Chapter 9

Lecture 29

Performance analysis V – Manoeuvres – 2

Topics

9.3.3 Factors limiting radius of turn and rate of turn

9.3.4 Determination of minimum radius of turn and maximum rate of turn at a chosen altitude

9.3.3 Factors limiting radius of turn and rate of turn

Turning flight is a very important item of performance evaluation, especially for the military airplanes. Minimum radius of turn and maximum rate of turn are important indicators of the manoeuverability of an airplane. From Eqs.(9.11) and (9.12) it is observed that, at a given altitude and flight velocity, a small radius of turn and a high rate of turn are achieved when the bank angle (ϕ) has the highest possible value. Equations (9.11a) and (9.12a) indicate that at a given altitude, the minimum radius of turn (r_{min}) and the maximum rate of turn ($\dot{\psi}_{max}$) are obtained when 'V' is low and 'n' is high. The following considerations limit the achievable values of r_{min} and $\dot{\psi}_{max}$.

(I)Limitation due to C_{Lmax} : From the above discussion we observe that the lift coefficient in a turning flight is higher than the lift coefficient required at the same speed in level flight. Let C_{LT} be the lift coefficient in the turning flight and C_{LL} be the lift coefficient in the level flight at the same speed.

Then, $C_{LT} = n\,W / (\tfrac{1}{2}\rho V^2 S) = n\,C_{LL}$

However, C_{LT} cannot be more than C_{Lmax} and this imposes limitations on the attainable values of load factor (n) and the bank angle (ϕ). Let these two values be denoted by $(n_{max})_{CLmax}$ and $(\phi_{max})_{CLmax}$. They can be expressed as :

$(n_{max})_{CLmax} = C_{Lmax} / C_{LL}$ (9.13)

Noting that $\phi = \cos^{-1}(1/n)$, $(\phi_{max})_{CLmax} = \cos^{-1}(C_{LL} / C_{Lmax})$ (9.14)

It may be noted that, at stalling speed (V_s), the value of C_{LL} equals C_{Lmax} or n = 1. Hence, turn is not possible at stalling speed .

(II) Limitation due to allowable load factor from structural consideration : The bank angle ϕ and the load factor in a turn are related by:

$\cos \phi = 1/n$.

However, n cannot exceed the value permitted by the structural design of the airplane. Let this value be denoted by $(n_{max})_{str}$. Hence, ϕ_{max} is limited to $\cos^{-1}\{1/(n_{max})_{str}\}$.

(III) The drag coefficient in a turning flight is higher than that in a level flight at the same speed. However, in a steady turn the thrust required cannot exceed the thrust available (T_a). This also imposes limitations on the attainable values of ϕ and n. Let these two values be denoted as $(\phi_{max})_{Ta}$ and $(n_{max})_{Ta}$.

It may be noted that, at $V = V_{max}$ and $(V_{min})_e$ the entire engine output is used in overcoming the drag in level flight. Hence, the steady level turn is not possible at these two speeds.

The lowest of the above three values viz $(n_{max})_{CLmax}$, $(n_{max})_{str}$ and $(n_{max})_{Ta}$ is the permissible value of n_{max} . Let this value be denoted by $(n_{max})_{perm}$. Substituting this value in Eqs.(9.11a) and (9.12a) gives r and $\dot{\psi}$.

9.3.4 Determination of minimum radius of turn and maximum rate of turn at a chosen altitude

In a general case, the drag polar and the thrust available are functions of Mach number. In such a case, the minimum radius of turn (r_{min}) and the maximum rate of turn ($\dot{\psi}_{max}$) at an altitude, can be obtained by using the following steps. The limitations stated in the previous subsection, are taken into account during the procedure.

(i) Choose an altitude. Obtain V_{max} and V_{min} at this altitude. Note that a steady level, co-ordinated-turn is possible only within this speed range.

(ii) Choose a flight speed (V) in between V_{max} and V_{min} and obtain C_{LL} as:

$$C_{LL} = 2W / (\rho SV^2)$$

Obtain Mach number (M) corresponding to the chosen V and the speed of sound at chosen altitude.

(iii) Obtain the C_{Lmax} at the chosen flight Mach number. It may be recalled from subsection 3.7.4, that for airplanes flying at high speeds, the C_{Lmax} depends on Mach number. Obtain the ratio C_{Lmax} / C_{LL}.

The ratio C_{Lmax} / C_{LL} gives the quantity $(n_{max})_{CLmax}$ defined above. If this value is smaller than the allowable load factor from structural consideration viz. $(n_{max})_{str}$, then the turn may be limited by C_{Lmax}. In this situation, choose $C_{LT1} = C_{Lmax}$. It may be mentioned that the procedure presented here, aims at obtaining the value of lift coefficient in the turn (C_{LT}) which satisfies all the three limitations on the turn mentioned above. The quantity C_{LT1} is the value of C_{LT} as limited by C_{Lmax}. This will be modified in the subsequent steps.

If C_{Lmax} / C_{LL} is more than $(n_{max})_{str}$, then the turn may be limited by $(n_{max})_{str}$. In this situation, choose C_{LT1} as $(n_{max})_{str} \times C_{LL}$.

(iv) Obtain from the drag polar, the drag coefficient C_{DT1}, corresponding to C_{LT1} and the chosen Mach number. Calculate the drag D_{T1} from:

$$D_{T1} = 1/2 \ \rho V^2 S C_{DT1}$$

If D_{T1} is greater than the available thrust (T_a), then the turn is limited by engine output. In this situation, obtain the maximum permissible value of drag coefficient in turning flight (C_{DT}) as limited by T_a . It is given as : $C_{DT} = T_a / (1/2\ \rho V^2 S)$

Corresponding to this value of C_{DT}, obtain the lift coefficient C_{LT} by referring to the drag polar.

If D_{T1} is smaller than T_a , then the turn is not limited by the engine output. In this situation, the turn is limited by C_{Lmax} or $(n_{max})_{str}$. Consequently, C_{LT} is the smaller of the two values obtained in step (iii).

(v) Once C_{LT} is known, ϕ is given by:

$\phi = \cos^{-1}(C_{LL}/C_{LT})$. Knowing ϕ and V, the radius of turn (r) and rate of turn ($\dot{\psi}$), at the chosen speed, can be calculated using Eqs.(9.11) and (9.12).

(vi) The previous steps should be repeated at various values of flight speeds ranging between V_{min} and V_{max}. Plotting these results, the values of r_{min} and $\dot{\psi}_{max}$ and the corresponding speeds V_{rmin} and $V_{\dot{\psi}max}$ can be determined at the chosen altitude.

(vii) Repeat steps (i) to (vi) at different altitudes.

The procedure is illustrated, at a chosen altitude, in example 9.3.

Example 9.3

A passenger airplane has a gross weight of 176,400 N and a wing area of 45 m^2. Obtain the variations of r and $\dot{\psi}$ with velocity at an altitude of 8 km from the following data.

$C_{Lmax} = 1.4$, $(n_{max})_{str} = 3.5$, $C_D = 0.017 + 0.05 C_L^2$

$V_{min} = 103$ m/s, $V_{max} = 274$ m/s and the thrust output (T_a) varies as given in the table below.

V (m/s)	105	115	125	145	165	185	205
T_a (N)	21100	21125	21150	21480	21580	21980	22270

Solution:

At 8 km altitude the value of ρ is 0.525 kg/m^3. The minimum radius of turn and $\dot{\psi}_{max}$ at various speeds are worked out in a tabular manner using the procedure outlined above.

V (m/s)	105	115	125	145	165	185	205
C_{LL}	1.354	1.129	0.955	0.710	0.548	0.436	0.355
C_{Lmax} / C_{LL}	1.034	1.240	1.466	1.972	2.553	3.21	3.94
C_{LT1}	1.4	1.4	1.4	1.4	1.4	1.4	1.243*
C_{DT1}	0.115	0.115	0.115	0.115	0.115	0.115	.0942
D_{T1}(N)	15000	17993	21258	28601	37042	46568	46852
T_a (N)	21100	21125	21150	21480	21580	21980	22270
C_{DT}	**	**	0.1114	0.0864	0.067	0.0543	0.0448
C_{LT}	1.4$	1.4$	1.396£	1.178£	1.08£	0.863£	0.745£
$\frac{C_{LT}}{C_{LL}}=n$	1.034	1.240	1.461	1.659	1.824	1.98	2.10
ϕ (degrees)	14.75	36.25	46.9	52.93	56.76	59.63	61.6
r (m)	4273	1838	1491	1619	1819	2043	2321
$\dot{\psi}$ (rad/s)	0.0246	0.0626	0.0838	0.0896	0.0907	0.0906	0.0883

The symbols in the above table have the following meanings:

* Turn is limited by load factor $(n_{max})_{str}$ hence $C_{LT1} = (n_{max})_{str}\, C_{LL}$.

** Thrust available is more than thrust required. Hence, $C_{LT} = C_{LT1}$

$ Turn is limited by C_{Lmax}

£ Turn is limited by T_a

Table E9.3 Variations of radius of turn (r) and rate of turn $(\dot{\psi})$ with flight velocity (V) for airplane in example 9.3

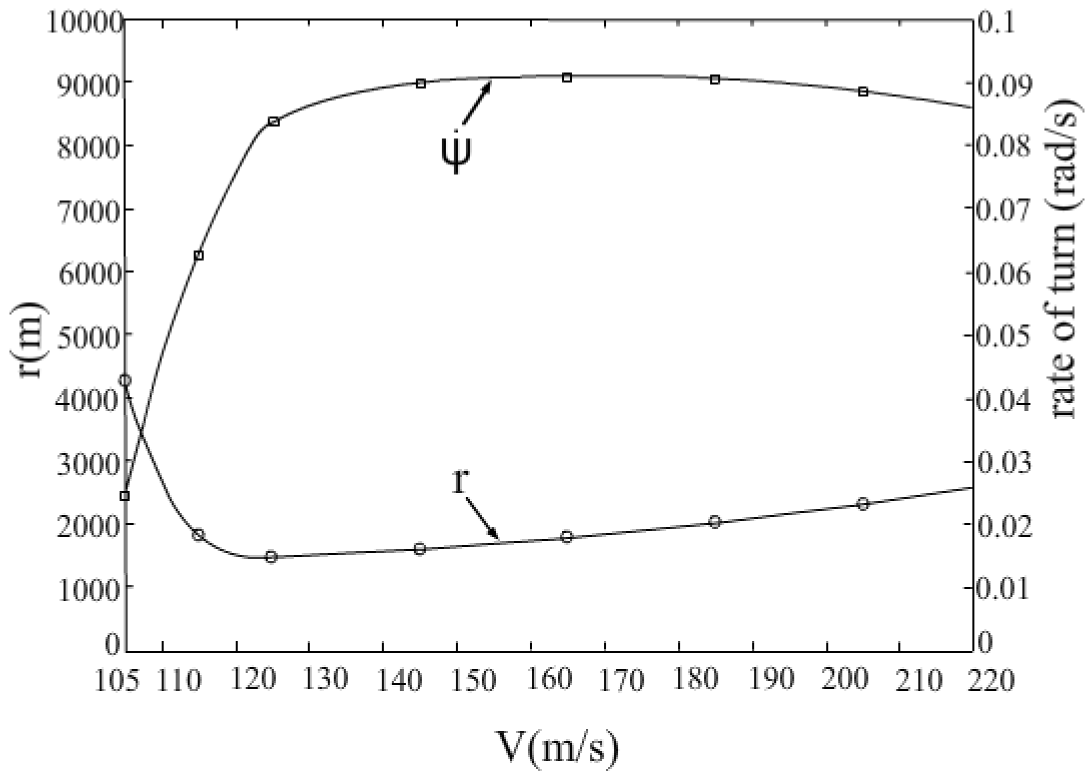

Fig.E9.3 Variations of radius of turn (r) and rate of turn $(\dot{\psi})$ with flight velocity (V) for the airplane in example 9.3

The plots of r vs V and $\dot{\psi}$ vs V are shown in Fig.E9.3. From these plots

r_{min} = 1.490 m and $\dot{\psi}_{max}$ = 0.0907 rad/s, V_{rmin} = 124 m/s and $V_{\dot{\psi}max}$ = 165 m/s

Answers:

Minimum radius of turn (r_{min}) = 1490 m at V_{rmin} = 124 m/s

Maximum rate of turn ($\dot{\psi}_{max}$) = 0.090 rad/s at $V_{\dot{\psi}max}$ = 165 m/s

Remarks:

i) Turning performance of a jet airplane :

Section 7 of Appendix B presents the turning performance of a jet airplane. Figures 9.5a and b show the variations of $\dot{\psi}$ and r with velocity at different altitudes for that airplane. Figures 9.5c and d present the variations of $V_{\dot{\psi}max}$ and V_{rmin} with altitude.

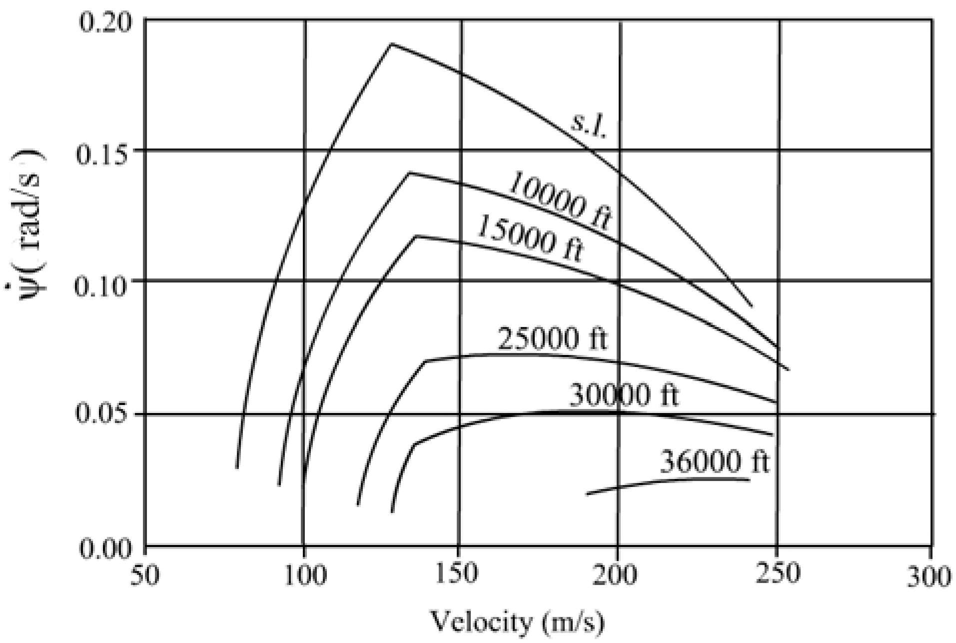

Fig.9.5a Turning performance of a jet transport – rate of turn ($\dot{\psi}$)

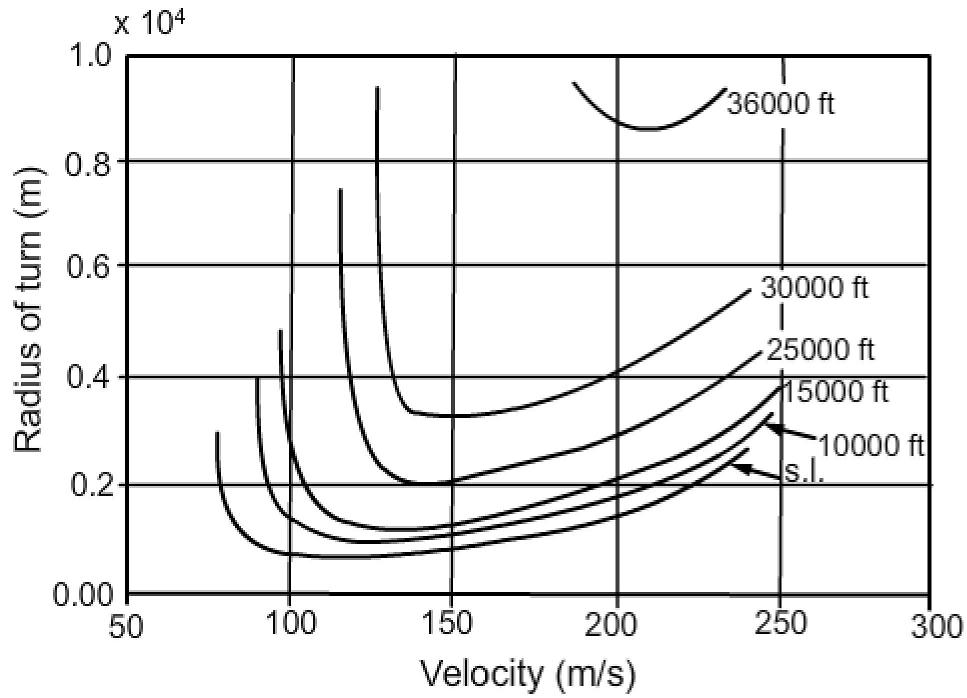

Fig.9.5b Turning performance of jet transport – radius of turn (r)

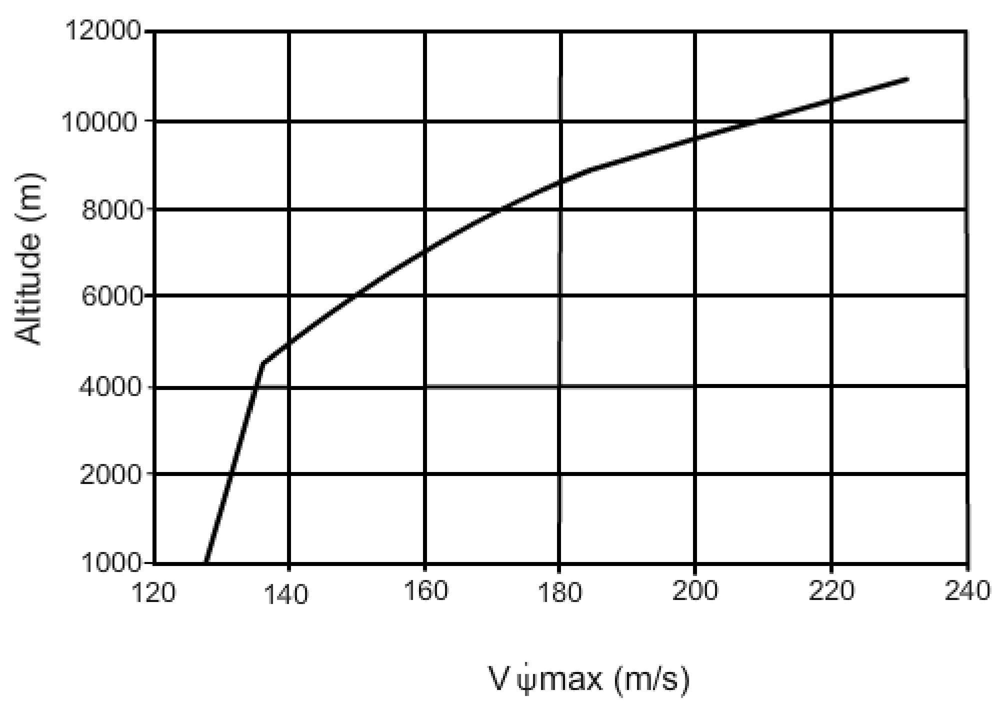

Fig.9.5c Turning performance of jet transport - variation of $V_{\dot{\psi}\,max}$

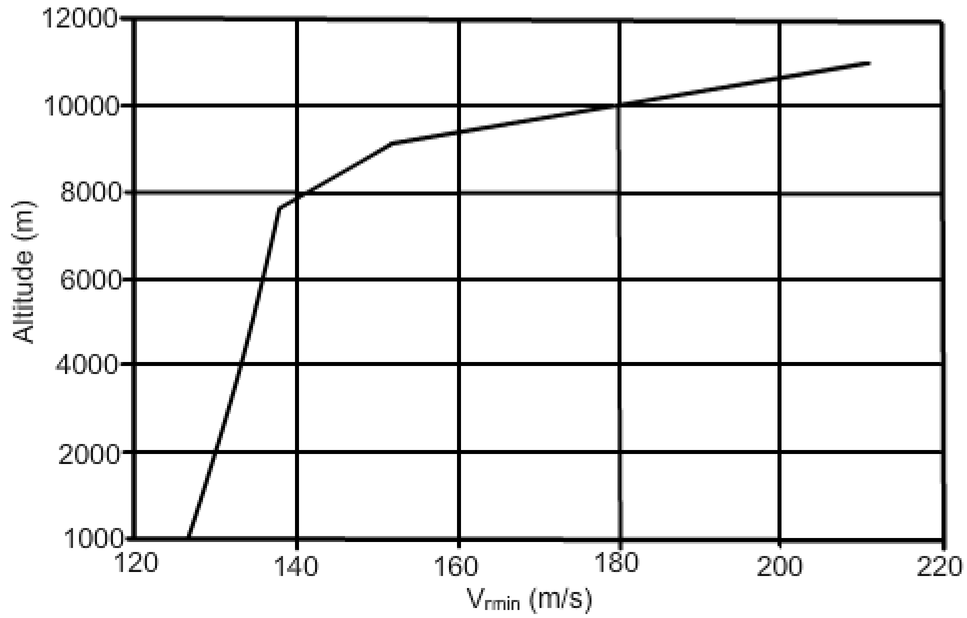

Fig.9.5d Turning performance of jet transport - variation of V_{rmin}

Note:

Some curves in Figs.9.5a,c and d show discontinuity in slope at certain points. This occurs when the criterion limiting the turning performance changes from $(n_{max})_{str}$ to $(n_{max})_{Ta}$.

ii) Turning performance of a piston engined airplane :

Section 7 of Appendix A presents the turning performance of a piston engined airplane. Figures 9.6a and b show the variations of r and $\dot{\psi}_{max}$ with velocity at different altitudes for that airplanes. Figures 9.7c and d present the variations of r_{min} and $\dot{\psi}_{max}$ with altitude. Figure 9.6e presents the variations of $V_{\dot{\psi}max}$ and V_{rmin} with altitude. Both these speeds increase with altitude. The two speeds come close to each other as absolute ceiling is approached. Minimum radius of turn (r_{min}) increases with altitude and $\dot{\psi}_{max}$ decreases with altitude. At absolute ceiling, the rate of turn becomes zero and the radius becomes infinite.

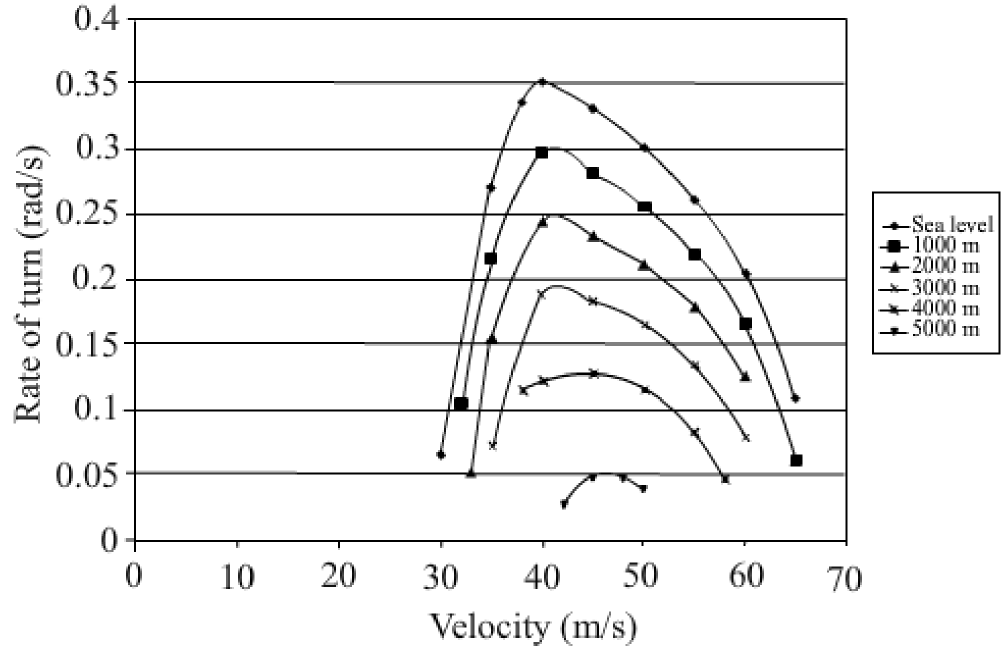

Fig.9.6a Turning performance of a piston engined airplane

- variation of rate of turn (ψ)

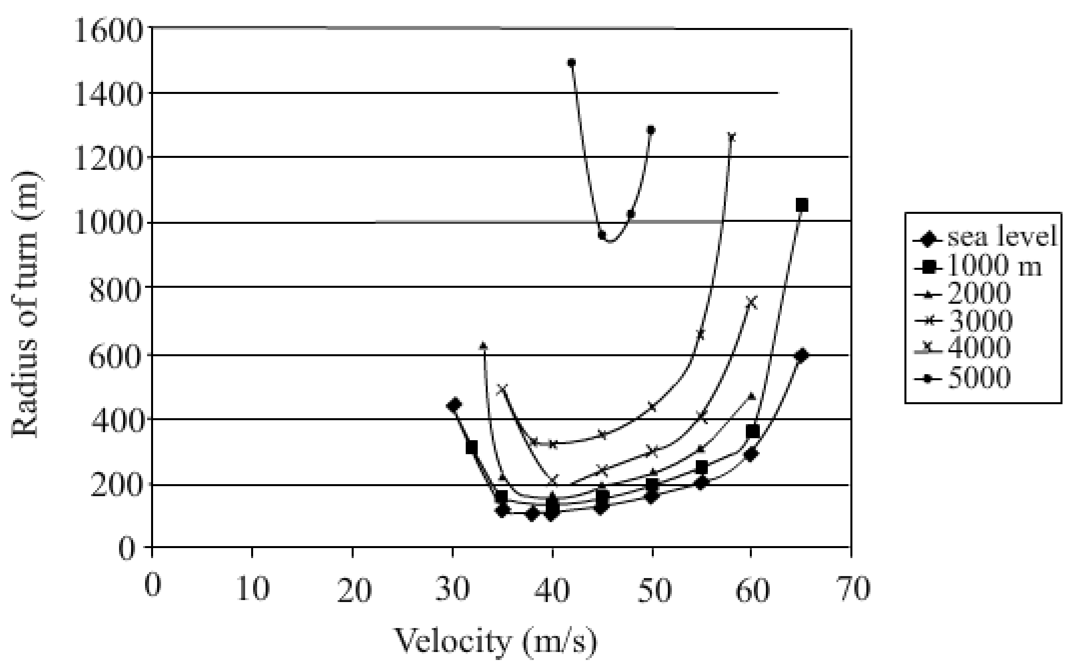

Fig.9.6b Turning performance of a piston engined airplane –

variation of radius of turn (r)

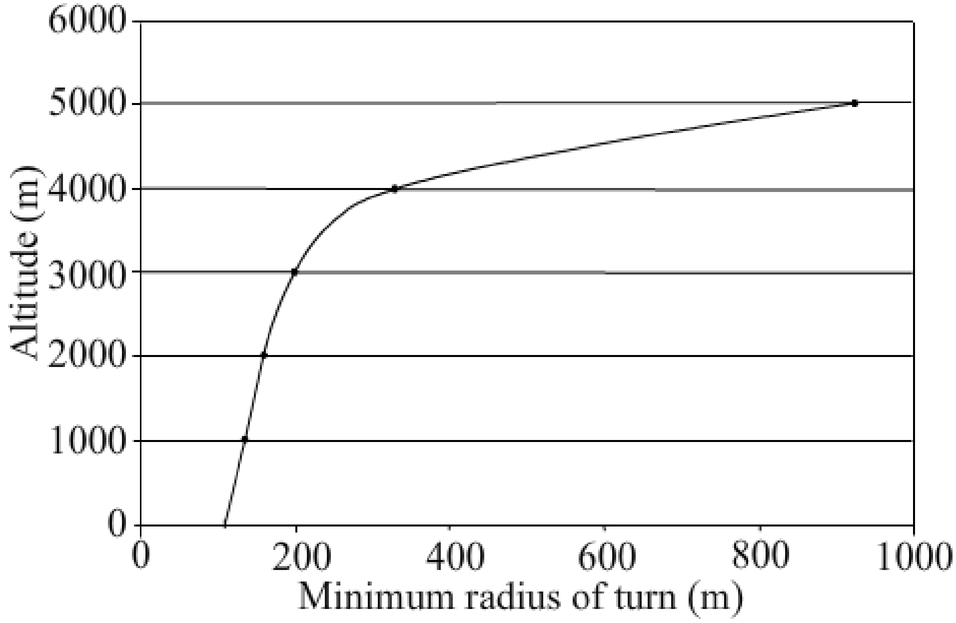

Fig.9.6c Turning performance of a piston engined airplane – variation of r_{min} with altitude

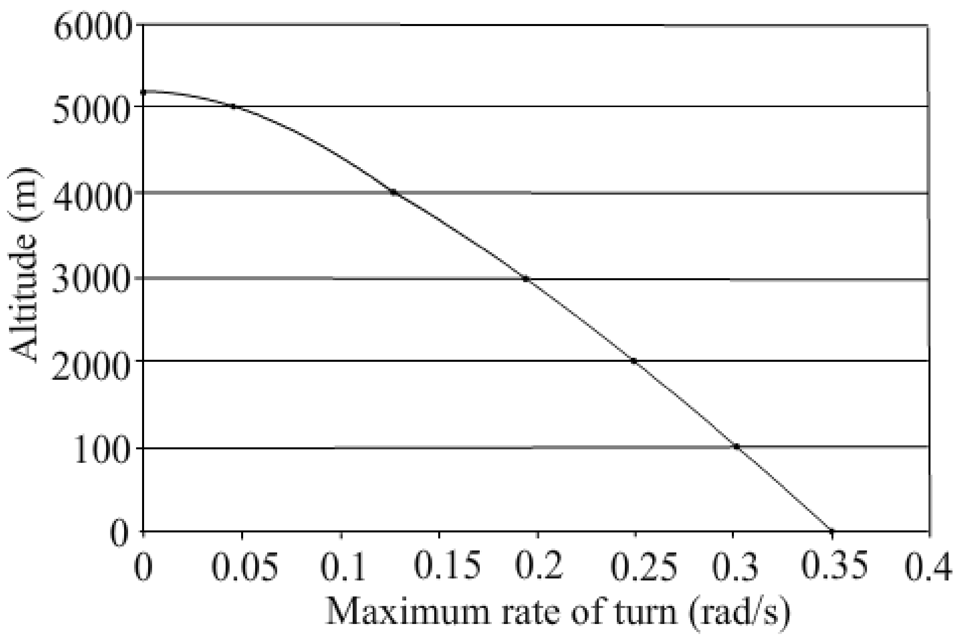

Fig.9.6d Turning performance of piston engined airplane – variation of $\dot{\psi}_{max}$ with altitude

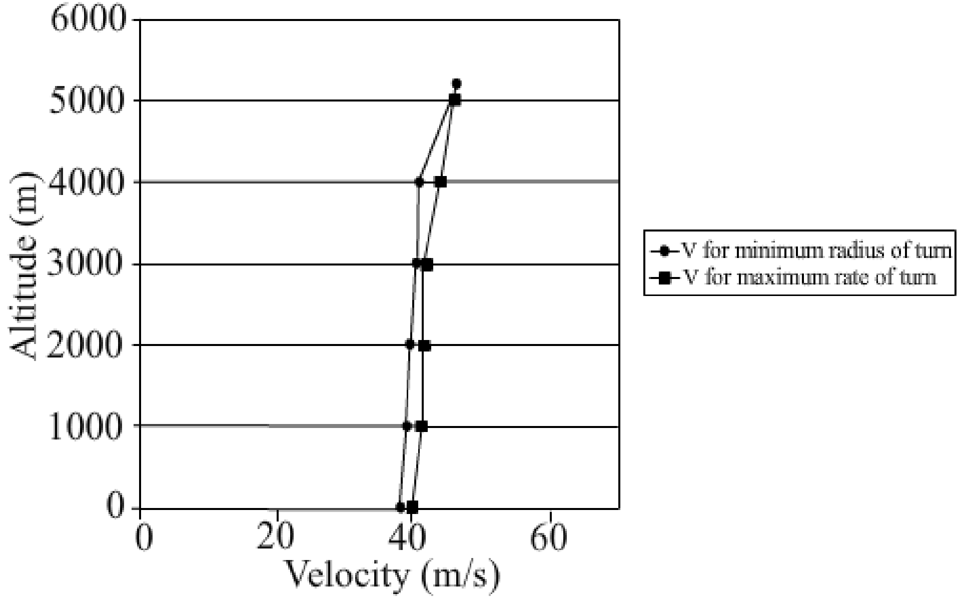

Fig.9.6e Turning performance of piston engined airplane – variations of V_{rmin} and $V_{\psi max}$ with altitude

iii) In many situations the minimum radius of turn in level flight is limited by the available engine output. This can be overcome and a smaller radius of turn can be obtained by allowing the airplane to descend during the turn. In this manner a loss of potential energy is used to increase the available energy during turn. Reference 1.12, chapter 2 may be consulted for additional details. See also subsection 9.3.6 for further information.