

Chapter 4

Lecture 14

Engine characteristics – 2

Topics

4.2.8 Parameters for describing propeller performance and typical propeller characteristics

4.2.9 Selection of propeller diameter for chosen application

4.2.8 Parameters for describing propeller performance and typical propeller characteristics

As pointed out at the end of the previous subsection, the momentum theory of propeller has limitations. Though the refined theories are helpful in design of propeller blades, the propeller characteristics obtained from the wind tunnel tests are used for estimation of airplane performance. These characteristics are presented in terms of certain parameters. First these parameters are defined and then typical characteristics of propellers are presented. The procedures for (a) selection of the propeller diameter and (b) obtaining the propeller efficiency for given h , v , BHP and N , are given in the next two subsections.

Following Ref.4.1 and Ref.3.7 chapter 16, the propeller performance is expressed in terms of the following coefficients. It may be pointed out that FPS units are used in these references whereas SI units are used here.

$$\text{Advance ratio : } J = V/nd \quad (4.14)$$

$$\text{Power coefficient: } C_P = P/\rho n^3 d^5; P \text{ in Watts} \quad (4.15)$$

$$\text{Thrust coefficient: } C_T = T/\rho n^2 d^4 \quad (4.16)$$

$$\text{Speed power coefficient: } C_s = V (\rho / P n^2)^{1/5} = J / \sqrt[5]{C_P} \quad (4.17)$$

$$\begin{aligned} \text{Propeller efficiency: } \eta_p &= TV / P; P \text{ in Watts} \\ &= J (C_T / C_P) \end{aligned} \quad (4.18)$$

$$\text{Torque coefficient: } C_Q = \frac{Q}{\rho n^2 d^5} \quad (4.19)$$

$$\text{Torque speed coefficient: } Q_s = J/\sqrt{C_Q} = V\sqrt{\rho d^3/Q} \quad (4.20)$$

Where, P = Power in watts, T = thrust (N); V = flight velocity (m/s), n = rotational speed (rev/s),

d = diameter of propeller (m)

Q = Torque (Nm) = $P/2\pi n$

In FPS units:

T = thrust (lbs); P = power (ft lbs/s) = 550 BHP

V = velocity (ft / s), BHP = brake horse power

The performance of a propeller is indicated by thrust coefficient (C_T), power coefficient (C_P) and efficiency (η_p). These quantities depend on advance ratio (J) and pitch angle (β). Based on Ref.4.1, the experimental characteristics of a two bladed propeller are presented in Figs. 4.5a to d.

Figure 4.5a presents the variation of η_p vs J with β as parameter. It is seen that

η_p is zero when V is zero; J is also zero in this case by virtue of its

definition(Eq.4.14). Equation (4.2) also indicates that η_p is zero when V is zero.

This is because even though the engine is working and producing thrust, no useful work is done when V is zero. This is like a person pressing an immovable wall. He spends muscular energy to push the wall but the output and hence the efficiency is zero as the wall does not move and no useful work is done.

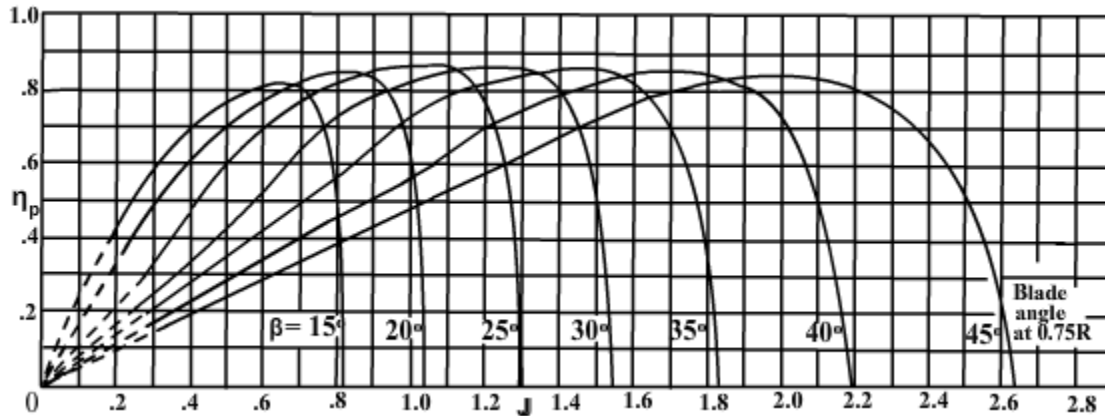


Fig 4.5a Propeller efficiency (η_p) vs advance ratio (J) with pitch angle (β) as parameter.

For a chosen value of β , the efficiency (η_p) increases as J increases. It reaches a maximum for a certain value of J and then decreases (Fig. 4.5a). The maximum value of η_p is seen to be around 80 to 85%. However, the value of J at which the maximum of η_p occurs, depends on the pitch angle β . This indicates that for a single pitch or fixed pitch propeller, the efficiency is high (80 to 85%) only over a narrow range of flight speeds (Fig. 4.5a). Keeping this behaviour in view, the commercial airplanes use a variable pitch propeller. In such a propeller the entire blade is rotated through a chosen angle during the flight and the pitch of all blade elements changes. Such propellers have high efficiency over a wide range of speeds. However, propellers with variable pitch arrangements are expensive and heavy. Hence, personal airplanes, where cost of the airplane is an important consideration, employ a fixed pitch propeller. As a compromise, in some designs, propellers with two or three pitch settings are employed.

Figure 4.5b presents the variation of power coefficient (C_p) vs J with β and C_T as parameters. This chart is useful to obtain η_p for given values of altitude, velocity, RPM and BHP (see subsection 4.2.10).

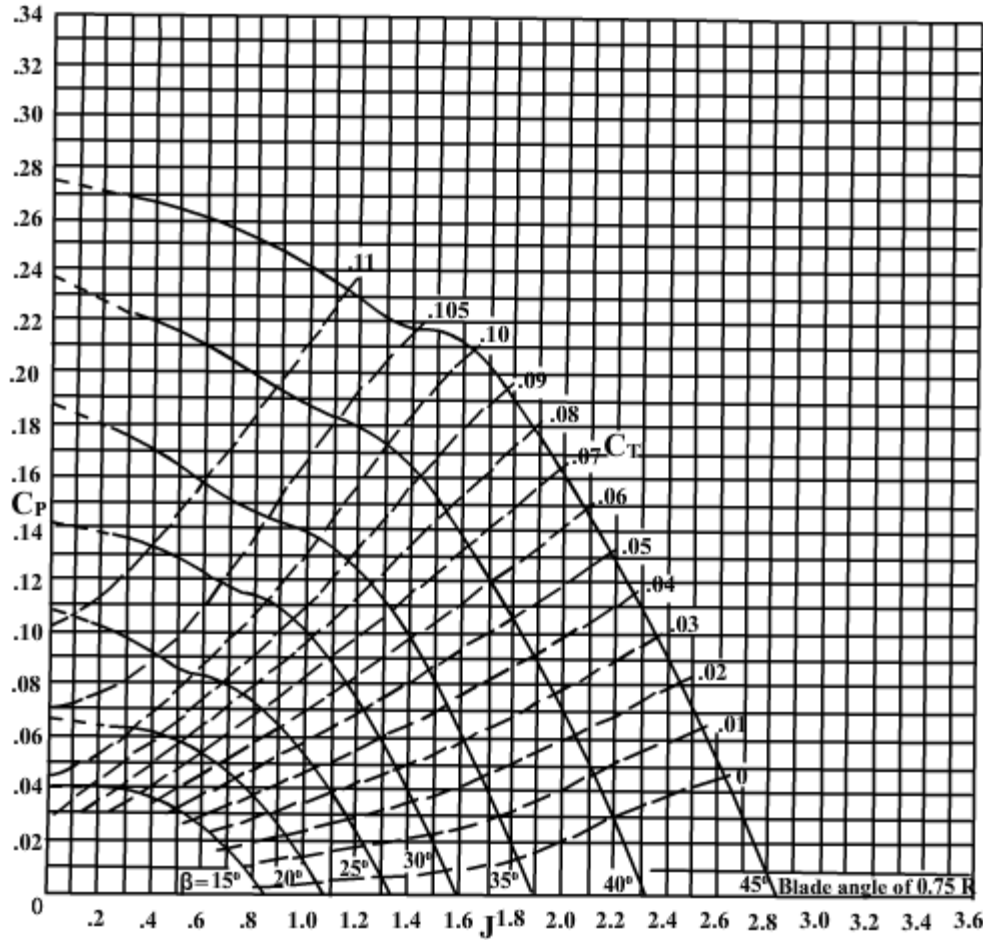


Fig 4.5b Power coefficient (C_p) vs advance ratio (J) with pitch angle (β) and thrust coefficient (C_T) as parameters.

Figure 4.5c presents the variations of C_s vs J and C_s vs η_p with β as parameter.

This figure is designated as 'Design chart' and is used for selection of the diameter of the propeller. A brief explanatory note on this topic is as follows.

Using definitions of J and C_p , the parameter C_s , defined below, is obtained. It is observed that this parameter does not involve the diameter (d) of the propeller.

$$C_s = \frac{J}{C_p^{1/5}} = V (\rho / P n^2)^{1/5} \quad (4.21)$$

It is also observed that the parameter C_s depends on V , ρ , P and N .

Consequently, this parameter can be evaluated when the power output (P), engine RPM(N) and flight condition viz. V and h are specified.

The design problem involves obtaining the value of J which would give the maximum value of η_p for a specified value of C_s . This is arrived at in the following manner.

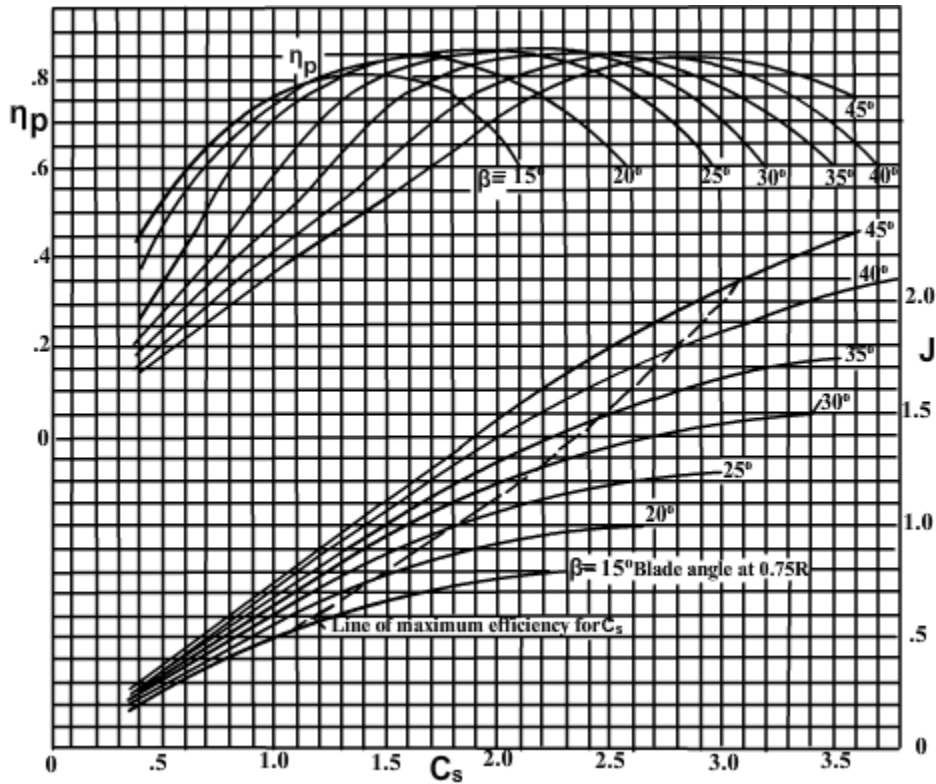


Fig 4.5c Design chart

Using the data in Figs 4.5b & a , the values of C_s can be obtained for constant values of J or β . For example, for $\beta = 15^\circ$ the values given in table 4.1 are obtained.

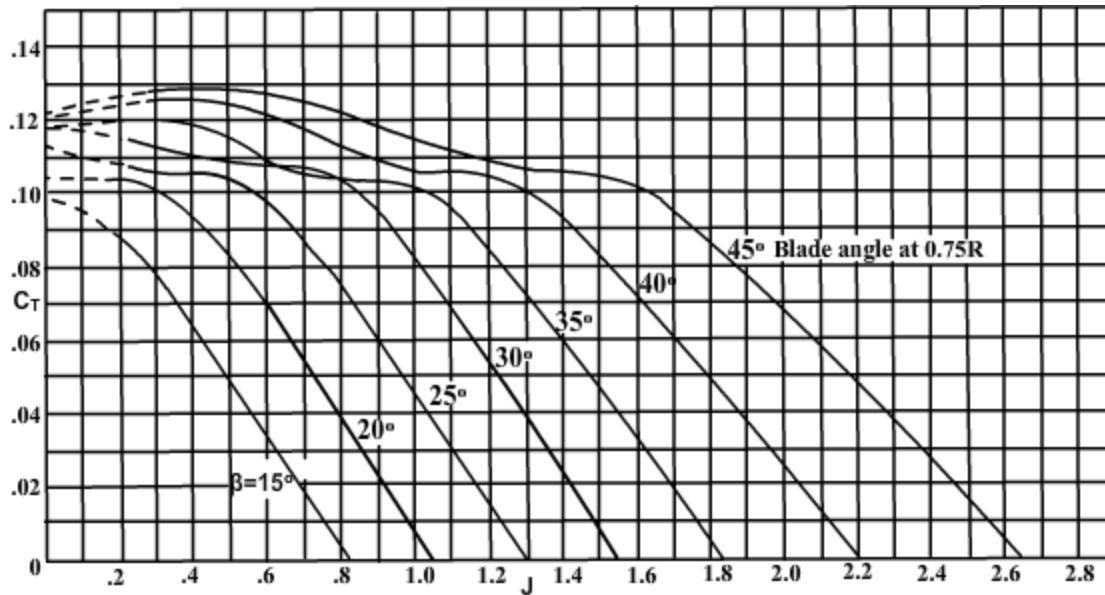
J	C_p From Fig 4.5b	C_s From Eq.(4.21)	η_p From Fig 4.5a
0	0.04	0	0
0.2	0.04	0.381	0.43
0.4	0.037	0.773	0.69
0.6	0.025	1.255	0.805
0.8	0.005	3.685	0.35

Table 4.1 variation of C_s with J for $\beta = 15^\circ$

Similar calculations at $\beta = 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ$ and 45° yield additional values. From these values the curves for C_s vs η_p and C_s vs J at different values of β can be plotted. These are shown in the upper and lower parts of Fig.4.5c. Based on these plots, the dotted line in the lower part of Fig.4.5c gives the values of J and β which would give maximum η_p . This line is designated as 'Line of maximum efficiency for C_s '. For example, corresponding to a value of $C_s = 1.4$, the dotted line gives J = 0.74 and $\beta = 20^\circ$. The upper part of the Fig.4.5c gives $\eta_p = 82\%$ for the chosen value of $C_s = 1.4$.

From the value of J, the propeller diameter is obtained as $d = V/(nJ)$; note that the values of V and n are already known. Subsection 4.2.9 gives additional details and example 4.4 illustrates the procedure to select the propeller diameter.

Figure 4.5d presents the variation of thrust coefficient (C_T) vs J with β as parameter. It is observed that when J is zero, C_T is not zero as the propeller produces thrust, even when 'V' is zero. The curves in Fig.4.5d are useful to estimate the thrust developed by the propeller especially during the take-off flight.



4.5d Thrust coefficient (C_T) vs advance ratio (J) with pitch angle β as parameter.

Fig 4.5 Typical characteristics of a two bladed propeller
(Adapted from Ref 4.1)

Remark :

Reference 4.1 contains information on propellers with three and four blades. Reference 3.7 chapter 16 contains information on six bladed propellers. Additional information can be obtained from Ref 4.2 which is cited in chapter 17 of Ref. 4.3.

4.2.9 Selection of propeller diameter for chosen application

A propeller is selected to give the best efficiency during a chosen flight condition which is generally the cruising flight for transport airplanes. Some companies may design their own propellers but it is an involved task. Hence, the general practice is to use the standard propellers and the charts corresponding to them. As a first step, the number of blades of the propeller is decided depending on the amount of power to be absorbed by the propeller.

The designer of a new airplane generally chooses the diameter of the propeller using the design chart (e.g. Fig.4.5c) appropriate to the propeller. Let us consider a two bladed propeller. Following steps are used to select the diameter of a propeller.

- (a) Choose a level flight condition i.e. altitude (h_c) and speed (V_c).
- (b) Obtain lift coefficient (C_L) in this flight using :
- $$C_L = W / (0.5 \rho V_c^2 S)$$
- Obtain the corresponding C_D from the drag polar of the airplane.
- (c) Obtain THP required during the flight using : $THP = (0.5 \rho V_c^3 S C_D) / 1000$
- (d) Assume $\eta_p = 0.8$.
- (e) Obtain $BHP = THP / 0.8$. Then RPM (N) which will give this power output at the chosen h_c with low BSFC is known from the engine curves e.g. Fig.4.2.
Calculate $n = N / 60$.
- (f) Calculate $C_s = V (\rho / P n^2)^{1/5}$.
- (g) From the design chart like Fig. 4.5c, obtain the value of J on the dotted line, corresponding to the value of C_s in step (f). Also obtain the value of β from the same curve. Obtain the value of η_p from the upper part of the design chart.
- (h) Since V, n and J are known, obtain propeller diameter (d) using : $d = V / n J$
- (i) If the value of η_p obtained in step (g) is significantly different from the value of 0.8 assumed in step (d), then iterate by using the value of η_p obtained in step (g).

Finally round-off the propeller diameter to nearby standard value.

Remark :

The choice of the parameters of the propeller like, diameter, pitch, blade size are also influenced by factors like noise level of the propeller, ground clearance, and natural frequency of the blade. Refer chapter 6 of Ref. 1.9.

Example 4.4

Consider the case of Piper Cherokee airplane dealt with in Appendix A and obtain the diameter of the propeller for this airplane. According to chapter 6 of Ref.1.9, the chosen speed and altitude for propeller design are 132 mph (212.4 kmph or 59 m/s) and sea level standard conditions respectively. The engine operates at 75% of the maximum power at an RPM of 2500.

Solution :

From Appendix 'A' the following data are obtained on Piper Cherokee airplane.

Weight of airplane = $W = 10673.28 \text{ N}$

Drag polar : $C_D = 0.0349 + 0.0755 C_L^2$

Wing area = $S = 14.864 \text{ m}^2$.

At sea level $\rho = 1.225 \text{ kg/m}^3$

$$C_L \text{ under chosen flight condition is } = \frac{10673.28}{\frac{1}{2} \times 1.225 \times 59^2 \times 14.864} = 0.3368$$

$$C_D = 0.0349 + 0.0755 \times 0.3368^2 = 0.04346$$

Hence, thrust horse power required (THP_r) is :

$$\text{THP}_r = \frac{\frac{1}{2} \times 1.225 \times 59^3 \times 14.864 \times 0.04346}{1000} = 81.26 \text{ kW}$$

As a first step, assume $\eta_p = 0.8$.

Consequently, the required BHP is :

$$\text{BHP}_r = 81.26/0.8 = 101.6 \text{ kW} = 101600 \text{ W}$$

Noting that at sea level the maximum power is 135 kW, the BHP_r of 101.6 kW is close to 75% of that value which is prescribed in the exercise.

$$N = 2500. \text{ Hence, } n = \text{revolutions per second} = 2500/60 = 41.67$$

$$\text{Consequently, } C_s = V \left(\rho / p n^2 \right)^{1/5} = 59 \left(1.225 / 101600 \times 41.67^2 \right)^{1/5} = 1.38$$

The airplane has a two bladed propeller of standard design and hence Fig 4.5c is applicable. From this figure, corresponding to C_s of 1.38, the dotted line gives

$$J = 0.74, \beta = 20^\circ, \eta_p = 0.83.$$

Consequently, the first estimate of propeller diameter is :

$$d = \frac{V}{nJ} = \frac{59}{41.67 \times 0.74} = 1.91 \text{ m}$$

Since, the value of η_p obtained is somewhat different from the value of 0.8 assumed earlier, the steps are repeated with $\eta_p = 0.83$.

$$\text{BHP}_r = 81.26/0.83 = 97.90 \text{ kW} = 97900 \text{ W}$$

$$C_S = 59 (1.225/97960 \times 41.47^2)^{1/5} = 1.390$$

From Fig. 4.5c corresponding to C_S of 1.39, the dotted line gives:

$$J = 0.75 \text{ and } \beta = 20^\circ \text{ and } \eta_p = 0.83.$$

Consequently, the second estimate of propeller diameter is :

$$d = \frac{59}{41.67 \times 0.75} = 1.89 \text{ m}$$

Since the latest value of η_p is same as the value with which the steps were repeated, the propeller diameter is taken as 1.89 m.

Remark:

The value of the propeller diameter obtained above is very close to the value of 1.88 m in the actual airplane.