Chapter 9

Cross-checks on design of tail surfaces - 2

Lecture 35

Topics

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9.2.4 Cross-check II – Adequacy of elevator with $|\delta_e| \leq 25^o$ in landing configuration with c.g. at the most forward location

Combining Eqs(9.2) and (9.22) yields.

$$C_{mcg} = C_{macw} + C_{L\alpha w}(i_w - \alpha_{oLw})\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{mo})_{f,n,p} - V_H \eta C_{L\alpha t}\{i_t - \varepsilon_0 + \tau_{tab}\delta_t\}$$

$$+ C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)\alpha + (C_{m\alpha})_{f,n,p}\alpha - V_H \eta C_{L\alpha t}\left\{\alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \tau\delta e\right\} \quad (9.31)$$

With flaps deflected and airplane near ground, Eq.(9.31) has the following modified form.

$$C_{mcg} = (C_{mac})_{at} + (\Delta C_{mac})_{flap} + C_{L\alpha wg}(i_W - \alpha_{oLW})\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{mo})_{f,n,p} - V_H \eta C_{L\alpha tg}(i_t - \varepsilon_{og} + \tau_{tab}\delta_t)$$

$$+ C_{L\alpha wg}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)\alpha + (C_{m\alpha})_{f,n,p}\alpha - V_H \eta C_{L\alpha tg}\left\{\alpha\left(1 - \frac{d\varepsilon_g}{d\alpha}\right) + \tau\delta e\right\} \quad (9.32)$$

where,

$(c_{mac})_{airfoil}$ = pitching moment coefficient of airfoil of wing

$(\Delta C_{mac})_{flap}$ = change in C_{mac} due to deflection of flap (see example 9.1 for details)

$C_{L\alpha wg}$ = slope of lift curve of wing in proximity of ground

$C_{L\alpha tg}$ = slope of lift curve of h.tail in proximity of ground

ε_{og} = downwash at tail in proximity of ground when $\alpha = 0$

$\frac{d\varepsilon_g}{d\alpha} = (d\varepsilon / d\alpha)$ in proximity of ground

To perform the cross-check II, substitute $C_{mcg} = 0$ in Eq.(9.32) and obtain $|\delta e|_{reqd}$. The value of $|\delta e|_{reqd}$ should be less than or equal to 25^o. If $|\delta e|_{reqd}$ is more than 25^o, the elevator area needs to be increased or the forward shift of c.g. needs to be decreased by suitable rearrangement of items in fuselage. Example 9.1 illustrates the steps to check the adequacy of elevator for the sixty seater turboprop airplane under design.

9.2.5 Cross-check III – Adequacy of elevator to develop sufficient pitching moment to enable nose wheel lift-off

The phases of take-off flight are shown in Fig.3.4. The details are described in section 10.3 of Ref.3.3.

During the ground run, the airplane starts from rest and accelerates to the take-off speed (V_{T0} or V_1). The flaps and engine(s) are adjusted for their take-off settings. In the case of an airplane with tricycle type of landing gear, all the three wheels remain in contact with the ground till a speed of about 85% of the V_{T0} is reached. This speed is called 'Nose wheel lift-off speed'. At this speed the pilot pulls the stick back and increases the angle of attack of the airplane so as to attain a lift coefficient corresponding to take-off (C_{LT0}). At this stage, the nose wheel is off the ground and the speed of the airplane continues to increase. As the speed exceeds the take-off speed the airplane gets airborne and the main landing gear wheels also leave the ground.

When the airplane has a tail wheel type of landing gear, the angle of attack is high at the beginning of the take-off run. However, the tail wheel is lifted-off the ground as soon as some speed is gained and the deflection of elevator can rotate the airplane about the main wheels. This action reduces the angle of attack and consequently the drag of the airplane during most of the ground run. As the take-off speed is approached the tail wheel is lowered to get the incidence corresponding to C_{LT0}. When V_{T0} is exceeded, the airplane gets airborne.

The point at which all the wheels have left the ground is called 'Unstick point'.

The requirement of the elevator in the case of airplane with tricycle landing gear is that it should produce enough pitching moment to cause rotation of airplane at $V = 0.85\ V_{TO}$ so that it (airplane) attains angle of attack corresponding to C_{LTO}.

The angular acceleration for rotation of the airplane, is about the main landing gear. This angular acceleration depends on :(a) elevator power ($C_{m\delta e}$), (b) area of h.tail (S_t), (c) h.tail arm (l_t), (d) distance between airplane c.g. and main landing gear, (e) airplane weight, (f) coefficient of friction between ground and tyres and (g) moment of inertia of the airplane about y-axis.

Carrying out the cross-check is rather complicated and interested reader can refer to Ref.1.24, chapter 12.

9.3 Cross-checks for directional static stability and control

From chapter 5 of Ref.3.1 the following points are noted.

(i) For trim about z-axis of the airplane:

$$C_n = 0 \tag{9.33}$$

(ii) For static stability about z-axis of the airplane :

$$(dC_n / d\beta) \text{ or } C_{n\beta} > 0 \tag{9.34}$$

$$C_n = \text{yawing moment coefficient} = \frac{N}{\frac{1}{2}\rho V^2 S b} \tag{9.35}$$

N = yawing moment about c.g. of airplane

$\frac{1}{2}\rho V^2$ = free stream dynamic pressure

S = wing area

b = wing span

β = angle of sideslip

(iii) C_n and $C_{n\beta}$ can be written as:

$$C_n = (C_n)_w + (C_n)_{f,n,p} + (C_n)_{vt} \quad (9.36)$$

$$C_{n\beta} = (C_{n\beta})_w + (C_{n\beta})_{f,n,p} + (C_{n\beta})_{v\,t} \quad (9.37)$$

where suffices w, f, n, p and vt indicate wing, fuselage, nacelle, power and v.tail respectively.

Chapter 5 of Ref.3.1 presents methods to estimate $(C_{n\beta})_w$, $(C_{n\beta})_{f,n,p}$ and $(C_{n\beta})_{v\,t}$.

Appendix 'C' of Ref.3.1 presents the calculation of $C_{n\beta}$ for a jet airplane.

Using these methods and the parameters of airplane under design, obained in chapters 2 to 8, the value of $C_{n\beta}$ can be estimated.

9.3.1 Desirable level of $C_{n\beta}$

In longitudinal static stability, the shift of c.g. has a profound effect on the level of static stability (i.e. $C_{m\alpha}$).This is because the contribution of wing to $C_{m\alpha}$ depends directly on (x_{cg} - x_{ac}). Thus, the shift in the position of c.g, during flight, almost decides the area of the horizontal tail. However, a shift of c.g. does not cause a significant change in $C_{n\beta}$ because such a change may only have a secondary effect by way of slightly affecting l_v. Hence, to arrive at the area of the vertical tail, a prescription for a desirable value of $C_{n\beta}$ is needed. Reference 4.7, chapter 8 gives:

$$(C_{n\beta})_{desirable} = 0.0005 \left(\frac{W}{b^2}\right)^{1/2} \text{deg}^{-1} \quad (9.38)$$

where, W = weight of the airplane in lbs, b = wing span in feet.

However, Appendix 'B' of Reference 1.22, gives characteristics of seven airplanes. It is observed that for the subsonic airplanes $C_{n\beta}$ lies between 0.0013 to 0.0026 deg^{-1}.

Reference 1.18, chapter 16 presents a curve for $(C_{n\beta})_{desirable}$ vs Mach number as a suggested guideline. Following table can be prepared from this curve.

M	0.1	0.5	0.8
$C_{n\beta}$ deg^{-1}	0.001	0.0015	0.0025

Table 9.1 Guidelines for desirable value of $C_{n\beta}$

If the $C_{n\beta}$ calculated for the airplane under design is not in the range of above mentioned values, then the area of the vertical tail needs to be changed. Example 5.2 in Ref.3.1 illustrate the procedure to obtain area of v.tail for desired value of $C_{n\beta}$. However, the final value of $C_{n\beta}$ is decided after the dynamic stability analysis. Chapter 9 of Ref.3.1, shows that a large value of $C_{n\beta}$ leads to some unacceptable response of the airplane to a disturbance during flight.

9.3.2 Area of rudder

Rudder must provide adequate control during the following situations.

a) Cross wind take-off and landing

b) One engine inoperative condition for multi-engined airplane.

c) Adverse yaw during roll

d) Spin recovery

1) Cross wind take-off and landing:

For an airplane, the critical condition during take-off, is with a cross wind equal to 20% of V_{T0}. This gives a yaw angle (ψ_{cross}) of 11.5^0 or 0.2 radian. The moment due to cross wind equals $C_{n\psi} \times \psi_{cross}$.This moment must be balanced by the rudder with a maximum deflection (δ_{rmax}) of 25^0. Note that moment due to rudder is given by:

$$(C_n)_{rudder} = - a_V\ \tau_r\ (S_V / S)\ (l_V / b)\eta_V\ \delta_{rmax} \qquad (9.39)$$

Using equation (9.39) obtain τ_r. Since, τ_r depends on (S_{rudder} / S_V), the rudder area can be calculated.

2) One engine inoperative case:

For a multi-engine airplane, when an engine is inoperative, the other engine will cause a yawing moment. This moment has to be balanced by the rudder. The yawing moment due to engine will be almost constant with the flight speed. However, the moment due to rudder is proportional to the square of the flight velocity. Hence, there is a speed below which the maximum rudder deflection (δ_{rmax}) would not be able to balance the yawing moment due to engine. The requirement from this consideration is that the rudder should be effective down to $V = 1.3V_s$. Subsection 5.8.3 of Ref.3.1, gives the details regarding achievement of control in the event of one engine being inoperative. Example 5.3 of Ref.3.1 illustrates the calculation of minimum control speed in this case.

3) Adverse yaw during role:

Subsection 5.8.1 of Ref.3.1 explains as to how adverse yaw is brought about and the value of C_n due to adverse yaw. This must be balanced by the rudder as in Eq.(9.39).

4) Spin recovery:

Spin is a flight condition in which the airplane wings are stalled and it moves rapidly downwards along a helical path. The only control that is still effective is the rudder. The way to come out of the spin is to stop the rotation, go into a dive and pull out. The rudder must be powerful enough to get the airplane out of spin.

Analysis of spin is complicated. Reference 9.1, chapters 7 and 8 be referred to for details.

A criteria regarding spin recovery, at the preliminary design stage, is that at least one third of the rudder should not be blanketted by the wake of the horizontal tail. As a rough guideline, the wake of the h.tail lies between two lines – one drawn at 30^o from the trailing edge of h.tail and the second drawn at 60^o from the leading edge of the h.tail. Figure 9.1 shows the two lines and the area of the rudder blanketed by wake.

The relative locations h.tail and v.tail should ensure that at least $(1/3)^{rd}$ of rudder area is unblanketed.

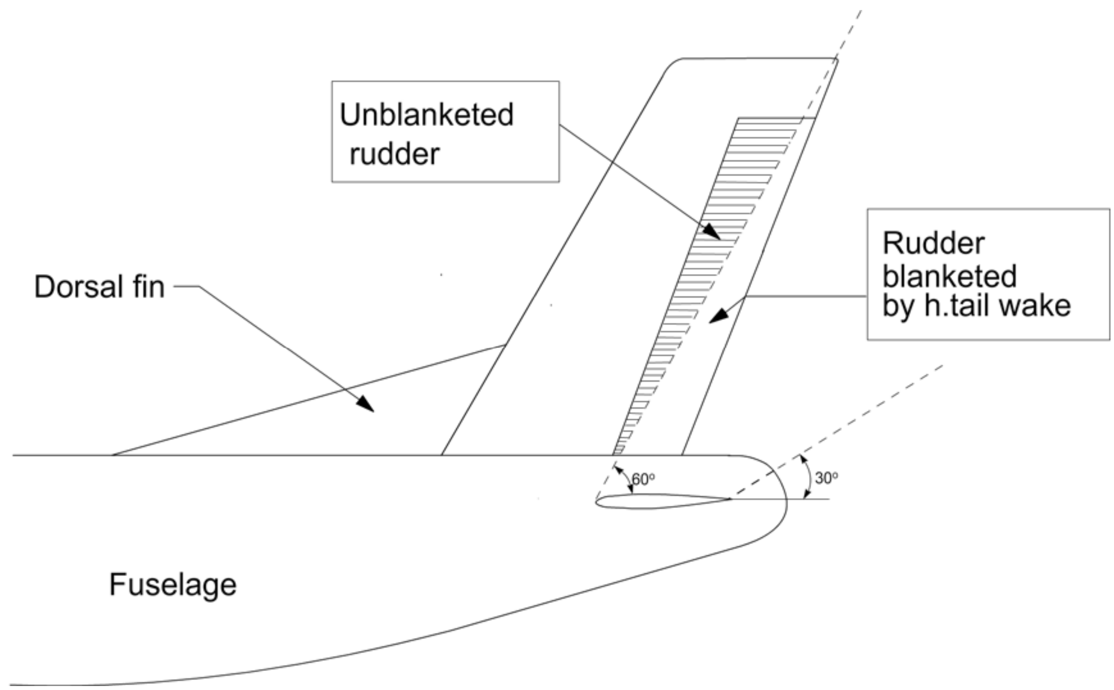

Fig.9.1 Location of the wake of h.tail. The region between the two dotted lines is the area covered by the wake of the h.tail.