp}}$ .

It is evident from the above discussion that the flight speeds for  $\gamma_{\text{min}}$  and  $(R/S)_{\text{min}}$  are different.

### 6.11 Glide hodograph

In section 6.6, the climb hodograph was discussed. Similarly, a glide hodograph is obtained when horizontal velocity ( $V_h$ ) is plotted on the x-axis and the rate of sink ( $V_d$ ) is plotted on the y-axis. A typical diagram is shown in Fig.6.7. Such a diagram gives complete information about glide performance at an altitude especially,  $\gamma_{\min}$ ,  $V_{\gamma\min}$ ,  $(R/S)_{\min}$ ,  $V_{(R/S)\min}$  and  $\gamma_{(R/S)\min}$ .

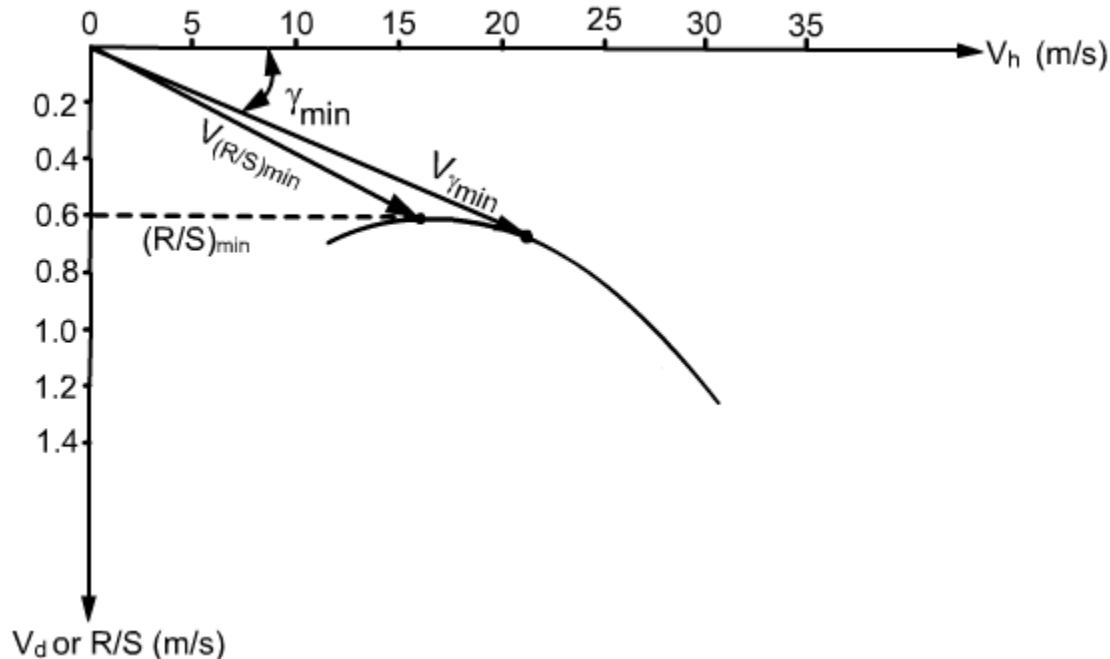


Fig.6.7 Glide hodograph

#### Example 6.4

A glider weighing 4905 N has a wing area of  $25 \text{ m}^2$ ,  $C_{D0} = 0.012$ ,  $A = 16$  and  $e = 0.87$ . Determine (a) the minimum angle of glide, minimum rate of sink and corresponding speeds under sea level standard conditions (b) the greatest duration of flight and the greatest distance that can be covered when glided from a height of 300 m. Neglect the changes in density during glide.

**Solution:**

$$(a) C_D = C_{D0} + \frac{C_L^2}{\pi A e} = 0.012 + \frac{1}{3.14 \times 16 \times 0.87} C_L^2 = 0.012 + 0.023 C_L^2$$

$$C_{L\gamma\min} = C_{Lmd} = (0.012/0.023)^{\frac{1}{2}} = 0.721$$

$$C_{D\gamma\min} = 2C_{DO} = 0.024$$

$$C_{L(R/S)\min} = C_{Lmp} = \sqrt{3} C_{Lmd} = 1.25$$

$$C_{D(R/S)\min} = 4C_{DO} = 0.048$$

Hence,

$$\gamma_{\min} = (C_D/C_L)_{\min} = 0.024 / 0.721 = 0.0332 \text{ radians or } 1.9^\circ$$

$$(R/S)_{\min} = \left( \frac{2W}{\rho S} \right)^{\frac{1}{2}} (C_D/C_L^{3/2})_{\min}$$

Noting that the density ( $\rho$ ) has been assumed to be constant and equal to that at sea level i.e.  $\rho = 1.225 \text{ kg/m}^3$ , the above equation gives :

$$(R/S)_{\min} = \left( \frac{2 \times 4905}{1.225 \times 25} \right)^{\frac{1}{2}} \left( \frac{0.048}{1.25^{3/2}} \right) = 0.615 \text{ m/s}$$

$$V_{\gamma\min} = \left( \frac{2 \times 4905}{1.225 \times 25 \times 0.721} \right)^{\frac{1}{2}} = 21.05 \text{ m/s.}$$

$$V_{(R/S)\min} = \left( \frac{2W}{\rho S C_{Lmp}} \right)^{\frac{1}{2}} = \left( \frac{2 \times 4905}{1.225 \times 25 \times 1.25} \right)^{\frac{1}{2}} = 16 \text{ m/s}$$

(b) The greatest distance, in descending from 300 m to sea level,  $(R_{\text{glide}})_{\max}$ , is (note  $\gamma$  is assumed constant during glide) :

$$(R_{\text{glide}})_{\max} = 300 / 0.0332 = 9040 \text{ m} = 9.04 \text{ km.}$$

Longest time taken in descending from 300 m to sea level  $(E_{\text{glide}})_{\max}$  is (note R/S is assumed constant during glide) :

$$(E_{\text{glide}})_{\max} = 300 / 0.615 = 487 \text{ s} = 8 \text{ min } 7 \text{ s.}$$

**Note:**

The rate of sink, in a flight when the greatest distance is covered, is higher than the minimum rate of sink. Hence, the time of flight will be shorter in this case than in a flight for longest endurance in glide. From the above data, the student may



show that  $(R/S)$  in the flight corresponding to  $(R_{\text{glide}})_{\text{max}}$  is 0.7015 m/s and the endurance in this flight is 427 s.

**Remarks:**

i) If the glide takes place from a sufficiently high altitude (as may happen for an airplane having an engine failure in cruise), the rate of sink  $(R/S)$  cannot be taken as constant during the descent. Equation (6.41) should be used to calculate the rates of sink at various altitudes.

ii) The time elapsed during glide ( $E_{\text{glide}}$ ), in a general case is given by:

$$E_{\text{glide}} = \int_{h_1}^{h_2} \frac{dh}{V_d} ; \quad (6.44)$$

where  $V_d$  is the rate of descent corresponding to an altitude 'h'.