# Chapter 4

# Estimation of wing loading and thrust loading- 2

## Lecture 10

## Topics



## 4.4 Selection of wing loading based on prescribed flight speed ($V_p$)

Reference 1.6 gives a procedure for optimizing the wing loading for $V_{max}$ at a prescribed altitude. However, guidelines for $V_{max}$, in terms of cruise speed ($V_{cr}$), are not given. Similarly, prescriptions for $V_{max}$ are also not found in other sources. However, Ref.1.12 in part I, chapter 2, while prescribing the mission specifications for three typical cases viz. (i) a twin-engine propeller airplane, (ii) a jet transport and (iii) a fighter, does mention about cruising speed ($V_{cr}$) or cruising Mach number ($M_{cr}$) at specified altitude (s). Later, in chapter three of the same reference, wing loading and thrust or power loading are also checked for cruising speed specification. In view of this, the optimization of wing loading for prescribed velocity ($V_p$) at prescribed altitude ($H_p$) is considered in this section. It may be added that for a passenger airplane, the power required for take-off and climb are generally are higher than that required during cruise. With an engine rating satisfying these requirements, a maximum speed ($V_{max}$) of 1.05 to 1.2 times the cruising speed is attainable at $H_{cr}$. For military airplanes, $V_{max}$ at sea level and at altitude may be prescribed along with the airplane configuration and these may be the critical requirements. In case of some military airplanes, the mission specification, in addition to altitude and Mach number, may also specify ability to accelerate through a certain Mach number range in a short duration of time.

The optimization from the consideration of $V_p$, aims at finding out the wing loading which will result in the lowest thrust requirement ($T_{vp}$) for a chosen $V_p$ at $H_p$. The steps are as follows.

In level flight, T = D

Let, $\bar{t}_v = T_{vp} / W \; ; \; T_{Vp} = \frac{1}{2}\rho V_p^2 S C_D$ (4.12)

$\rho$ = atmospheric density at altitude $H_p$

Hence,

$$\bar{t}_v = \frac{\frac{1}{2}\rho V_p^2 S C_D}{W}$$

Denoting $\frac{1}{2}\rho V_p^2$ as $q_p$ and (W/S) as p yields:

$$\bar{t}_v = \frac{C_D\, q_p}{p} \qquad (4.13)$$

The drag coefficient ($C_D$) is generally written as:

$$C_D = C_{D0}(M) + K(M)C_L^2 \qquad (4.14)$$

where, $C_{D0}$ = Parasite drag coefficient. $C_{D0}$ is nearly constant for M < $M_{crit}$ and then it becomes a function of M.

$KC_L^2 = C_{Di}$ = Lift-dependent drag coefficient. K is nearly constant for M < $M_{crit}$, and then it becomes a function of M.

### 4.4.1 Alternate break-up of drag polar

In the remark at the end of section 4.2, it is mentioned that in the approach of Ref.1.6 the changes in drag polar, as a result of the changes in wing loading are taken care of by an alternate representation of the drag polar. This representation is explained in this section.

The drag polar is commonly written as:

$$C_D = C_{D0} + K C_L^2 \tag{4.15}$$

where, $C_{DO}$ = parasite drag coefficient $= \frac{D_o}{\frac{1}{2}\rho V^2 S}$ , $D_o$ = parasite drag

$K = \frac{1}{\pi A e}$, e = Oswald efficiency factor; A = Aspect ratio of wing

The expressions for $C_{D0}$ and K for subsonic airplanes with jet engine, turbo-prop engine and piston engine are given by Eqs.(3.57),(3.58) and (3.59). These expressions include influence of parameters like wing aspect ratio and wing sweep. These expressions are expected to give reasonable drag polars for the present stage of the preliminary design process. The steps to obtain the alternate break up of drag polar are as follows.

(1) Noting that the parasite drag ($D_0$) is the sum of parasite drags of the major components plus the effect of interference, it can be expressed as:

$$D_0 = (D_0)_{wing} + (D_0)_{fuselage} + (D_0)_{nacelle} + (D_0)_{ht} + (D_0)_{vt} + (D_0)_{etc} + (D_0)_{int} \tag{4.16}$$

where, $(D_0)_{etc}$ is the parasite drag of components like landing gear, external fuel tanks and armament mounted on the airplane. $(D_0)_{int}$ is the incremental parasite drag due to the interference between flows past wing and fuselage, tail and fuselage etc.

$C_{D0} = D_0 / (0.5\, \rho\, V^2\, S)$

Hence,

$$C_{D0} = \frac{\frac{1}{2}\rho V^2 S}{\frac{1}{2}\rho V^2 S}(C_{D0})_w + \frac{\frac{1}{2}\rho V^2 S_{fuse}}{\rho V^2 S}(C_{D0})_{fuse} + \frac{\frac{1}{2}\rho V^2 S_{nac}}{\frac{1}{2}\rho V^2 S}(C_{Do})_{nac}$$

$$+ \frac{\frac{1}{2}\rho V^2 S_{ht}}{\frac{1}{2}\rho V^2 S}(C_{Do})_{ht} + \frac{\frac{1}{2}\rho V^2 S_{vt}}{\frac{1}{2}\rho V^2 S}(C_{Do})_{vt} + \frac{\frac{1}{2}\rho V^2 S_{etc}}{\frac{1}{2}\rho V^2 S}(C_{Do})_{etc} + (C_{Do})_{int}$$

Note that $(C_{Do})_{etc}$ and $S_{etc}$ may include more than one items.

$$\text{Or } C_{D0} = (C_{D0})_W + (C_{D0})_{fuse}\frac{S_{fuse}}{S} + (C_{D0})_{nac}\frac{S_{nac}}{S} + (C_{D0})_{ht}\frac{S_{ht}}{S}$$

$$+ (C_{D0})_{vt}\frac{S_{vt}}{S} + (C_{D0})_{etc}\frac{S_{etc}}{S} + C_{Dint} \qquad (4.17)$$

(2) Some simplification is introduced based on the following considerations. The parasite drag coefficients for wing, horizontal tail and vertical tail are nearly same. Further the areas of the horizontal tail and vertical tail are proportional to the wing area. Hence, in Eq.(4.17) the contributions of wing, horizontal tail and vertical tail to $C_{D0}$, can be clubbed together. i.e.

$$(C_{Do})_W + (C_{Do})_{ht}\frac{S_{ht}}{S} + (C_{Do})_{vt}\frac{S_{vt}}{S} \approx (C_{Do})_W\left(1+\frac{S_{ht}}{S}+\frac{S_{vt}}{S}\right) = K_t\,(C_{Do})_W \qquad (4.18)$$

$$\text{where, } K_t = (1+\frac{S_{ht}}{S}+\frac{S_{vt}}{S}) \qquad (4.19)$$

(3) The quantities $(C_{Do})_{fuse}$, $(C_{Do})_{nac}$, $(C_{Do})_{etc}$, $C_{Doint}$ and $S_{fuse}$, $S_{nac}$, $S_{etc}$ do not change, when the wing area changes as a result of the change of wing loading.

Hence, $(C_{Do})_{fuse}\frac{S_{fuse}}{S} + (C_{Do})_{nac}\frac{S_{nac}}{S} + (C_{Do})_{etc}\frac{S_{etc}}{S} + C_{Dint}$ is written as $(C_{Do})_{other}\frac{S_{other}}{S}$ i.e.

$$(C_{Do})_{fuse}\frac{S_{fuse}}{S} + (C_{Do})_{nac}\frac{S_{nac}}{S} + (C_{Do})_{etc}\frac{S_{etc}}{S} + C_{Dint} = (C_{Do})_{other}\frac{S_{other}}{S} \qquad (4.20)$$

The parasite drag coefficient can also be expressed as:

$$C_{Do} = C_{fe}\frac{S_{wet}}{S} \qquad (4.21)$$

where, $C_{fe}$ = equivalent skin friction drag coefficient and

$S_{wet}$ = wetted area of airplane

**Example 4.2**

Consider a high subsonic jet airplane with an initial estimate of gross weight ($W_0$) as 60,000 kgf and wing loading of 5500 N/m$^2$. Estimate $C_{fe}$.

**Solution :**

In this case the wing area would be :

$$S = W / (W/S) = \frac{60000 \times 9.81}{5500} = 107.02\,m^2$$

From Eq.(3.57) an approximate value of $C_{Do}$ would be

$C_{Do}$ = 0.02686 $S^{-0.1}$ = 0.02686 x $107.02^{-0.1}$ = 0.0168

Further, from section 3.5.8 of Ref.1.15, for a jet airplane,

$$\frac{S_{Wet}}{S} = 5.5$$

Hence, $C_{fe} = C_{Do} \frac{S_{wet}}{S}$ = 0.0168/5.5 = 0.003055

**Answer: $C_{fe}$= 0.003055**

(4) Using the definition of $C_{fe}$ in Eq.(4.21), the right hand side of Eq.(4.18) is expressed as :

$$(C_{Do})_W \left(1+\frac{S_{ht}}{S}+\frac{S_{vt}}{S}\right) = K_t (C_{Do})_W = K_t C_{fe} \frac{(S_{wet})_w}{S} \quad (4.22)$$

$(S_{wet})_w$ = wetted area of wing.

$(S_{wet})_w \approx 2\, S_{exposedwing}$ {1 + 1.2 $(t/c)_w$ }. The quantity $S_{exposed\ wing}$ can be obtained as the wing parameters and the fuselage width are roughly known from the preliminary three view stage. This is illustrated in example 4.3.

The quantity $K_t\, C_{fe} \frac{(S_{wet})_w}{S}$ is denoted by $f_1$ i.e.

$$F_1 = K_t C_{fe} \frac{(S_{wet})_w}{S} \quad (4.23)$$

(5) The quantity $(C_{Do})_{other} \frac{S_{other}}{S}$ of Eq.(4.20) is expressed as :

$$(C_{Do})_{other} \frac{S_{other}}{S} = (C_{Do} - F_1) \quad (4.24)$$

It may be pointed out that $S_{other}$ remains same but 'S' changes as the wing loading changes. Let the current value S be denoted as $S_{current}$ and 'S' based on preliminary three-view stage as $S_{old}$ .

Rewriting Eq.(4.24) yields:

$$(C_{Do})_{other}\frac{S_{other}}{S_{current}} = (C_{Do})_{other}\frac{S_{other}}{S_{old}}\times\frac{S_{old}}{S_{current}}$$

$$=(C_{Do})_{other}\frac{S_{other}}{S_{old}}\times\frac{S_{old}}{W}\times\frac{W}{S_{current}} \qquad (4.25)$$

Now, $\frac{W}{S_{old}}$ = wing loading from the preliminary three-view drawing stage which may be denoted as $(W/S)_{old}$.

$\frac{W}{S_{current}}$ = current value of wing loading, which following Ref.1.6 is denoted as p.

Further, from Eq.(4.24), $(C_{Do})_{other}\frac{S_{other}}{S_{old}} = (C_{Do} - F_1)$

Hence,

$$(C_{Do})_{other}\frac{S_{other}}{S_{current}} = (C_{Do} - F_1)\frac{1}{(W/S)_{old}}p \qquad (4.26)$$

Combining Eqs.(4.23) and (4.26) gives:

$$C_{Do} = F_1 + (C_{Do} - F_1)\frac{1}{(W/S)_{old}}p = F_1 + F_2 p;\ F_2 = \frac{(C_{D0} - F_1)}{(W/S)_{old}} \qquad (4.27)$$

(6) The lift-dependant drag coefficient ($C_{Di}$) is expressed as :

$$C_{Di} = \frac{1}{\pi Ae}C_L^2$$

Noting that $C_L = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{p}{q}$, $p = \frac{W}{S_{current}}$, $q = \frac{1}{2}\rho V^2$

$$\text{Hence, } C_{Di} = \frac{1}{\pi Ae}\frac{p^2}{q^2} = F_3 p^2; F_3 = \frac{1}{\pi Aeq^2} \qquad (4.28)$$

Consequently, the alternate representation of the drag polar is:

$$C_D = C_{Do} + C_{Di} = F_1 + F_2 p + F_3 p^2 \qquad (4.29)$$

$$\text{where, } f_1 = K_t\ C_{fe}\frac{(S_{wet})_w}{S_{old}};\ F_2 = \frac{C_{Do} - F_1}{(W/S)_{old}};\ F_3 = \frac{1}{\pi Aeq^2} \qquad (4.30)$$

**Example 4.3**

Obtain the drag polar in the alternate representation for the airplane in example 4.2 with the following additional data.

Wing: A = 9; $\lambda$ = 0.24; $\Lambda_{\frac{c}{4}}$ = 25°; (t/c) of airfoil = 0.14.

Fuselage diameter where, wing is located = 3.79 m.

$S_{ht}/S$ = 0.31, $S_{vt}/S$ = 0.21.

**Solution:**

The data from example 4.2 are :

$W_0$ = 60,000 kgf, $(W/S)_{old}$ = 5500 N/m$^2$, $S_{old}$ = 60000 x 9.81/5500 = 107.02 m$^2$

$C_{Do}$ = 0.0168, $C_{fe}$ = 0.003055.

Wing span = b = $\sqrt{AS} = \sqrt{9.3\times107.02}$ = 31.55 m

Further, $S = \frac{b}{2}(c_r + c_t) = \frac{b}{2}c_r(1+\lambda)$

Hence, $c_r = \frac{2\times107.02}{31.55\times1.24} = 5.47\,\text{m}.$

$c_t$ = 1.31 m.

Hence, equation for local chord (c) is

$$c = c_r - \frac{c_r - c_t}{b/2}y = 5.47 - 0.264\ y$$

Half of the width of fuselage = 3.79/2 = 1.895 m.

Hence, semispan of exposed wing = $\frac{31.55}{2} - 1.895 = 13.89$ m

Root chord of exposed wing = 5.47 – 0.264 x 1.895 = 4.97 m

Tip chord of exposed wing = 1.31 m

Consequently, $S_{\text{exposed wing}}$ = 13.89 (4.97+1.31) = 87.23

$(S_{wet})_w$ = 2 x $S_{\text{exposedwing}}$ x {1 + 0.2(t/c)}

= 2 x 87.23 {1 + 0.2 x (0.14)} = 203.76 m$^2$

$K_t$ = 1 + 0.31 + 0.21 = 1.52

$$\text{Hence, } F_1 = K_t C_{fe}\frac{(S_{wet})_w}{S_{old}} = 1.52\times0.003055\times\frac{203.76}{107.02} = 0.00884$$

$$F_2 = \frac{C_{Do} - F_1}{(W/S)_{old}} = \frac{0.0168 - 0.00864}{5500} = 1.447\times10^{-6}\ \text{m}^2/\text{N}$$

From Eq.(3.57)

$$K = \frac{1}{\pi A}\left(1.0447 + \frac{0.2078}{\cos^2\Lambda_{\frac{1}{4}}}\right)$$

$$= \frac{1}{3.14\times 9.3}\left(1.0447 + \frac{0.2078}{\cos^2 25^\circ}\right) = 0.0444$$

Hence, $F_3 = \frac{0.0444}{q^2}$

Finally, the drag polar in alternate form is:

$$C_D = 0.00884 + 1.447\times 10^{-6}\,p + \frac{0.0444}{q^2}p^2$$

**Answer: The drag polar in alternate form is:**

$$\mathbf{C_D = 0.00884 + 1.447\times 10^{-6}\,p + \frac{0.0444}{q^2}p^2}$$

Having estimated the drag polar the wing loading can be optimised for the prescribed flight speed $V_p$ at altitude $H_p$.

Equation(4.13) can be rewritten as :

$$\bar{t}_v = \frac{T_{vp}}{W} = \frac{C_D q_p}{p} = \frac{q_p}{p}\{F_1 + F_2\,p + F_3\,p^2\}$$

$$\bar{t}_v = q_p\left(\frac{F_1}{p} + F_2 + F_3\,p\right) \qquad (4.31)$$

The wing loading ($p_v$) which will minimize $\bar{t}_p$, for chosen $V_p$ , is obtained by differentiating Eq.(4.7) with p and equating to zero i.e.

$$\frac{d\bar{t}_v}{dp} = 0 \text{ or } 0 = q_p\left(-\frac{F_1}{p_v^2} + F_3\right)$$

$$\text{Or}\,p_v = \sqrt{F_1/F_3} = q_p\sqrt{F_1\,\pi\,Ae} \qquad (4.32a)$$

.

A typical plot of $t_v$ vs p is shown in Fig.4.5. The value of optimum wing loading from consideration of $V_p$ is marked in the figure. The minimum thrust loading is indicated by $(t_v)_{min}$. Allowing 5% increase ($\Delta t$) above this minimum, and solving, Eq.(4.31) gives two values of wing loading. These values are also indicated in Fig.4.5 and if the wing loading is in this range, then the thrust required would be within 5% of the minimum thrust required for $V_p$.

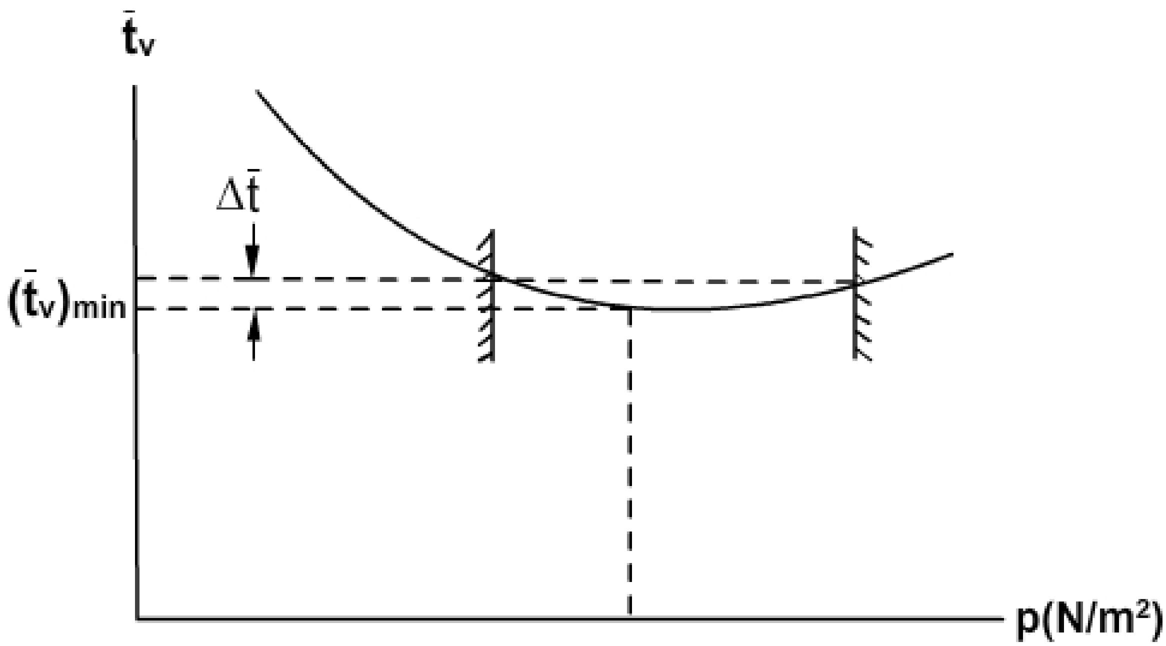


Fig.4.5 Thrust required for prescribed $V_p$ vs wing loading

**Remark:**

To select an engine, its rating needs to be prescribed. Further, the rating of an engine is the sea level static take-off thrust produced by it. Hence, to get the required rating of the engine, which can give $\bar{t}_v$ at the chosen altitude, the value of $(\bar{t}_v)_{min}$ needs to be multiplied by a factor based on typical engine characteristics. If the ratio of the sea level static thrust $(T_{s.l})_{static}$ to the thrust available at $V_p$ and $H_p$ be $R_{tslbhvp}$, then the required thrust loading is given by:

$$\frac{T_{s.l.}}{W} = R_{tslbhvp}\,\bar{t}_v \qquad (4.33)$$

For example, let $\bar{t}_v$ be 0.06 when $V_p$ corresponds to a Mach number of 0.8 and the flight altitude be 35000 ft (10670 m). Now, from Fig.4.22, which shows typical characteristics of a turbofan engine, the sea level static thrust for cruise rating is

44000 lbs. However, from Ref.3.3, chapter 6, the sea level static take-off thrust, which is the engine rating, is 55600 lbs. From Fig.4.22 the thrust at M = 0.8 and h = 35000 ft is 12300 lbs. Thus, $R_{tslbhvp}$ is (55600 / 12300) or 4.52, and hence the thrust to weight ratio, based on sea level static thrust, is:

$$\frac{T_{s.l.}}{W} = 4.52 \times 0.06 = 0.271$$

**Example 4.4**

Consider the airplane in example 4.3 and obtain (a) the optimum wing loading for cruising at M = 0.8 at h = 11 km. (b) the range of wing loadings if an increase by 5 % is permitted above the minimum value of thrust required (c) the thrust loading from cruising consideration, if the ratio of sea level static take-off thrust to that at cruising condition, $R_{tslbhvp}$ , be 5.

**Solution :**

The data are :

$M_{cr}$ = 0.8, $h_{cr}$ = 11 Km

Drag polar is $C_D = 0.00884 + 1.447 \times 10^{-6}\, p + \frac{0.0444}{q^2} p^2$

At 11 km altitude, speed of sound (a) is 295.1 m/s

$V_c = V_p = 0.8 \times 295.1 = 236.10$ m/s

$\rho$ at 11 km is 0.364 kg/m$^3$

$$q_p = \frac{1}{2} \times 0.364 \times 236.1^2 = 10145.3 \text{ N/m}^2$$

Hence, $F_3 = 0.0444/10145.3^2 = 4.314 \times 10^{-10}$ m$^2$/N$^2$

Hence, optimum wing loading from consideration of $V_p$ is

$$p_v = \sqrt{F_1/F_3} = \sqrt{\frac{0.00884}{4.314 \times 10^{-10}}} = 4526.9 \text{ N/m}^2$$

$$\left(\bar{t}_v\right)_{min} = q_p\left(\frac{F_1}{p} + F_2 + F_3 p\right)$$

$$= q_p\left(2\frac{F_1}{p} + F_2\right)$$

$$= 10145.3\left(2\times\frac{0.00884}{4526.9}+1.447\times10^{-6}\right) = 0.0543$$

(b) If a 5% increase is permitted over $(\bar{t}_v)_{min}$ then $\bar{t}_v$ can be :

0.0543 x 1.05 = 0.05701.

From Eq.(4.31):

$$0.05701 = q_p\left(\frac{F_1}{p}+F_2+F_3 p\right)$$

Or $0.05701 = 10145.3\left(\frac{0.00884}{p}+1.447\times10^{-6}+4.314\times10^{-10}\,p\right)$

Simplifying,

$p^2 - 9671.2\,p + 20.49 \times 10^6 = 0$

Or p = 3135 or 6536.4 N/m$^2$

Thus, if the wing loading is between 3135 to 6536 N/m$^2$ then $\bar{t}_{vp}$ will be with 5% of $\left(\bar{t}_v\right)_{min}$.

(c) The ratio $R_{tslphp}$ is given as 5.0. Hence, thrust loading for this airplane would be $\frac{T}{W} = 5.0\times0.0543 = 0.2715$.

**Answers: (a) Optimum wing loading from consideration of $V_p$ is 4527 N/m$^2$.**

**(b) Range of wing loading when an increase by 5 % is allowed over $\left(\bar{t}_v\right)_{min}$ : 3135 to 6536 N/m$^2$. (c) $\frac{T}{W} = 0.2715$.**

**Remarks:**

(i) During the discussion in section 4.4.2 and in example 4.4, the prescribed flight speed ($V_p$) is considered as $V_{cr}$. However, if the prescribed speed is above $V_{cr}$ then, in the case of high subsonic jet airplanes, the value of $C_{DO}$ may be higher due to compressibility effects. This increase in $C_{DO}$ should be taken into account and the value of $F_1$ increased suitably. Section 3.3.5 of Ref.3.3 gives guidelines for estimating changes in $C_{DO}$ and K in the transonic Mach number range.

(ii) In example 4.4 the value of optimum wing loading from cruise consideration is 4527 N/m$^2$. This value is close to wing loading of actual jet airplanes. In this background the following is pointed out.

Reference 1.12, Part I , chapter 3 gives a procedure to select optimum wing loading from consideration of $V_{cr}$. It amounts to considering $F_1$ as $C_{DO}$. Consequently, the value of optimum wing loading from consideration of $V_{cr}$ is much higher than the wing loading of airplanes of this category.

This brings out the advantage of the alternate break up of drag polar.

(iii) Reference 1.12, part I chapter 3, points out that the thrust available at $h_{cr}$ must be more than that required for level flight $\left(\bar{t}_{level}\right)$ at $V_{cr}$. This is because the airplane must have the ability to climb to service ceiling. The rate of climb at $h_{cr}$ is generally 500 ft/min (or 152.4 m/min or 2.54 m/s) for jet airplanes. Hence, the thrust to weight ratio required at $h_{cr}$ is

$$\bar{t} = \frac{V_c}{V_{cr}} + \bar{t}_{level}$$

In example 4.4, this would give :

$$\bar{t} = \frac{2.54}{236.1} + 0.0543 = 0.0651$$

Taking $R_{tslbvp}$ = 5.0

$$\frac{T}{W} = 5 \times 0.0651 = 0.326$$

This value of T/W is typical of airplanes with engines with bypass ratio of 6.

## 4.5 Selection of wing loading based on absolute ceiling ($H_{max}$)

At absolute ceiling the flight is possible only at one speed at which $T_{req} = T_{min} = D_{min}$. Hence, for a jet airplane:

$$\bar{t}_{Hmax} = \frac{D_{min}}{W} = \frac{1}{(L/D)_{max}}$$

For a parabolic polar i.e. $C_D = C_{D0} + KC_L^2$, the following relations are already known.

$(C_L)_{(L/D)max} = (C_{D0} / K)^{1/2}$

$(C_D)_{(L/D)max} = 2C_{D0}$

Hence,

$$\bar{t}_{Hmax} = \frac{1}{(L/D)_{max}} = \frac{2C_{D0}}{\sqrt{C_{D0} / K}} = \sqrt{4\, C_{D0}\, K} \tag{4.34}$$

$$= \sqrt{4\, K\, (F_1 + F_2\, p)} \tag{4.35}$$

From Eq.(4.27) $C_{D0} = F_1 + F_2$ p and consequently, it varies with p (see paragraph on alternate break up of drag polar in Section 4.4.1). The variation of $\bar{t}_{Hmax}$ with p is shown as curve 'A' in Fig.4.6. It is seen that $\bar{t}_{Hmax}$ increases as p increases. Sometimes, the flight velocity ($V_{hmax}$) may be prescribed at absolute ceiling. In this case, the optimization of wing loading is carried out in the following manner. Note: that $q_{hmax} = ½\, \rho_{hmax} V_{hmax}^2$

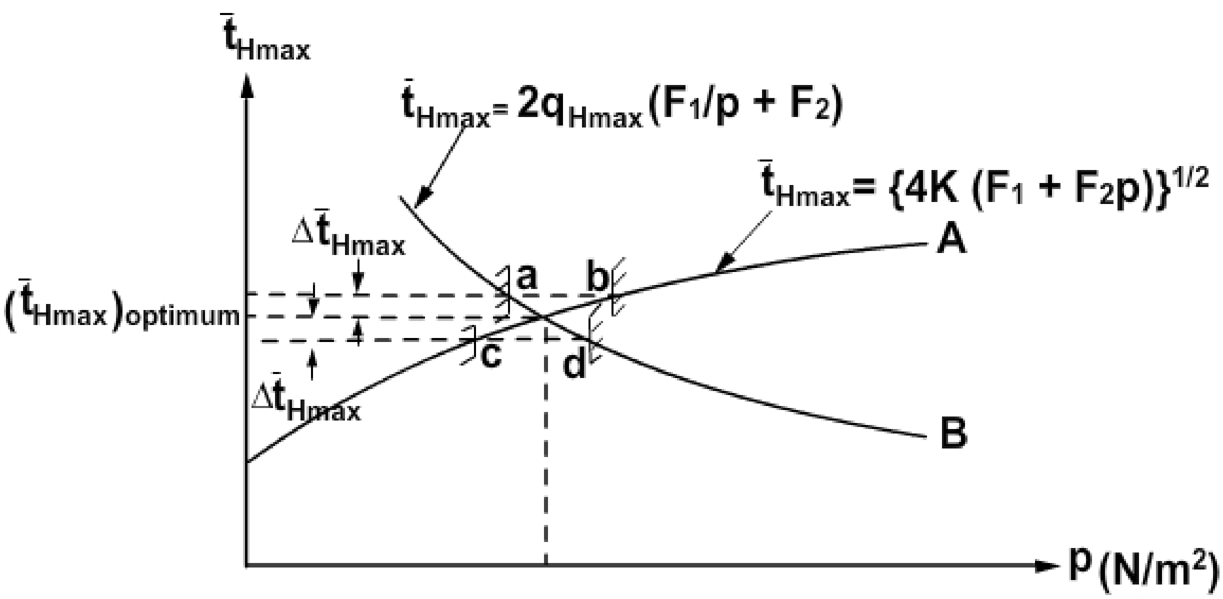


Fig.4.6 Thrust required for chosen ceiling vs wing loading

$$\bar{t}_H = \frac{T_{req}}{W} = q_{Hmax}\frac{S(C_D)_{(L/D)max}}{W} = q_{Hmax}\ 2C_{D0} / p \text{ ; note that } (C_D)_{(L/D)max} = 2\ C_{DO}$$

Substituting for $C_{D0}$ as $F_1 + F_2\,p$, gives:

$$\bar{t}_H = 2\ q_{Hmax}\left(\frac{F_1}{p} + F_2\right) \qquad (4.36)$$

For a prescribed $q_{Hmax}$, $\bar{t}_H$ the variation of $\bar{t}_H$ with p can be obtained from Eq.(4.36). It is plotted as curve B in Fig.4.6. Thus, if both viz. (a) V at $H_{max}$ and (b) $H_{max}$ are prescribed, then $p_{opt}$ is given by the intersection point of curves A and B in Fig.4.6. Allowing $\Delta t_{Hmax}$ as deviation from the optimum value, the permissible upper and lower limits on p are obtained as shown in Fig.4.6.

**Example 4.5**

For the airplane in example 4.4 obtain the optimum wing loading from the consideration of absolute ceiling. It is prescribed that the absolute ceiling be 12 km and velocity be corresponding to $(L/D)_{max}$. Further, obtain the range of wing loadings if it is permitted that $\bar{t}_{Hmax}$ can vary between $\pm 5\%$ from that corresponding to the aforesaid optimum.

**Solution:**

From Eqs.(4.35) and (4.36) :

$$\bar{t}_{Hmax} = \sqrt{4K(F_1 + F_2 p)} \qquad (E4.5.1)$$

$$\bar{t}_H = 2q_{Hmax}\left(\frac{F_1}{p} + F_2\right) \qquad (E4.5.2)$$

From example 4.4, $F_1 = 0.00884$ and $F_2 = 1.447 \times 10^{-6}$. $C_{Do} = 0.0168$, $K = 0.0444$ .

From example 4.2, $(W/S)_{old} = 5500\ N/m^2$. It is prescribed that $q_{Hmax}$ corresponds to $V = V_{(L/D)max}$

$$(C_L)_{(L/D)max} = \sqrt{C_{DO}/K} = \sqrt{0.0168/0.0444} = 0.615$$

$$q_{Hmax} = \frac{W/S}{(C_L)_{(L/D)max}} = \frac{5500}{0.615} = 8943.1\ N/m^2$$

Solving Eqs.(E 4.5.1) and (E 4.5.2) gives $p_{Hmax} = 5500\ N/m^2$ as it should be.

$\bar{t}_{Hmax}$ corresponding to $p_{Hmax}$ is :

$$\sqrt{4\times0.0444(0.00884+1.447\times10^{-6}\times5500)} = 0.0546$$

Allowing $\pm5\%$ variation in $\bar{t}_{Hmax}$, the permissible limits on $\bar{t}_{Hmax}$ or:

$$\bar{t}_{Hmax1} = 0.05187$$

$$\bar{t}_{Hmax2} = 0.05733$$

Solving Eq.(E.4.5.1) for the above values of $\bar{t}_{Hmax1}$ and $\bar{t}_{Hmax2}$ gives:

$$\left.\begin{array}{l} p_{Hmax1} = 4360.2\ \text{N/m}^2 \\ p_{Hmax2} = 6680.3\ \text{N/m}^2 \end{array}\right\} \qquad \text{(E4.5.3)}$$

Solving Eq.(E.4.5.2) for the above values of $\bar{t}_{Hmax1}$ and $\bar{t}_{Hmax2}$ gives:

$$\left.\begin{array}{l} p_{Hmax1} = 6084.0\ \text{N/m}^2 \\ p_{Hmax2} = 5027.7\ \text{N/m}^2 \end{array}\right\} \qquad \text{(E4.5.4)}$$

Hence, to satisfy both the above criteria the range of permissible wing loading is given by :

(a)higher of the lower limits from the two considerations and (b) lower of the upper limits from the two considerations.

From Eqs.(E4.5.3) and (E4.5.4) the range of values is which satisfies these limitations is :

$$5028 \le p \le 6084\ \text{N/m}^2$$

**Answer: The range of wing loading to satisfy $H_{max}$ requirement is** :

**5028 to 6084 N/m$^2$**.

**Remark:**

This example again bring out the fact that the alternate representation of drag polar permits examination of the effect of wing loading on $H_{max}$.