Chapter 5

Lecture 20

Performance analysis I – Steady level flight – 4

Topics

5.10.2 Parameters influencing V_{max} of a jet airplane

5.10.3 Airplane with engine-propeller combination

5.11 Special feature of steady level flight at supersonic speeds

5.10.2 Parameters influencing V_{max} of a jet airplane

From Eq.(5.27), an analytical expression for V_{max} can be deduced when it is assumed that the thrust available (T_a) ,C_{DO} and K remain constant with flight speed. The derivation is as follows.

$$T_a = \frac{1}{2}\rho V^2 S\, C_{DO} + K\left(\frac{2W^2}{\rho V^2 S}\right) \tag{5.27}$$

or $AV^4 – BV^2 + C = 0$

$$\text{where, } A = \frac{1}{2}\rho SC_{DO}\text{, } B = T_a \text{ and } C = \frac{2KW^2}{\rho S}. \tag{5.27a}$$

When T_a, C_{DO} and K have constant values, Eq.(5.27a) gives :

$$V = \left\{\frac{B \pm \sqrt{B^2 - 4AC}}{2A}\right\}^{\frac{1}{2}}$$

Consequently, V_{max} being the larger of the two solutions, is :

$$V_{max} = \left\{\frac{B + \sqrt{B^2 - 4AC}}{2A}\right\}^{\frac{1}{2}}$$

Substituting for A, B and C from Eq.(5.27a) yields :

$$V_{max} = \left\{\frac{T_a}{\rho SC_{DO}} + \sqrt{\frac{T_a^2}{\rho^2 S^2 C_{DO}^2} - 4\frac{W^2}{S^2}\frac{K}{\rho^2 C_{DO}}}\right\}^{\frac{1}{2}} \tag{5.27b}$$

Multiplying and dividing some of the terms in Eq.(5.27b) by 'W' gives:

$$V_{max} = \left\{ \frac{(T_a/W)(W/S)}{\rho C_{DO}} + \sqrt{\frac{(T_a/W)^2 (W/S)^2}{\rho^2 C_{DO}^2} - 4\left(\frac{W}{S}\right)^2 \frac{K C_{DO}}{\rho^2 C_{DO}^2}} \right\}^{\frac{1}{2}}$$

Simplifying yields :

$$\text{Or} \qquad V_{max} = \left\{ \frac{(T_a/W)(W/S) + \left(\frac{W}{S}\right)^2 \sqrt{(T_a/W)^2 - 4C_{DO}K}}{\rho C_{DO}} \right\}^{1/2} \qquad (5.27c)$$

Equation (5.27c) shows that V_{max} depends on thrust to weight ratio (T_a /W), wing loading (W/S), C_{DO} , K and ρ .The maximum speed (V_{max}) increases with increase of (T_a /W) and (W/S) and decreases with increase of C_{DO} and K. The term 'ρ' in the denominator of Eq.(5.27c) indicates that V_{max} would be higher at higher altitudes because ρ decreases with altitude. In section 4.5 it is pointed out that the thrust output decreases as $\sigma^{0.7}$. Taking this into account, Eq.(5.27c) indicates that V_{max} would increase slightly upto a certain altitude as shown in Fig.5.9.

The trend of V_{max}, decreasing after a certain altitude, observed in Fig.5.9, can be explained as follows.

From atmospheric characteristics (Chapter 2), it is observed that, with the increase of altitude the speed of sound decreases. Thus for a given V_{max} the Mach number corresponding to it would increase with altitude. When the Mach number exceeds the critical Mach number, C_{DO} & K would no longer be constant but actually increase. This would result in lowering of V_{max} as compared to that obtained with constant values of C_{DO} and K. In section 4.2 of Appendix 'B' the values of V_{max} at different altitudes are obtained by a graphical procedure which takes into account the changes in C_{DO} and K when Mach number is greater than 0.8.

5.10.3 Airplane with engine-propeller combination

The steps to calculate V_{max} and $(V_{min})_e$ in this case, are as follows.

(1) Assume an altitude 'h'. Let P_a be the THP available in kW at this altitude.

(2) From Eq.(5.15) :

$$P_r = P_a = \frac{1}{2000}\rho V^3 S C_{DO} + \frac{1}{500}\frac{KW^2}{\rho VS}$$

or $A_1V^4 - B_1V + C_1 = 0$ (5.28)

where, $A_1 = \frac{1}{2000}\rho S C_{DO}, B_1 = P_a , C_1 = \frac{1}{500}\frac{KW^2}{\rho S}$.

Equation (5.28) is not a quadratic. An iterative method of solving Eq.(5.28) is given in example 5.3. Equation (5.28) has two solutions V_1 and V_2. The higher of these two gives V_{max} and the lower value gives $(V_{min})_e$. The minimum speed at the chosen altitude is higher of $(V_{min})_e$ and V_s (see example 5.3).

Remark:

Obtain power available at V_1 calculated above and denote it by P_{a1}. If P_a assumed at the beginning of the calculation in step (1), is significantly different from P_{a1}, then the calculations would need to be revised with the new value of P_{a1}. However, it is expected that the calculations would converge in a few iterations.

Example 5.2

For the airplane in example 5.1 obtain the maximum and minimum speed in steady level flight at sea level.

Solution:

The given data are :

W = 100,000N, T = 20,000N, S = 25 m^2, $C_D = 0.016 + 0.064\, C_L^2$, $C_{Lmax} = 1.5$

In this case, T / W = 20000 / 100000 = 0.2 = C_D / C_L

$$0.2 = \frac{0.016}{C_L} + 0.064 C_L$$

Or $0.064\, C_L^2 - 0.2\, C_L + 0.016 = 0$

Solving the above equation gives: C_L = 3.04 and 0.0822. The corresponding speeds are :

$$V_{max} = \sqrt{\frac{2 \times 100000}{1.225 \times 25 \times 0.0822}} = 281.8 \text{ m/s}$$

and $$(V_{min})_e = \sqrt{\frac{2 \times 100000}{1.225 \times 25 \times 3.04}} = 45.4 \text{ m/s}$$

Since V_S , as calculated in example 5.1, is 66.0 m/s, the minimum speed is decided by V_S and equals 66.0 m/s.

The Mach number corresponding to V_{max} is :

281.8 / 340.29 = 0.828.

This value of Mach number is likely to be greater than M_{crit}. As a possible assumption let us assume M_{cruise} = 0.8 and obtain ΔC_{DO} and ΔK from Eqs.3.50a and 3.51a. Consequently,

$\Delta C_{DO} = - 0.001 (M - 0.8) + 0.11 (M - 0.8)^2$ and $\Delta K = (M - 0.8)^2 + 20(M - 0.8)^3$

For M = 0.828, ΔC_{DO} = 0.000055 and ΔK = 0.00122

Hence, the drag polar at M = 0.828 is likely to be:

$C_D = (0.016 + 0.000055) + (0.064+0.00122)C_L^2 = 0.016055 + 0.06522\, C_L^2$

Using this polar and revising the calculations, gives: V_{max} = 281.3 m/s

This revised value of V_{max} is very close to the value of 281.8 m/s obtained earlier and hence further revision is not needed.

(Answers: V_{max} = 281.3 m/s =1012.7 kmph, V_{min} = 66.00 m/s = 237.6 kmph)

Example 5.3

A piston-engined airplane has the following characteristics.

W = 11,000 N, S = 11.9 m^2, $C_D = 0.032 + 0.055\, C_L^2$, C_{Lmax} = 1.4.

Obtain the maximum and minimum speeds in level flight at an altitude of 3 km assuming that the engine BHP is 103 kW and the propeller efficiency is 83%.

Solution :

W = 11,000 N, S = 11.9 m^2, $C_D = 0.032 + 0.055\, C_L^2$

C_{Lmax} = 1.4, ρ at 3km altitude = 0.909 kg/m^3,

$P_a = \eta \times BHP = 0.83 \times 103 = 85.5$ kW

From Eq.(5.15):

$$P_a = P_r = \frac{1}{2000}\rho V^3 S\, C_{Do} + \frac{2K}{1000}\frac{W^2}{\rho S V}$$

$$\text{Or } 85.5 = \frac{1}{2000} \times 0.909 \times 11.9 \times 0.032 \times V^3 + \frac{2}{1000} \times \frac{0.055 \times 11000^2}{0.909 \times 11.9 \times V}$$

$$= 1.731 \times 10^{-4} V^3 + \frac{1230.5}{V} \qquad (5.28a)$$

Equation (5.28a) is not a quadratic. However, it can be solved for V_{max} and $(V_{min})_e$ by an iterative procedure.

Solution for V_{max}:

When solving for V_{max}, by an iterative procedure, it is assumed that the first approximation (V_{max1}) is obtained by retaining only the term containing the highest power of V in Eq.(5.28a) i.e.

1st approximation: $85.5 = 1.731 \times 10^{-4} V^3_{max1}$

This gives $V_{max1} = 79.05$ m/s

To obtain the 2nd approximation, substitute V_{max1} in the second term on RHS of Eq.(5.28a). Note that this term was ignored in the first approximation.

$$85.5 = 1.731 \times 10^{-4} V^3_{max2} + \frac{1230.5}{79.05}$$

Or $V_{max2} = 73.93$ m/s

To obtain the 3rd approximation, substitute V_{max2} in the second term on RHS of Eq.(5.28a), i.e.

$$85.5 = 1.731 \times 10^{-4} V^3_{max3} + \frac{1230.5}{73.93}$$

Or $V_{max3} = 73.54$ m/s

To obtain the 4th approximation, substitute V_{max3} in the second term on RHS of Eq.(5.28a), i.e.

$$85.5 = 1.731 \times 10^{-4} V_{max4}^3 + \frac{1230.5}{73.54}$$

Or V_{max4} = 73.51 m/s

Since the 3rd and 4th approximations are close to each other, V_{max} is taken as 73.51 m/s.

Solution for $(V_{min})_e$:

When solving for $(V_{min})_e$, by an iterative procedure, it is assumed that the first approximation $(V_{min})_{e1}$, is obtained by retaining only the term containing the lowest power of V in Eq.(5.28a) i.e.

$$85.5 = \frac{1230.5}{(V_{min})_{e1}}$$

Or $(V_{min})_{e1}$ = 14.4 m/s.

To obtain the 2nd approximation, substitute $(V_{min})_{e1}$ in the first term on RHS of Eq.(5.28a). Note that this term was ignored in the first approximation.

$$85.5 = 1.731 \times 10^{-4} \times 14.4^3 + \frac{1230.5}{(V_{min})_{e2}}$$

Or $(V_{min})_{e2}$ = 14.48 m/s

Since the second approximation is very close to the first one,

$(V_{min})_e$ is taken as 14.48 m/s

The stalling speed at 3 km altitude is :

$$V_s = \sqrt{\frac{2W}{\rho S C_{Lmax}}} = \sqrt{\frac{2 \times 11000}{0.909 \times 11.9 \times 1.4}} = 38.2\text{m/s}$$

Since V_s is greater than $(V_{min})_e$, the minimum speed is 38.2 m/s.

Answers:

At 3 km altitude:

V_{max} = 73.51 m/s = 265.0 kmph , V_{min} = 38.20 m/s = 137.4 kmph

5.11 Special features of steady level flight at supersonic speeds

At transonic and supersonic speeds the variations of C_{D0}, K and T_a with Mach number do not permit simple mathematical treatment of the performance analysis. The thrust required (T_r) increases rapidly as the Mach number approaches unity (Fig.5.11).

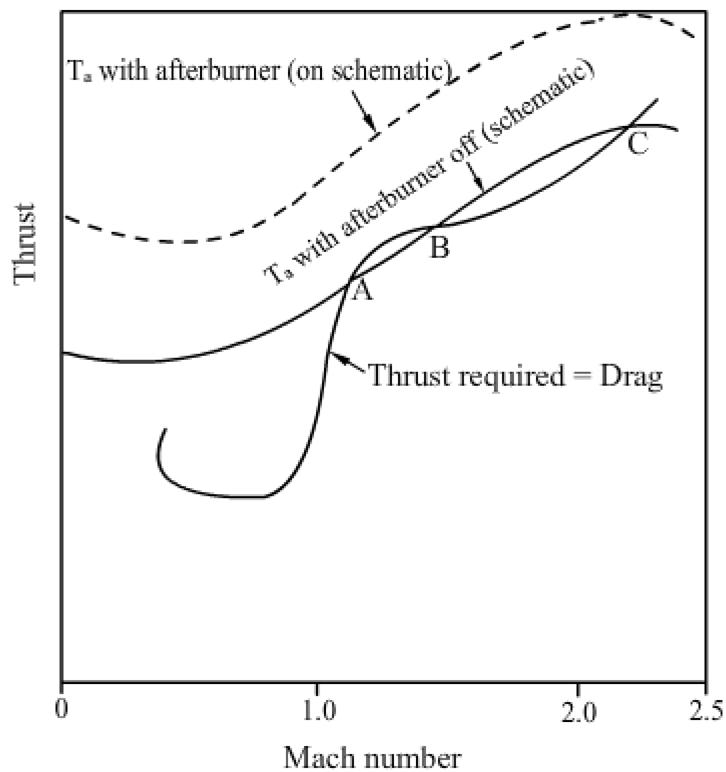

Fig.5.11 Level flight performance at high speeds

The thrust available also increases but the increase is not as fast as that of T_r and the thrust available and thrust required curves may intersect at many points (points A, B, and C in Fig.5.11). It is interesting to note that if the airplane can go past Mach number represented by point B in Fig.5.11, then it can fly up to Mach number represented by point C with the same engine. To overcome the rapid drag rise in transonic region (Fig.5.11), the afterburning operation of the engine is resorted to. It may be mentioned that an afterburner duct is located between the turbine and the nozzle (Fig.4.8b). When the afterburner is on, additional fuel is burnt in the afterburner duct. This gives additional thrust. However, the specific fuel consumption is very high with afterburner on and hence this operation is resorted to only for a short duration.

The thrust with afterburner on is shown schematically by a dotted line in Fig.5.11. It is observed that the thrust available is more than the thrust required and airplane can accelerate beyond point B. When the Mach number is close to that represented by point C, the afterburner can be shut down and the airplane runs with normal engine operation.