

## Chapter 4

### Estimation of wing loading and thrust loading - 7

#### Lecture 15

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#### 4.12 Introductory remarks on choice of engine

After arriving at the thrust or power rating required, the next step is to choose the engine.

The engine is selected from the engines available in literature (for example Ref.1.21). The development of a new engine generally takes much longer than the development of the airframe and hence, general practice is to design the airframe around an available engine. These engines would have ratings as per their design. Hence, again a compromise would be called for while selecting the engine from the available ones (see sections 3.9 to 3.12 of Appendix 10.2).

The weight, frontal area and SFC of the engine would also have to be taken in to account while arriving at the final choice. It may be pointed out that a higher frontal area would result in increased parasite drag; and a heavier engine would cause higher induced drag.

After choosing the engine, the variations of thrust or power and TSFC or BSFC with altitude and velocity need to be obtained. In the case of airplanes with engine-propeller combination, the variation of propeller efficiency also needs to be calculated. Before discussing further, a review of the engine characteristics is presented in section 4.13.

### 4.13 Engines considered for airplane applications

Following power plants are considered for airplane applications.

- (a) Piston engine-propeller combination.
- (b) Gas turbine engines - turboprop, turbofan and turbojet.
- (c) Ramjets.
- (d) Rockets.
- (e) Combination power plants like ramrocket and turboramjet.

At present, piston engine-propeller combination and gas turbine engines are the power plants used for airplanes. Ramjets offer simplicity of construction and have been proposed for hypersonic airplanes. However, a ramjet cannot produce any thrust when flight speed is zero. Hence, it is proposed to use a rocket or turbojet engine to bring it (ramjet) to a flight speed corresponding to Mach number ( $M$ ) of 2 or 3 and then the ramjet engine would take over. Consequently, the combination power plants viz. ramrocket or turboramjet have been proposed.

#### **Remark:**

Reference 1.20, chapters 14 and 18 briefly mentions about solar powered airplanes. In these cases, the solar energy is converted into electricity which is stored in batteries or fuel cells. Electrical motors, using this power, would run propellers.

#### 4.14 Piston engine-propeller combination

In this case the output of the engine viz. brake horse power (BHP) is available at the engine shaft and is converted into thrust by the propeller.

##### 4.14.1 Operating principle of a piston engine

A few relevant facts about the operation of piston engines, used on airplanes, are mentioned here. In these engines a certain amount for fuel-air mixture is taken in, it is compressed, then ignition, due to a spark, takes place which is followed by the power stroke and the exhaust stroke (Fig.4.11).

##### Remarks:

i ) The piston engine in which the ignition is caused by a spark from the spark plug is called a spark-ignition engine. There are other types of piston engines in which the pressure and temperature at the end of the compression stroke are high enough to cause spontaneous ignition. Such, engines do not need a spark plug and are known as compression-ignition engines.

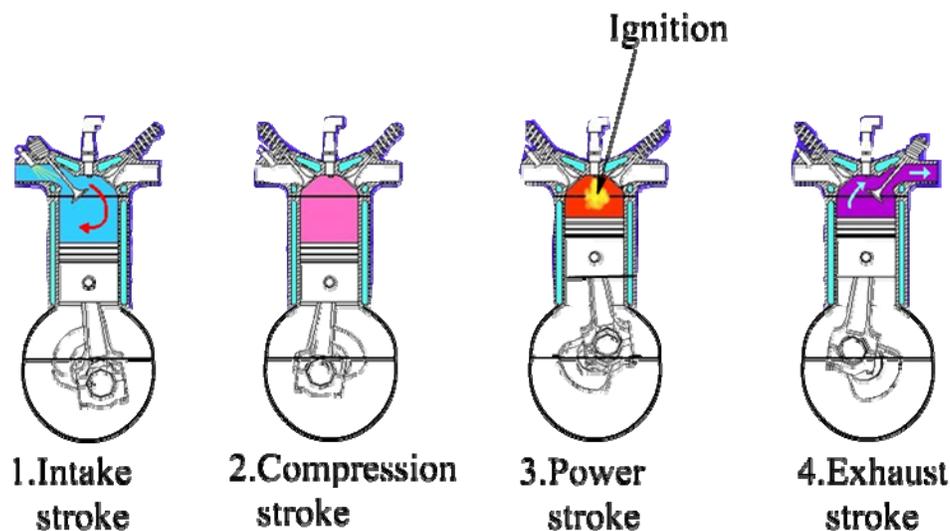


Fig.4.11 Four stroke cycle of a spark-ignition engine

ii) The volume of the air-fuel mixture taken in, is almost equal to the swept volume i.e., product of the area of cross-section of the engine cylinder and the

length of the piston stroke. The mass of fuel taken in, per power stroke is thus approximately equal to:

$$(\text{swept volume}) \times (\text{density of air}) / (\text{air-fuel ratio}).$$

#### 4.14.2 Effect of flight speed on the output of a piston engine

For a given altitude and r.p.m. (N) the power output changes only slightly with flight speed. This is because the piston engines are generally used at low speeds ( $M < 0.3$ ) and at these low Mach numbers, the increase in manifold pressure due to the deceleration of air in the engine manifold is negligible. Hence, power output increases only slightly with flight speed. This increase is generally ignored.

#### 4.14.3 Effect of altitude on the output of a piston engine

To understand the effect of altitude on the output of the piston engine, the following three facts are noted.

(a) As stated at the end of the subsection 4.14.1, the mass of fuel taken in per stroke is equal to the product of swept volume and density of air divided by air-fuel ratio.

(b) For complete combustion of fuel, the air-fuel ratio has a definite value (around 15, the stoichiometric ratio).

(c) As the flight altitude increases, the density of air decreases.

Thus, for a given engine r.p.m. and air-fuel ratio, the mass of air and consequently, that of the fuel taken in, decreases as the altitude increases. Since, the power output of the engine depends on the mass of the fuel taken in, it (power output) decreases with altitude. The change in power output (P) with altitude is roughly given as (Ref.1.5, Appendix 1 A-5, and Ref.1.20, Chapter 14):

$$(P / P_0) = 1.13\sigma - 0.13 \quad (4.104)$$

where,  $P_0$  is the power output at sea level under ISA conditions and  $\sigma$  is the density ratio.

#### Remarks:

(i) Reference 1.15, chapter 3, gives the following alternate relationship for decrease of power output with altitude :

$$(P / P_0) = \sigma^{1.1} \quad (4.105)$$

(ii) Figure 4.12 shows the performance for a typical piston engine. To prepare such a performance chart, the engine manufacturer carries out certain tests, on each new engine. During these tests the engine is run at a chosen RPM and different loads are applied. The throttle setting is adjusted to get steady conditions. The quantities like (a) engine RPM(N), (b) torque developed, (c) manifold air pressure(MAP) and (d) the fuel consumed in a specific interval of time, are measured. These tests are conducted at different RPM's. From these test data, the power output and the fuel flow rate per hour are calculated. The data are also corrected for any difference between the ambient conditions during the test, and the sea level standard conditions. The left side of Fig.4.12 presents the sea level performance of a Lycoming engine. The upper part of the figure shows the power output at different MAP's with RPM as parameter. The lower part of the figure shows the fuel flow rate in US gallons per hour.

To obtain the effect of altitude on the engine output, the power output is measured at different RPM's and MAP's during flight tests at different altitudes. Typical altitude performance of Lycoming engine is presented in the right side of Fig.4.12.

From such a chart, the output of the engine and the fuel flow rate can be obtained for a chosen combination of altitude, RPM, MAP and ambient temperature. The steps to obtain these are explained with the help of examples 4.17 and 4.18

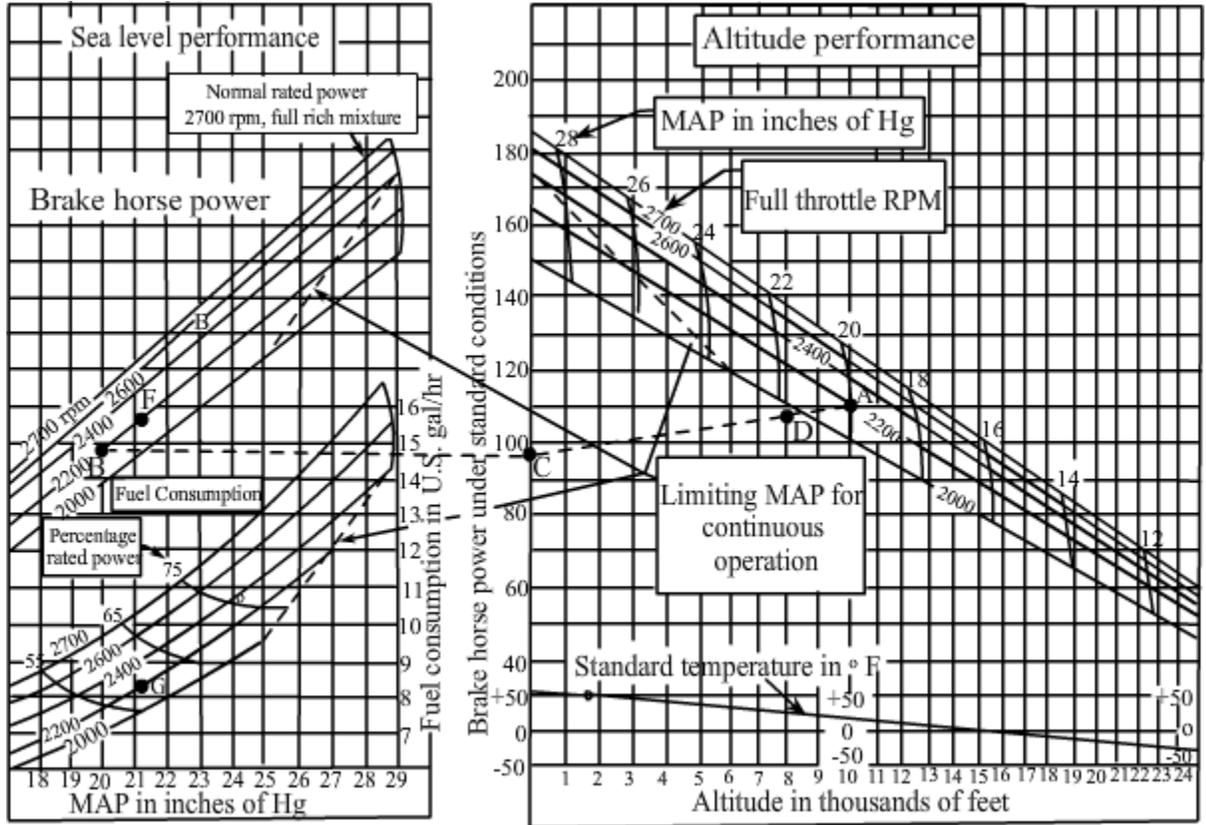


Fig.4.12 Typical piston engine performance (Lycoming O-360-A)  
(with permission from Lycoming aircraft engines )

It may be added that the units used in Fig.4.12, which is reproduced from manufacturer's catalogue, are in FPS system. However, SI units are used in this and the subsequent chapters.

#### 4.14.4 Specific fuel consumption (SFC)

In engine performance charts, the fuel consumption is presented as fuel flow rate per hour. However, in engineering practice the fuel consumption is expressed as specific fuel consumption (SFC). It is defined as :

$$SFC = \frac{\text{Fuel flow rate in Newton per hour}}{\text{BHP in kW}} \quad (4.106)$$

**Remarks :**

(i) The output of a piston engine or turboprop engine is available as power at the engine shaft. It is called BHP and measured in HP when FPS system is used. In SI units the output is measured in kW. On the other hand, the output of a turbofan or a turbojet engine is available as thrust, which is measured in 'lb' in FPS system and in Newton in SI units.

The specific fuel consumption of a jet engine is defined as:

$$\text{SFC} = \frac{\text{Fuel flow rate in Newton per hour}}{\text{Thrust in Newton}} \quad (4.107)$$

(ii) To distinguish the specific fuel consumption of a piston or a turboprop engine, from that of a jet engine, the SFC defined by Eq.(4.106), is denoted as BSFC i.e.

$$\text{BSFC} = \frac{\text{Fuel flow rate in Newton per hour}}{\text{BHP in kw}} \quad \text{with units of N/kW-hr} \quad (4.108)$$

The specific fuel consumption of a turbofan or a turbojet engine is denoted by TSFC i.e.

$$\text{TSFC} = \frac{\text{Fuel flow rate in Newton per hour}}{\text{Thrust in Newton}} \quad \text{with units of hr}^{-1} \quad (4.109)$$

(iii) BSFC in metric units is also expressed as mg/W-s

**Example 4.17**

Obtain the power output and BSFC for the Lycoming engine when operating at sea level at an RPM(N) of 2400 and MAP of 24" of mercury (Hg).

**Solution :**

From plots in the left side of Fig.4.12, for N = 2400 and MAP = 24" of Hg the power output is 136 HP and the fuel flow rate is 10.7 US gallons/hr.

Taking 1 US gallon = 3.78 litre and density of petrol as 0.76 kg/m<sup>3</sup> gives:

$$\begin{aligned} 1 \text{ gallon per hour of petrol} &= 3.78 \times 0.76 \text{ kg/hr} \\ &= 3.78 \times 0.76 \times 9.81 \text{ N/hr} \\ &= 28.18 \text{ N/hr of petrol} \end{aligned}$$

Hence, the fuel flow rate in the case under study is :

$$10.7 \times 28.18 = 301.5 \text{ N/hr.}$$

Noting that  $1 \text{ lb/hr} = 4.45 \text{ N/hr}$ , The fuel flow rate in this case is  $67.75 \text{ lbs/hr}$ .

Further,  $1 \text{ HP}$  is  $0.7457 \text{ kW}$ . Hence, power output of  $136 \text{ HP}$  equals  $101.4 \text{ kW}$ .

Hence, BSFC in SI units is:  $301.5/101.4 = 2.973 \text{ N/kW-hr}$

In FPS units it is:  $67.75/136 = 0.498 \text{ lb/HP-hr}$

**Answers:**

**For the given engine, the power output, fuel flow rate and BSFC at  $N = 2400$  and  $\text{MAP} = 24''$  of Hg under sea level standard conditions are :**

**(i) Power output =  $101.4 \text{ kW} = 136 \text{ HP}$ , (ii) Fuel flow rate =  $10.7 \text{ US gallons/hr}$  or  $301.5 \text{ N/hr}$  or  $67.75 \text{ lb/hr}$  of petrol (iii) BSFC =  $2.973 \text{ N/kW-hr} = 0.498 \text{ lb/HP-hr}$**

**Example 4.18**

Obtain the power output and BSFC for the Lycoming engine when operating at  $8000'$  altitude, RPM (N) of  $2200$  and  $\text{MAP}$  of  $20''$  of Hg.

**Solution :**

Reference 3.4 Chapter 6, gives the following procedure to obtain the output and fuel flow rate using left and right sides of Fig.4.12.

(i) At sea level for  $N = 2200$  and  $\text{MAP}$  of  $20''$  of Hg the output would be  $97.5 \text{ BHP}$ .

This is indicated by point 'B' in the left hand side of Fig.4.12. This side of the diagram is also called sea level performance.

(ii) Transfer this point to the right hand side of Fig.4.12 at sea level which is indicated by point 'C'. The right side of the diagram is also called altitude performance.

(iii) Locate a point on the altitude curve corresponding to  $N = 2200$  and  $\text{MAP}$  of  $20''$  of Hg. This point is indicated by 'A'.

(iv) Join points C and A by a dotted line. The value at  $8000'$  on this line (the point 'D') is the output at  $h = 8000'$  corresponding to  $N = 2200$  and  $\text{MAP} = 20''$  of Hg. It is seen that the value is  $107 \text{ HP}$ .

(v) To get the fuel flow rate, mark a point 'F' on the sea level performance at  $107 \text{ HP}$  and  $N = 2200$ . The  $\text{MAP}$  at this point is observed to be  $21.2''$  of Hg. The fuel flow rate corresponding to  $N = 2200$  and  $\text{MAP}$  of  $21.2''$  of Hg, from the lower

part of figure in the left side, is 8.25 gallons per hour. This point is indicated by 'G'. Hence, at  $h = 8000'$ ,  $N = 2200$  and MAP of  $20''$ , the output is 107 HP (79.79 kW) and the fuel flow rate is 8.25 gallons / hr (232.5 N/hr or 52.2 lbs/hr of petrol).

$$\text{Consequently, BSFC} = \frac{232.5}{79.79} = 2.914 \text{ N/kW-hr}$$

$$\text{Or in FPS units, BSFC} = \frac{52.2}{107} = 0.488 \text{ lb/HP-hr}$$

**Answers :**

**At  $h = 8000'$ ,  $N = 2200$  and MAP =  $20''$  of Hg :**

**Output = 79.79 kW = 107 HP and BSFC = 2.914 N/kW-hr = 0.488 lb/BHP-hr**

**Note : Reference 3.4, chapter 6 may be referred to obtain the correction to the output if the ambient temperature is different from that in ISA.**

#### 4.14.5 The propeller

The output of the engine is converted into thrust by the propeller. A typical engine with a two bladed propeller is shown in Fig.4.13. Depending on the engine power and the operating conditions, the propeller may have two to four blades. Special propellers with five or six blades have also been used in practice when required.



Fig.4.13 Typical engine-propeller combination

(Source: www.flickr.com)

The propeller blade, as seen in Fig.4.13, is like a wing with significant amount of twist. Refer Fig.A 2.1.1 for geometric parameters of a wing. The geometry of the propeller is defined by the following features. (a) The variation of the chord, shape and thickness of the airfoil section (also called blade element) over the span of the blade. (b) The angle between the chord of the blade element and the plane of rotation. This is also one of the definitions of the pitch angle ( $\beta$ ).

The pitch angle ( $\beta$ ) varies along the span of the blade for the following reason. Since, the propeller blade moves forward as it rotates, the blade element has a forward velocity of  $V_\infty$  and a circumferential velocity of  $2\pi rn$ ; where 'r' is the radius of the blade element and 'n' is the revolutions per second of the propeller. The blade element experiences a relative wind which is resultant of the forward and circumferential velocities. As 'r' varies from the root to the tip, the blade elements at various spanwise locations of the propeller are subjected to a relative wind which varies significantly, in magnitude and direction, along the span. Further, each blade element being an airfoil, must operate at a moderate angle of attack. These two considerations require that the blade elements along the span of the blade make different angles to the plane of rotation or have different pitch angles ( $\beta$ ). The pitch of the blade is generally the pitch of the blade element at  $r/R = 0.75$ , where R is the radius of the blade.

For other definitions of pitch consult Ref.1.2 and chapter 6 of Ref.3.4. For details of the geometry of propellers refer chapter 6 of Ref.3.4; chapter 16 of Ref.1.5 and Ref.4.8.

#### 4.14.6 Propeller efficiency

Consider that an engine, located in an airplane, is developing certain output indicated as BHP. The propeller attached to this engine produces a thrust T when the airplane moves with a speed  $V_\infty$ . In this situation, the power output called 'Thrust horse power (THP)', is  $(TV_\infty / 1000)$  in kW. The efficiency of the propeller is therefore defined as:

$$\eta_p = \text{THP} / \text{BHP} = TV_\infty / (1000 \times \text{BHP}) \quad (4.110)$$

**Note:** T is in Newton,  $V_\infty$  is in m/s and THP and BHP are measured in kW.

The efficiency of a propeller can be estimated by analysing the flow through a propeller. The momentum theory of propeller is briefly discussed in the next subsection. Subsequent subsections deal with determination of propeller efficiency from experimental results.

#### 4.14.7 Momentum theory of propeller

As the name suggests, this theory is based on the idealization that the thrust produced by the propeller is the result of the increase in momentum imparted to the airstream passing through the propeller. It is assumed that the propeller can be thought of as an actuator disc. This disc is an idealised device which produces a sudden pressure rise in a stream of air passing through it. This pressure rise integrated over the disc gives the thrust developed by the propeller. Figure 4.14 shows the actuator disc and the flow through it. It is assumed that :  
(i) the flow is incompressible and inviscid, (ii) the increase in pressure is constant over the disc (iii) there is no discontinuity in flow velocity across the disc and (iv) in the flow behind the disc, called slipstream, there is no swirl.

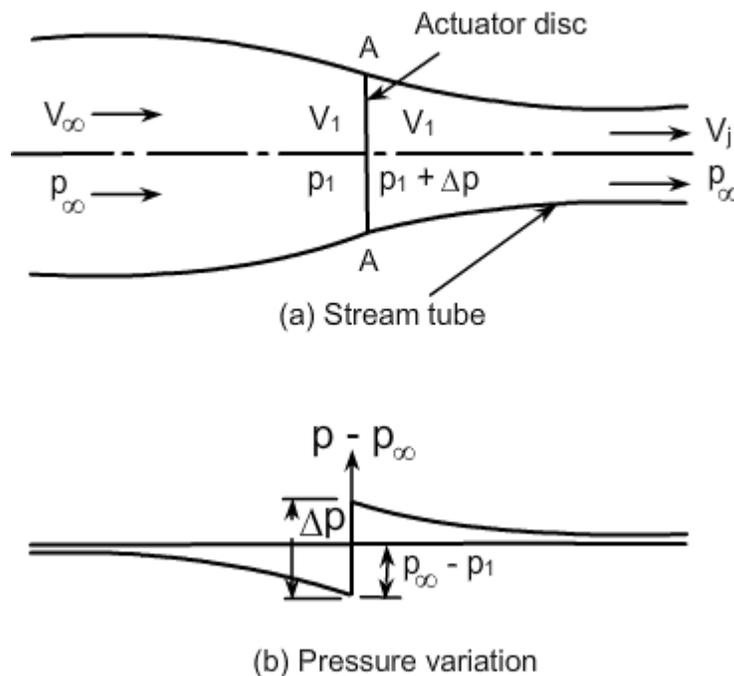


Fig.4.14 Flow through an actuator disc  
(a) Stream tube (b) Pressure variation

In Fig.4.14 the actuator disc is located at plane AA. Far upstream, the velocity is  $V_\infty$  and the pressure  $p_\infty$  is the atmospheric pressure. The velocity  $V_\infty$  equals the forward speed of the airplane on which the propeller is mounted. A stream tube enclosing the disc is also shown in Fig.4.14. As the stream approaches the front face of the disc the fluid velocity reaches a value  $V_1$  at the disc. As the flow is assumed to be inviscid and incompressible, Bernoulli's equation is valid till the front face of the disc and the pressure decreases, to a value  $p_1$ . At the disc, energy is added in the form of increase in pressure by an amount  $\Delta p$  while the velocity remains the same as  $V_1$  through the disc (Fig.4.14a). After the disc the pressure gradually returns to the atmospheric value of  $p_\infty$ . Bernoulli's equation is again valid behind the disc and the fluid velocity increases to a value  $V_j$ . The changes in pressure and velocity are shown in Fig.4.14a.

Applying Bernoulli's equation ahead and behind the disc gives :

$$\text{Total head ahead of disc} = H = p_\infty + \frac{1}{2}\rho V_\infty^2 = p_1 + \frac{1}{2}\rho V_1^2 \quad (4.111)$$

$$\text{Total head behind the disc} = H_1 = (p_1 + \Delta p) + \frac{1}{2}\rho V_1^2 = p_\infty + \frac{1}{2}\rho V_j^2 \quad (4.112)$$

$$\text{Consequently, } \Delta p = H_1 - H = p_\infty + \frac{1}{2}\rho V_j^2 - p_\infty - \frac{1}{2}\rho V_\infty^2 = \frac{1}{2}\rho(V_j^2 - V_\infty^2) \quad (4.113)$$

Since,  $\Delta p$  is the change in pressure over the disc, the thrust acting on the disc is:

$$T = A \Delta p = A \frac{\rho}{2}(V_j^2 - V_\infty^2) \quad (4.114)$$

where,  $A = \text{area of disc} = \frac{\pi}{4}d^2$  ;  $d = \text{diameter of the propeller}$

Alternatively, the thrust produced can also be obtained as the rate of change of momentum of the stream i.e.

$$T = \dot{m}(V_j - V_\infty) \quad (4.115)$$

$$\text{where, } \dot{m} = \text{rate of mass flow through the disc} = \rho A V_1 \quad (4.116)$$

$$\text{Hence, } T = \rho A V_1 (V_j - V_\infty) \quad (4.117)$$

Equating Eqs.(4.114) and (4.117) yields :

$$\rho A V_1 (V_j - V_\infty) = A \frac{\rho}{2} (V_j^2 - V_\infty^2)$$

$$\text{Or } V_1 = \frac{V_j + V_\infty}{2} \quad (4.118)$$

Thus, the momentum theory shows that the velocity at the disc ( $V_1$ ) is the average of  $V_j$  &  $V_\infty$ . In other words, half of the increase in velocity takes place ahead of the disc and the remaining half behind it.

The efficiency of the actuator disc can be obtained by considering the ratio of power output to the power input.

$$\text{The power output} = \text{work done} = T V_\infty = \dot{m} (V_j - V_\infty) V_\infty \quad (4.119)$$

The power input is the energy imparted to the fluid stream. This is the energy of the stream far behind the disc minus the energy of the stream far ahead of the disc. i.e.

$$\text{Power input} = \frac{1}{2} \dot{m} V_j^2 - \frac{1}{2} \dot{m} V_\infty^2 \quad (4.120)$$

Hence, propeller efficiency is:

$$\eta_p = \frac{\text{power output}}{\text{energy input}} = \frac{\dot{m} V_\infty (V_j - V_\infty)}{\frac{\dot{m}}{2} (V_j^2 - V_\infty^2)} = \frac{2 V_\infty}{V_\infty + V_j} = \frac{2}{1 + \frac{V_j}{V_\infty}} \quad (4.121)$$

#### Remarks:

(i) Equation (4.121) gives the propeller efficiency under ideal conditions and represents an upper limit on efficiency obtainable. In practical situations, the efficiency would be lower due to losses associated with (a) profile drag of blades, (b) swirl in slip stream and (c) the pressure at the blade tips being the same ahead and behind the disc.

(ii) For production of thrust,  $V_j$  must be greater than  $V_\infty$ . But for high propeller efficiency  $V_j$  must be only slightly higher than  $V_\infty$ . Hence to get adequate amount

of thrust with high propeller efficiency a large mass of air should be given a small velocity increment.

(iii) Propeller theories like blade element theory, and vortex theory take into account effects of drag of blades, finite span of blade etc. For details of these theories refer to chapter 6 of Ref.3.4.

#### 4.14.8 Parameters describing propeller performance and typical propeller characteristics

As pointed out at the end of the previous subsection, the momentum theory of propeller has limitations. Though the refined theories are helpful in design of propeller blades, the propeller characteristics obtained from the wind tunnel tests are used for estimation of airplane performance. These characteristics are presented in terms of certain parameters. First these parameters are defined and then typical characteristics of propellers are presented. The procedures for

(a) selection of the propeller diameter and

(b) obtaining the propeller efficiency for given  $h$ ,  $v$ , BHP and  $N$ , are given in the next two subsections.

Following Ref.4.8 and Ref.1.5 chapter 16, the propeller performance is expressed in terms of the following coefficients. It may be pointed out that FPS units are used in these references whereas SI units are used here.

$$\text{Advance ratio : } J = V/nd \quad (4.122)$$

$$\text{Power coefficient: } C_P = P/\rho n^3 d^5; P \text{ in Watts} \quad (4.123)$$

$$\text{Thrust coefficient: } C_T = T/\rho n^2 d^4 \quad (4.124)$$

$$\text{Speed power coefficient: } C_s = V (\rho / P n^2)^{1/5} = J / \sqrt[5]{C_P} \quad (4.125)$$

$$\begin{aligned} \text{Propeller efficiency: } \eta_p &= TV / P; P \text{ in Watts} \\ &= J (C_T / C_P) \end{aligned} \quad (4.126)$$

$$\text{Torque coefficient: } C_Q = \frac{Q}{\rho n^2 d^5} \quad (4.127)$$

$$\text{Torque speed coefficient: } Q_s = J / \sqrt{C_Q} = V \sqrt{\rho d^3 / Q} \quad (4.128)$$

where,  $P$  = power in Watts,  $T$  = thrust (N);  $V$  = flight velocity (m/s),  $n$  = rotational speed (rev/s),

$d$  = diameter of propeller (m)

$Q$  = torque (Nm) =  $P / 2\pi n$

In FPS units:

$T$  = thrust (lbs);  $P$  = power (ft lbs/s) = 550 BHP

$V$  = velocity (ft / s), BHP = brake horse power

The performance of a propeller is indicated by thrust coefficient ( $C_T$ ), power coefficient ( $C_P$ ) and efficiency ( $\eta_p$ ). These quantities depend on advance ratio ( $J$ ) and pitch angle ( $\beta$ ). Based on Ref.4.8, the experimental characteristics of a four bladed propeller are presented in Figs.4.15a to d.

Figure 4.15 a presents the variation of  $\eta_p$  vs  $J$  with  $\beta$  as parameter. It is seen that  $\eta_p$  is zero when  $V$  is zero;  $J$  is also zero in this case by virtue of its definition (Eq.4.122). Equation (4.110) also indicates that  $\eta_p$  is zero when  $V$  is zero. This is because even though the engine is working and producing thrust, no useful work is done when  $V$  is zero. This is like a person pressing an immovable wall. He spends muscular energy to push the wall but, the output and hence the efficiency is zero as the wall does not move and no useful work is done.

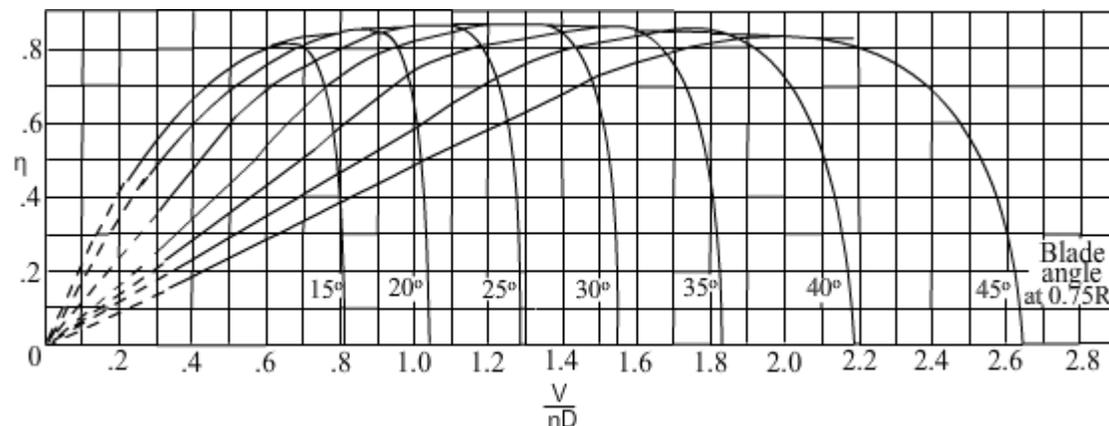


Fig.4.15a Propeller efficiency ( $\eta_p$ ) vs advance ratio ( $J$ ) with pitch angle ( $\beta$ ) as parameter.

For a chosen value of  $\beta$ , the efficiency ( $\eta_p$ ) increases as  $J$  increases. It reaches a maximum for a certain value of  $J$  and then decreases (Fig.4.15a). The

maximum value of  $\eta_p$  is seen to be between 81 to 86%. However, the value of  $J$  at which the maximum of  $\eta_p$  occurs, depends on the pitch angle  $\beta$ . This indicates that for a single pitch or fixed pitch propeller, the efficiency is high (81 to 86%) only over a narrow range of flight speeds (Fig.4.15a). Keeping this behaviour in view, the commercial airplanes use a variable pitch propeller. In such a propeller the entire blade is rotated through a chosen angle during the flight and the pitch of all blade elements changes. Such propellers have high efficiency over a wide range of speeds. However, propellers with variable pitch arrangements are expensive and heavy. Hence, personal airplanes, where cost of the airplane is a very important consideration, employ a fixed pitch propeller. As a compromise, in some designs, propellers with two or three pitch settings are employed.

Figure 4.15b presents the variation of power coefficient ( $C_p$ ) vs  $J$  with  $\beta$  and  $C_T$  as parameters. This chart is useful to obtain  $\eta_p$  for given values of altitude, velocity, RPM and BHP (see subsection 4.14.10).

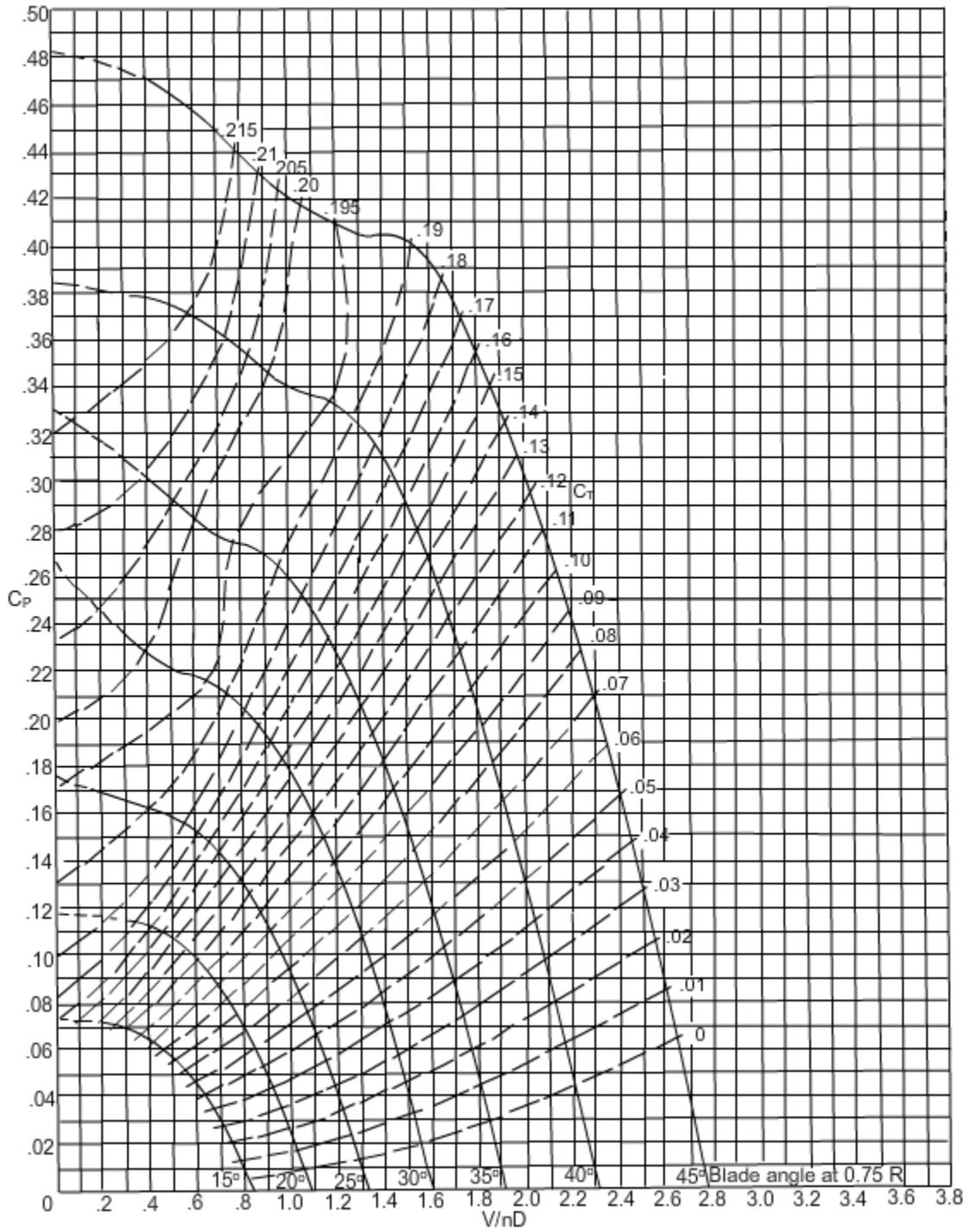


Fig.4.15b Power coefficient ( $C_p$ ) vs advance ratio ( $J$ ) with pitch angle ( $\beta$ ) and thrust coefficient ( $C_T$ ) as parameters.

Figure 4.15c presents the variations of  $C_s$  vs  $J$  and  $C_s$  vs  $\eta_p$  with  $\beta$  as parameter.

This figure is designated as 'Design chart' and is used for selection of the diameter of the propeller. A brief explanatory note on this topic is as follows.

Using definitions of  $J$  and  $C_p$ , the parameter  $C_s$ , defined below, is obtained. It is observed that this parameter does not involve the diameter ( $d$ ) of the propeller.

$$C_s = \frac{J}{C_p^{1/5}} = V (\rho / P n^2)^{1/5} \quad (4.129)$$

It is also observed that the parameter  $C_s$  depends on  $V$ ,  $\rho$ ,  $P$  and  $N$ .

Consequently, this parameter can be evaluated when the power output ( $P$ ), engine RPM( $N$ ) and flight condition viz.  $V$  and  $h$  are specified.

The design problem involves obtaining the value of  $J$  which would give the maximum value of  $\eta_p$  for a specified value of  $C_s$ . This is arrived at in the following manner.

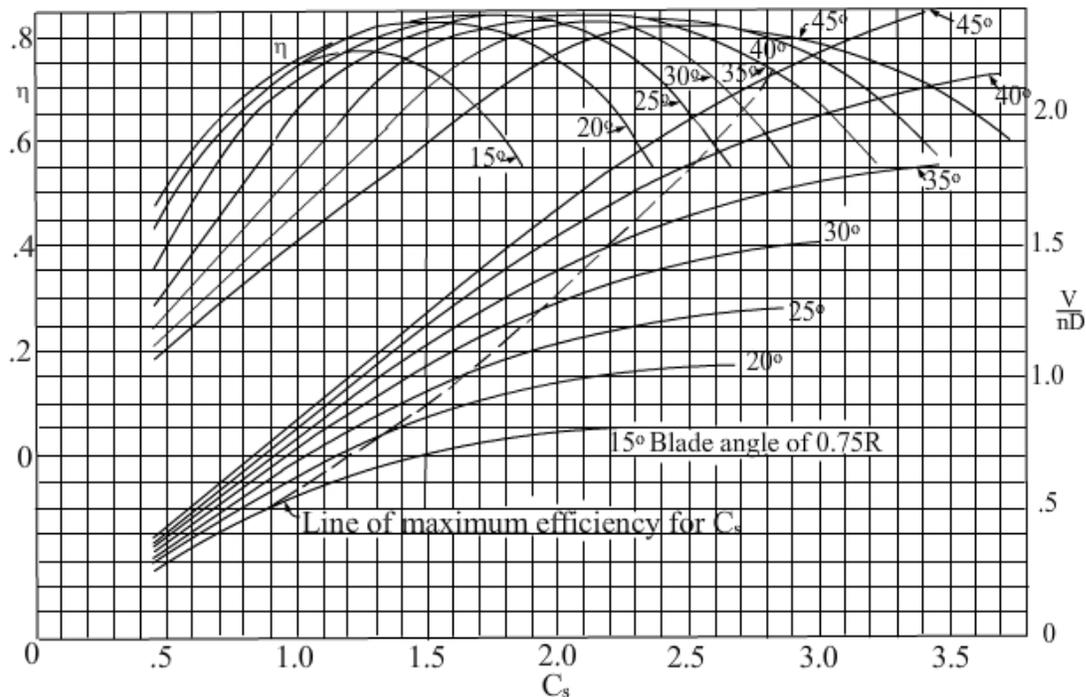


Fig.4.15c Design chart

Using the data in Figs.4.15b and a , the values of  $C_s$  can be obtained for constant values of  $J$  or  $\beta$  . For example, for  $\beta = 20^\circ$  the values given in Table 4.2 are obtained.

J	$C_p$ From Fig.4.15b	$C_s$ From Eq.(4.129)	$\eta_p$ From Fig.4.15a
0	0.119	0	0
0.2	0.118	0.307	0.34
0.4	0.114	0.618	0.6
0.5	0.108	0.780	0.685
0.6	0.100	0.951	0.755
0.7	0.0862	1.143	0.810
0.8	0.070	1.362	0.840
0.9	0.050	1.639	0.83
1.0	0.024	2.108	0.64

Table 4.2 Variation of  $C_s$  with  $J$  for  $\beta = 20^\circ$

Similar calculations at  $\beta = 15^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ$  and  $45^\circ$  yield additional values. From these values the curves for  $C_s$  vs  $\eta_p$  and  $C_s$  vs  $J$  at different values of  $\beta$  can be plotted. These are shown in the upper and lower parts of Fig.4.15c. Based on these plots, the dotted line in the lower part of Fig.4.15c gives the values of  $J$  and  $\beta$  which would give maximum  $\eta_p$  . This line is designated as 'Line of maximum efficiency for  $C_s$  '. For example, corresponding to a value of  $C_s = 2.0$ , the dotted line gives  $J = 1.32$  and  $\beta = 31^\circ$  . The upper part of the Fig.4.15c gives  $\eta_p = 85\%$  for the chosen value of  $C_s = 2.0$ .

From the value of  $J$ , the propeller diameter is obtained as  $d = V/(nJ)$  ; note that the values of  $V$  and  $n$  are already known. Subsection 4.14.9 gives additional

details and example 4.19 illustrates the procedure to select the propeller diameter.

Figure 4.15d presents the variation of thrust coefficient ( $C_T$ ) vs  $J$  with  $\beta$  as parameter. It is observed that when  $J$  is zero,  $C_T$  is not zero as the propeller produces thrust, even when 'V' is zero. The curves in Fig.4.15d are useful to estimate the thrust developed by the propeller especially during the take-off flight.

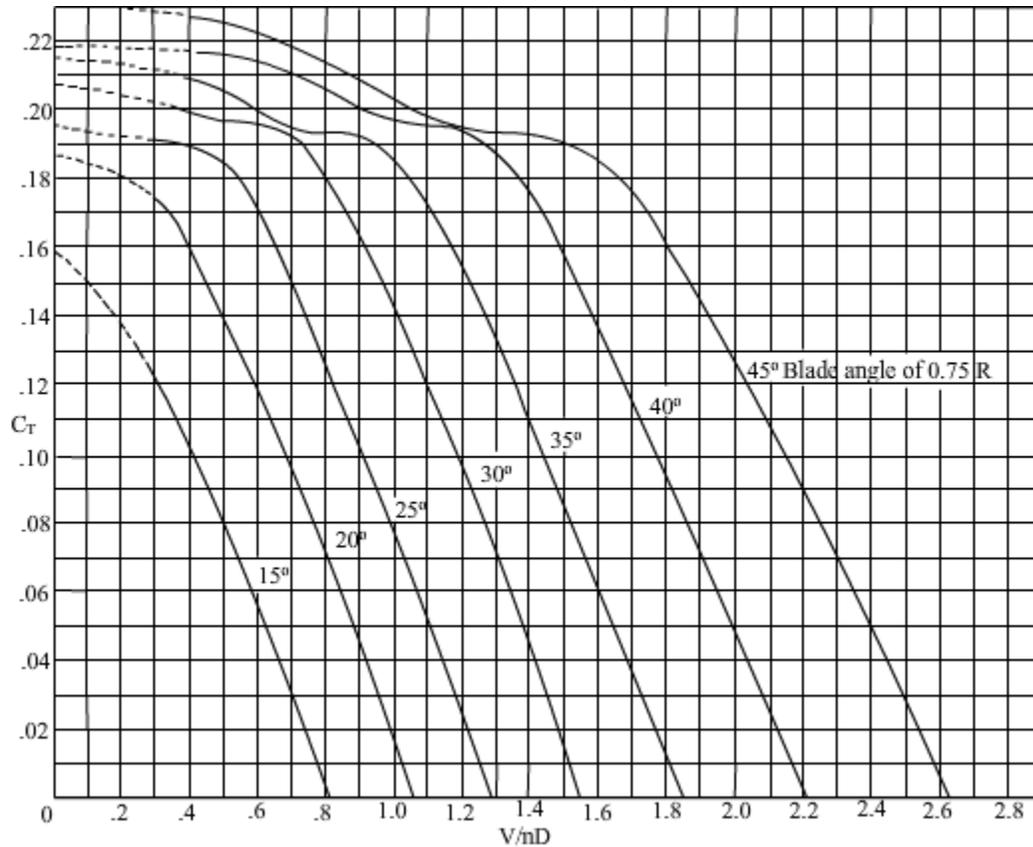


Fig.4.15d Thrust coefficient ( $C_T$ ) vs advance ratio ( $J$ ) with pitch angle  $\beta$  as parameter.

Fig.4.15 Typical characteristics of a four bladed propeller  
(Adapted from Ref.4.8)

**Remark :**

Reference 4.8 also contains information on propellers with two and three blades. Reference 1.5, chapter 16 contains information on six bladed propellers. Additional information can be obtained from Ref.4.9 which is cited in chapter 17 of Ref.1.20.