

## Chapter 3

### Lecture 11

#### Drag polar – 6

#### Topics

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#### 3.3.4 Parabolic drag polar at high speeds

The foregoing sections indicate that the drag coefficients of major airplane components change as the Mach number changes from subsonic to supersonic. Consequently, the drag polar of an airplane, being the sum of the drag coefficients of major components, will also undergo changes as Mach number changes from subsonic to supersonic. However, it is observed that the approximation of parabolic polar is still valid at transonic and supersonic speeds, with  $C_{D0}$  and K becoming functions of Mach number i.e.:

$$C_D = C_{D0}(M) + K(M)C_L^2 \quad (3.49)$$

Detailed estimation of the drag polar of a subsonic jet airplane is presented in section 2 of Appendix B.

### 3.3.5 Guidelines for variations of $C_{D0}$ and $K$ for subsonic jet transport airplanes

Subsonic jet airplanes are generally designed in a manner that there is no significant wave drag up to the cruise Mach number ( $M_{\text{cruise}}$ ). Further, the drag polar of the airplane for Mach numbers upto  $M_{\text{cruise}}$  can be estimated, using the methods for subsonic airplanes. Section 2 of Appendix B illustrates the procedure for estimation of such a polar. However, to calculate the maximum speed in level flight ( $V_{\text{max}}$ ) or the maximum Mach number  $M_{\text{max}}$ , guidelines are needed for the increase in  $C_{D0}$  and  $K$  beyond  $M_{\text{cruise}}$ . Such guidelines are obtained in this subsection by using the data on drag polars of B727-100 airplane at Mach numbers between 0.7 to 0.88.

Reference 3.18 part VI, chapter 5, gives drag polars of B727-100 at  $M = 0.7, 0.76, 0.82, 0.84, 0.86$  and  $0.88$ . Values of  $C_D$  and  $C_L$  corresponding to various Mach numbers were recorded and are shown in Fig.3.29 by symbols. Following the parabolic approximation, these polars were fitted with Eq.(3.49) and  $C_{D0}$  and  $K$  were obtained using least square technique. The fitted polars are shown as curves in Fig.3.29. The values of  $C_{D0}$  and  $K$  are given in Table 3.5 and presented in Figs.3.30 a & b.

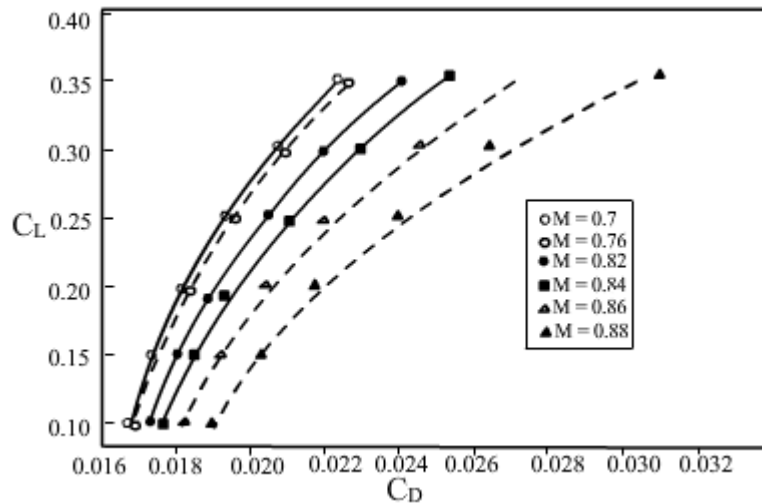


Fig.3.29 Drag polars at different Mach numbers for B727-100

M	$C_{D0}$	K
0.7	0.01631	0.04969
0.76	0.01634	0.05257
0.82	0.01668	0.06101
0.84	0.01695	0.06807
0.86	0.01733	0.08183
0.88	0.01792	0.103

Table 3.5 Variations of  $C_{D0}$  and K with Mach number

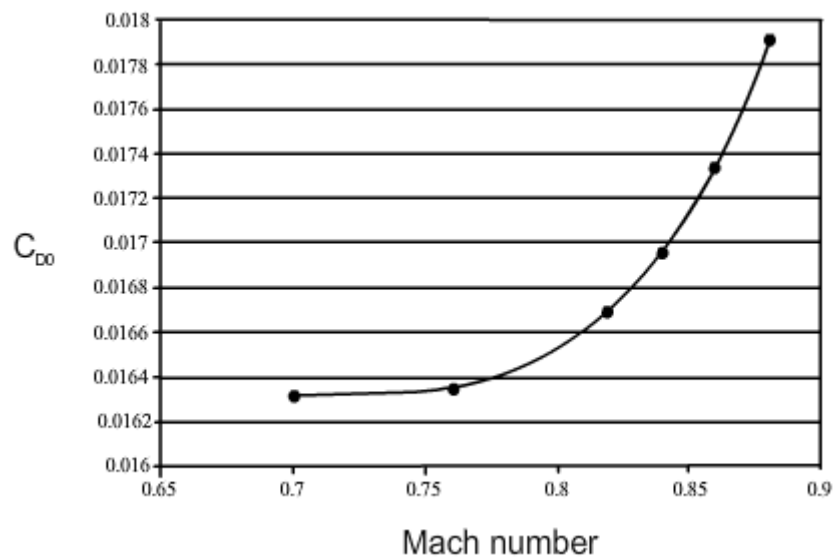


Fig.3.30a Parameters of drag polar -  $C_{D0}$  for B727-100

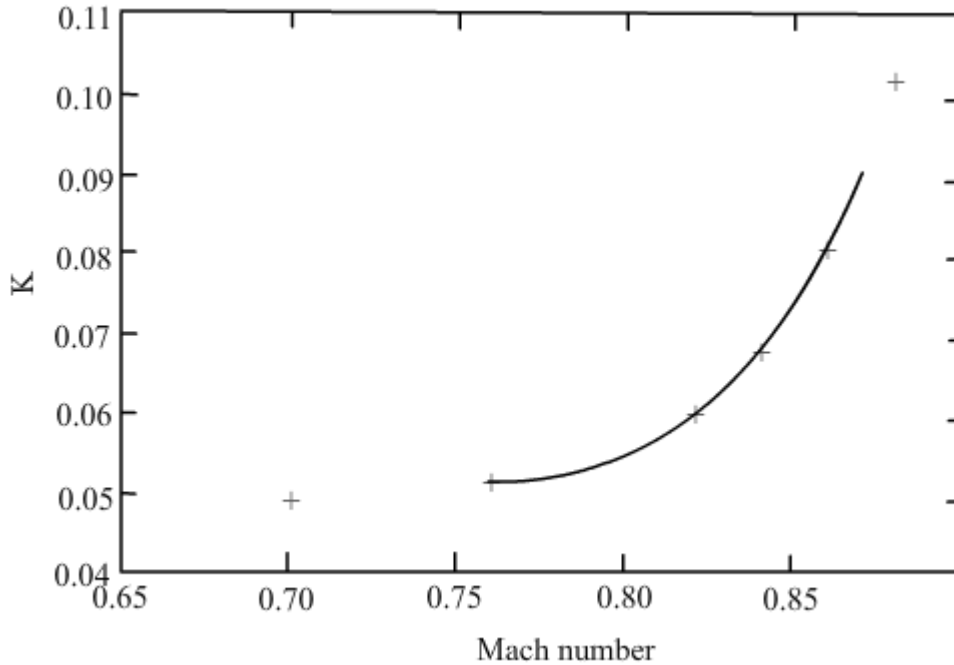


Fig.3.30b Parameters of drag polar - K for B727-100

It is seen that the drag polar and hence  $C_{D0}$  and K are almost constant up to  $M = 0.76$ . The variations of  $C_{D0}$  and K between  $M = 0.76$  and 0.86, when fitted with polynomial curves, give the following equations (see also Figs.3.30 a & b).

$$C_{D0} = 0.01634 - 0.001(M - 0.76) + 0.11(M - 0.76)^2 \quad (3.50)$$

$$K = 0.05257 + (M - 0.76)^2 + 20.0(M - 0.76)^3 \quad (3.51)$$

**Note:** For  $M \leq 0.76$ ,  $C_{D0} = 0.01634$ ,  $K = 0.05257$

Based on these trends, the variations of  $C_{D0}$  and K beyond  $M_{cruise}$  but upto  $M_{cruise} + 0.1$  are expressed by the following two equations.

$$C_D = C_{D0cr} - 0.001(M - M_{cruise}) + 0.11(M - M_{cruise})^2 \quad (3.50 a)$$

$$K = K_{cr} + (M - M_{cruise})^2 + 20.0(M - M_{cruise})^3 \quad (3.51 a)$$

where  $C_{D0cr}$  and  $K_{cr}$  are the values of  $C_{D0}$  and K at cruise Mach number for the airplane whose  $V_{max}$  or  $M_{max}$  is required to be calculated. It may be pointed out that the value of 0.01634 in Eq.(3.50) has been replaced by  $C_{D0cr}$  in Eq.(3.50a).

This has been done to permit the use of Eq.(3.50a) for different types of airplanes which may have their own values of  $C_{D0cr}$  (see section 4.2 of Appendix B). For the same reason the value of 0.05257 in Eq.(3.51) has been replaced by  $K_{cr}$  in Eq.(3.51a).

Section 4.2 of Appendix B illustrates the application of the guidelines given in this subsection.

### 3.3.6 Variations of $C_{D0}$ and $K$ for a fighter airplane

Reference 1.10, chapter 2 has given drag polars of F-15 fighter airplane at  $M = 0.8, 0.95, 1.2, 1.4$  and  $2.2$ . These are shown in Fig 3.31. These drag polars were also fitted with Eq.(3.49) and  $C_{D0}$  and  $K$  were calculated. The variations of  $C_{D0}$  and  $K$  are shown in Figs.3.32a & b. It is interesting to note that  $C_{D0}$  has a peak and then decreases, whereas  $K$  increases monotonically with Mach number. It may be recalled that the Mach number, at which  $C_{D0}$  has the peak value, depends mainly on the sweep of the wing.

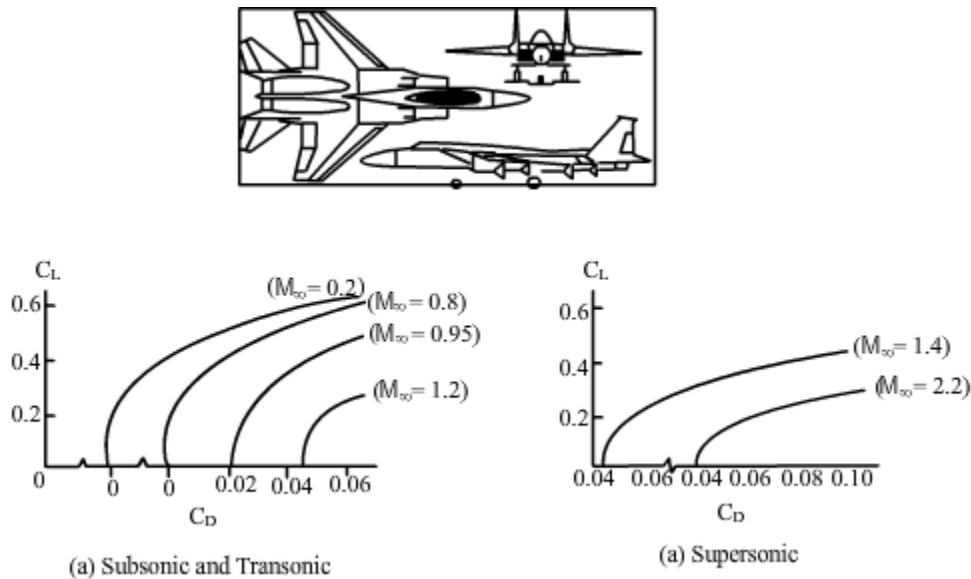


Fig.3.31 Drag polars at different Mach numbers for F15 (Reproduced from Ref.1.10, chapter 2 with permission from McGraw-Hill book company)

**Please note:** The origins for polars corresponding to different Mach numbers are shifted.

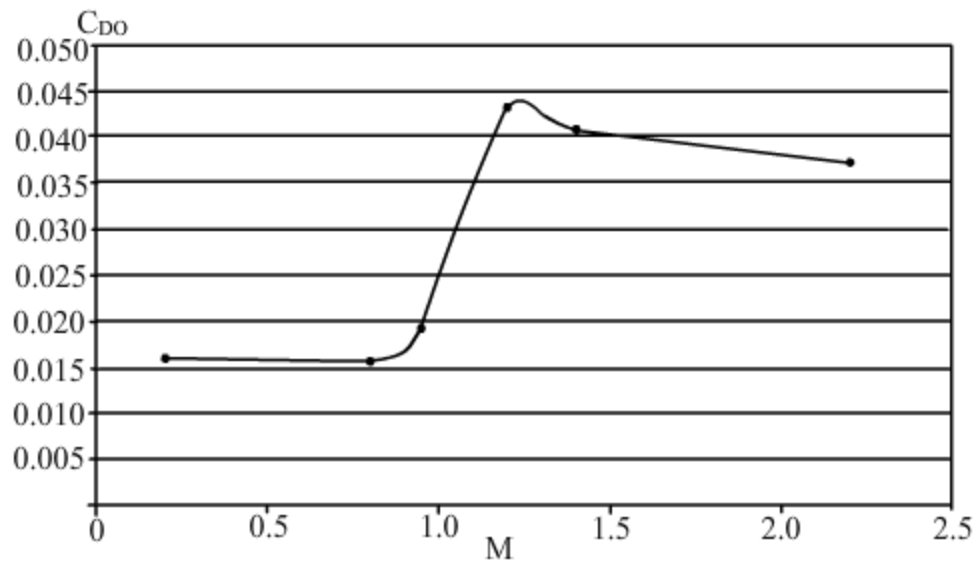


Fig.3.32a Typical variations of  $C_{D0}$  with Mach number for a fighter airplane

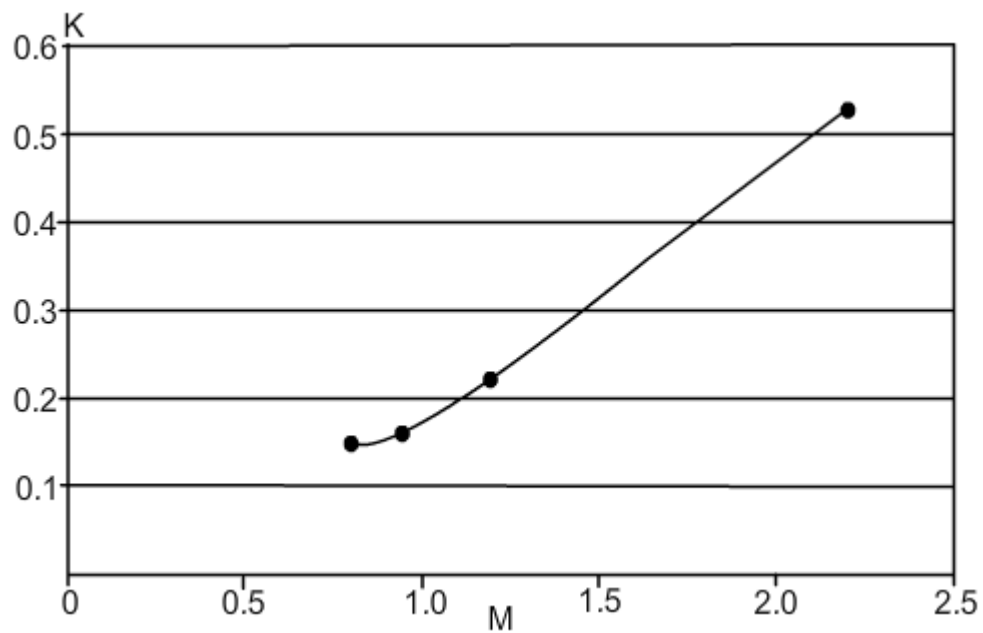


Fig.3.32b Typical variations of  $K$  with Mach number for a fighter airplane

### 3.3.7 Area ruling

The plan view of supersonic airplanes indicates that the area of cross section of fuselage is decreased in the region where wing is located. This is called area ruling. A brief note on this topic is presented below.

It was observed that the transonic wave drag of an airplane is reduced when the distribution of the area of cross section of the airplane, in planes perpendicular to the flow direction, has a smooth variation. In this context, it may be added that the area of cross section of the fuselage generally varies smoothly. However, when the wing is encountered there is an abrupt change in the cross sectional area. This abrupt change is alleviated by reduction in the area of cross section of fuselage in the region where the wing is located. Such a fuselage shape is called 'Coke-bottle shape'. Figure 3.33c illustrates such a modification of fuselage shape.

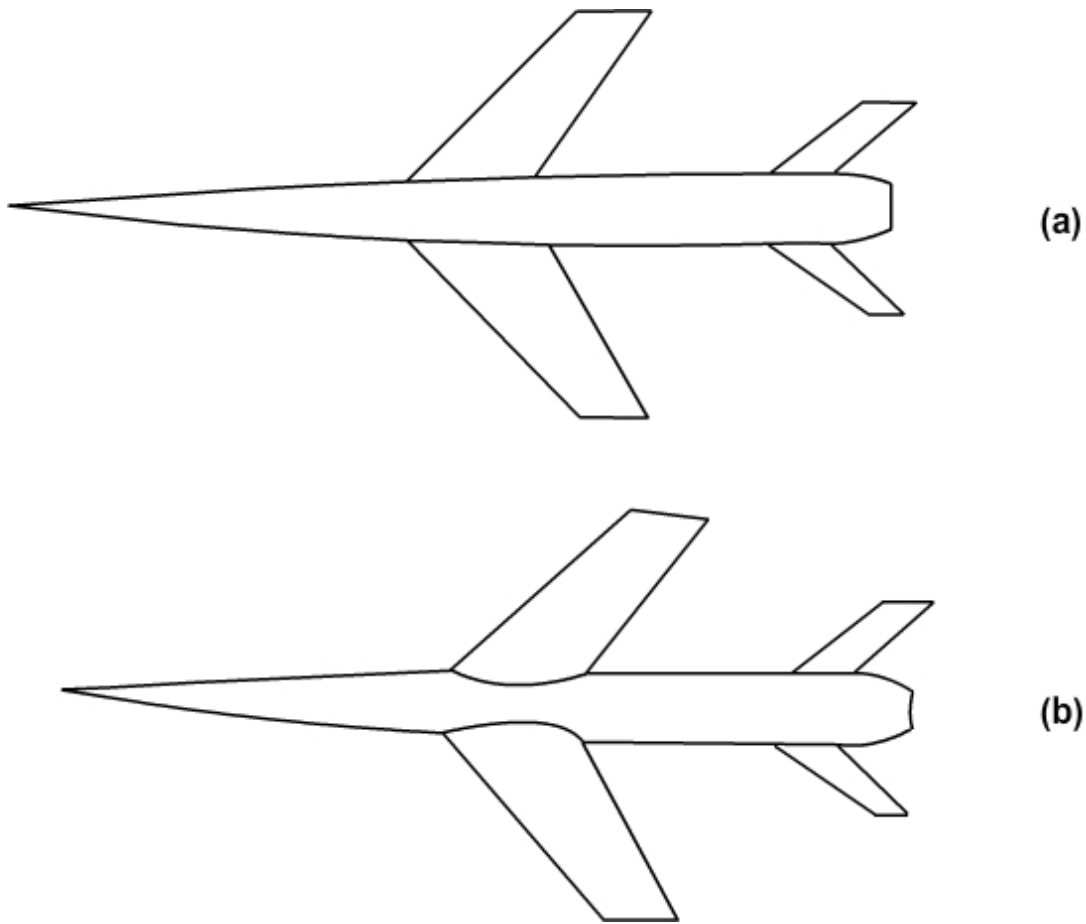


Fig.3.33 Design for low transonic wave drag

- (a) Abrupt change in cross sectional area at wing fuselage junction
- (b) Coke-bottle shape

Figure 3.34, based on data in Ref.1.9 , chapter 5, indicates the maximum wave drag coefficient, in transonic range, for three configurations viz (i) a body of revolution (ii) a wing-body combination without area ruling and (iii) a wing-body combination with area ruling (Ref. 1.9, chapter 5 may be referred to for further details). Substantial decrease in wave drag coefficient is observed as a result of area ruling. Figure 3.35. presents a practical application of this principle.



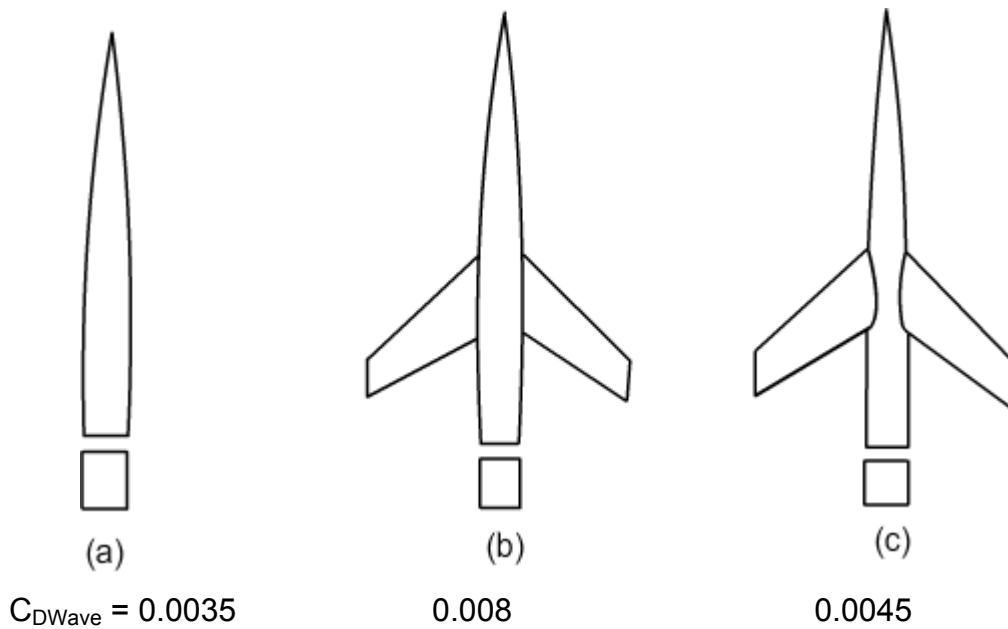


Fig.3.34 Maximum transonic wave drag coefficient of three different shapes  
(a) body of revolution (b) wing-body combination without area ruling (c) wing-body combination with area ruling

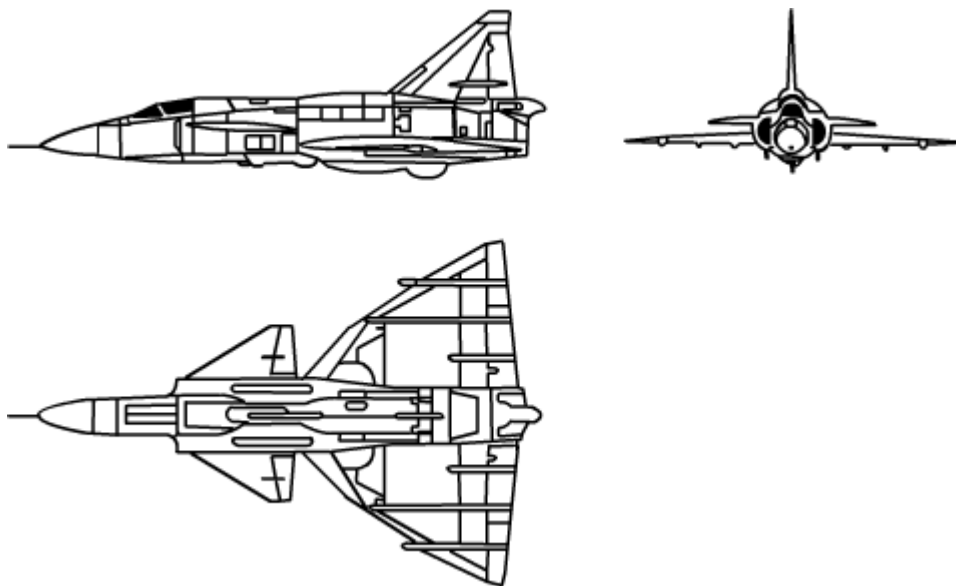


Fig.3.35 An example of area ruling - SAAB VIGGEN  
(Adapted from <http://upload.wikimedia.org>)

### 3.4 Drag polar at hypersonic speeds

When the free stream Mach number is more than five, the changes in temperature and pressure behind the shock waves are large and the treatment of the flow has to be different from that at lower Mach numbers. Hence, the flows with Mach number greater than five are termed hypersonic flows. Reference 3.19 may be referred to for details. For the purpose of flight mechanics it may be mentioned that the drag polar at hypersonic speeds is given by the following modified expression (Ref. 1.1, chapter 6).

$$C_D = C_{D0}(M) + K(M)C_L^{3/2} \quad (3.52)$$

Note that the exponent of the  $C_L$  term is 1.5 and not 2.0.

### 3.5 Lift to drag ratio

The ratio  $C_L / C_D$  is called lift to drag ratio. It is an indicator of the aerodynamic efficiency of the design of the airplane. For a parabolic drag polar  $C_L / C_D$  can be worked out as follows.

$$C_D = C_{D0} + KC_L^2$$

$$\text{Hence, } C_D / C_L = (C_{D0} / C_L) + KC_L \quad (3.53)$$

Differentiating Eq.(3.53) with  $C_L$  and equating to zero gives  $C_{Lmd}$  which corresponds to minimum of  $(C_D / C_L)$  or maximum of  $(C_L / C_D)$ .

$$C_{Lmd} = (C_{D0} / K)^{1/2} \quad (3.54)$$

$$C_{Dmd} = C_{D0} + K(C_{Lmd})^2 = 2 C_{D0} \quad (3.55)$$

$$(L/D)_{\max} = (C_{Lmd} / C_{Dmd}) = \frac{1}{2\sqrt{C_{D0} K}} \quad (3.56)$$

#### Note:

To show that  $C_{Lmd}$  corresponds to minimum of  $(C_D / C_L)$ , take the second derivative of the right hand side of Eq.(3.53) and verify that it is greater than zero.

### 3.6 Other types of drag

Subsections 3.1.1, 3.2.2, 3.2.14, 3.2.17 and 3.3.2. dealt with the skin friction drag, pressure drag (or form drag), profile drag, interference drag, parasite drag, induced drag, lift dependent drag and wave drag. Following additional types of drags are mentioned briefly to conclude the discussion on this topic.

### 3.6.1 Cooling drag

The piston engines used in airplanes are air cooled engines. In such engines, a part of free stream air passes over the cooling fins and accessories. This causes some loss of momentum and results in a drag called cooling drag.

### 3.6.2 Base drag

If the rear end of a body terminates abruptly, the area at the rear is called a base. An abrupt ending causes flow to separate and a low pressure region exists over the base. This causes a pressure drag called base drag.

### 3.6.3 External stores drag

Presence of external fuel tank, bombs, missiles etc. causes additional parasite drag which is called external stores drag. Antennas, lights etc. also cause parasite drag which is called protuberance drag.

### 3.6.4 Leakage drag

Air leaking into and out of gaps and holes in the airplane surface causes increase in parasite drag called leakage drag.

### 3.6.5 Trim drag

In example 1.1 it was shown that to balance the pitching moment about c.g. ( $M_{cg}$ ), the horizontal tail which is located behind the wing produces a lift ( $-L_T$ ) in the downward direction. To compensate for this, the wing needs to produce a lift ( $L_w$ ) equal to the weight of the airplane plus the downward load on the tail i.e.  $L_w = W + L_T$ . Hence, the induced drag of the wing, which depends on  $L_w$ , would be more than that when the lift equals weight. This additional drag is called trim drag as the action of making  $M_{cg}$  equal to zero is referred to as trimming the airplane. It may be added that a canard surface is located ahead of the wing and the lift on it, to make  $M_{cg}$  equal to zero, is in upward direction. Consequently, the lift

produced by the wing is less than the weight of the airplane. SAAB Viggen shown in Fig.3.35, is an example of an airplane with canard. Reference 1.15 and internet ([www.google.com](http://www.google.com)) may be consulted for details of this airplane.