Chapter 9

Cross-checks on design of tail surfaces - 3

Lecture 36

Topics

9.4 Cross-checks for lateral stability and control

9.4.1 Selection of dihedral angle

9.4.2 Aileron design

Example 9.1 (part I)

9.4 Cross-checks for lateral stability and control

From chapter 6 of Ref.3.1 the following points are noted.

(i) For trim about x-axis of the airplane :

$$C_l' = 0 \tag{9.39}$$

$$C_l' = \text{Rolling moment coefficient} = \frac{L'}{\frac{1}{2}\rho V^2 Sb} \tag{9.40}$$

L' = Rolling moment about c.g. of airplane, the quantities, $\frac{1}{2}\rho V^2$, S and b have been defined earlier in (Eq.9.35)

(ii) In steady level flight the bank angle (Φ) is zero. When the airplane is given a bank angle (Φ) gently, the aerodynamic field remains symmetric about the x-axis and no restoring moment is produced. Thus, the airplane is neutrally stable about x-axis. However, when the airplane is banked the component on weight W sin Φ causes airplane to sideslip(β). If airplane has a rate of roll 'p' then it develops a sideslip due to adverse yaw (subsection 5.8.1 of Ref.3.1 be referred for details). When an airplane has sideslip, it develops both rolling moment and yawing moment. The rolling moment due to sideslip, which tends to restore the airplane to Φ= 0, is a stabilizing effect. Rolling moment due to sideslip is called dihedral effect.

For a stabilizing dihedral effect

$$C_{l\beta}' = \frac{dC_l'}{d\beta} \text{ should be negative} \tag{9.41}$$

(iii) The quantity $C'_{l\beta}$ can be written as :

$$C'_{l\beta} = (C'_{l\beta})_w + (C'_{l\beta})_{f,n,p} + (C'_{l\beta})_{vt} \tag{9.42}$$

where, the suffices w, f, n, p and vt denote as in Eq.(9.37).

Sometimes the contributions of the wing and the body to ($C'_{l\beta}$) are considered together and referred to as $(C'_{l\beta})_{WB}$, where suffix WB stands for "Wing body combination".

Chapter 6 of Ref.3.1 presents methods to estimate contributions of wing (due sweep angle and dihedral angle), fuselage and vertical tail to $C'_{l\beta}$. The contribution of nacelle and power are generally neglected. Appendix 'C' of Ref.3.1 presents the calculation of $C'_{l\beta}$ for a jet airplane.

Using these methods and the parameters of the airplane under design, obtained in chapters 2 to 8, the contributions of fuselage, v.tail and wing sweep to $C'_{l\beta}$ can be estimated.

9.4.1 Selection of dihedral angle

As noted earlier, the levels of longitudinal and directional static stability ($C_{m\alpha}$ and $C_{n\beta}$) can be adjusted by changing the areas of the horizontal tail (S_t) and the vertical tail (S_v) respectively. The level of $C'_{l\beta}$, can be adjusted by choosing an appropriate dihedral angle. To arrive at the dihedral angle needed for an airplane, the contributions due to wing sweep, fuselage, power plant and vertical tail are first calculated. Then, the difference between the sum of these contributions and the desirable level of $C'_{l\beta}$ is provided by choosing an appropriate dihedral angle.

Reference 4.7, chapter 9 and Ref.1.18,chapter 16, provide a rough guideline as:

$$(C'_{l\beta})_{desirable} = -\frac{1}{2}(C_{n\beta}/2) \tag{9.43}$$

However, the data on seven airplanes given in Appendix 'B' of Ref. 1.22 indicates that Eq.(9.43) may be approximately valid for military airplanes. For other airplanes, $C'_{l\beta}$ could be equal to or higher than $C_{n\beta}$. Actual values of $C'_{l\beta}$ and

$C_{n\beta}$ are arrived at after carrying out the lateral dynamic stability analysis (refer to chapter 9 of Ref.3.1).

9.4.2 Aileron design

Section 6.10 and the subsections 6.10.1 to 6.10.3 of Ref.3.1 discuss: (a) aileron, differential aileron and spoiler aileron, (b) rolling moment due to aileron, (c) damping moment due to wing and (d) rate of roll achieved. The rate of roll is expressed in non-dimensional form as (pb/2V); where p is the desired rate of roll. Reference 4.7 chapter 9 gives:

$$\frac{pb}{2V} = 0.07 \text{ for cargo airplanes}$$

$$= 0.09 \text{ for military airplane}$$

However, the current practice, according to Ref.1.24, chapter 12 is to calculate the time required to achieve a desired angle of bank in a prescribed time duration.

Remark:

Section 16.10, of Ref.1.18 deals with additional topics like handling qualities, lateral control departure parameter (LCDP) and spin recovery parameter. These are suggested as topics for self-study.

Example 9.1 (part I)

For the sixty seater airplane considered in examples 3.1, 4.19, 5.1, 6.1, 6.2,6.3,8.1 and 8.3, obtain the following.

(a) Location of stick-free neutral point in cruise.

(b) Tail setting.

(c) Elevator angle required for trim under landing configuration.

Assume $W_{land} = W_{TO}$

Solution :

For the sake of convenience, part I of example 9.1 deals with contributions of wing and fuselage to the longitudinal static stability. The other contributions and analysis of the stability and control are carried out in part II.

I) The following data are obtained from various examples already worked out.

A) From example 3.1, the gross weight of the airplane is : 21280 kgf or 208757 N.

B) From example 5.1, the geometric/aerodynamic parameters of the wing and the flight condition are :

$S = 58.48\ m^2$, $b = 26.49$ m, Aspect ratio = A = 12,

Mean aerodynamic chord = $\bar{c}_w = 2.295$ m, $c_r = 2.636$ m, $c_t = 1.318$ m.

Root chord (c_{re}) of equivalent trapazoidal wing (ETW) = 3.1 m

Taper ratio (λ_e)of ETW = 1.318/3.1 = 0.425

Quarter chord sweep of outboard wing $(\Lambda_{\frac{1}{4}})_{outboard} = 3.9^o$

Wing setting (i_w) = 2.9^o

From Fig.5.5e : C_{mac} of airfoil = -0.07, angle of zerolift = - 1.8^o

Aerodynamic centre of wing at 0.25 $\bar{c}_w$

Cruise flight conditions:

V_{cr} = 500 kmph = 138.9 m/s, h_{cr} = 4.5 km

$\rho_{cr} = 0.7768\ kg/m^3$, Speed of sound at h_{cr} = 322.57 m/s

$M_{cr} = 138.9/322.57 = 0.431;\ \beta = \sqrt{1-M_{cr}^2} = \sqrt{1-0.431^2} = 0.902$

$(C_{L\alpha})_W$ at M_{cr} is 5.793 rad^{-1} or 0.1011deg^{-1}

$$C_{Lcruise} = \frac{2\times 208757}{0.7768\times 138.9^2\times 58.48} = 0.476$$

(C) Fuselage (From example 6.1):

l_f = 25.07 m

Width and height of fuselage at maximum cross section = 2.88 m both

The width of fuselage at different locations along the length are given in Tables E 9.1 a & E 9.1 b.

(D) Parameters of horizontal tail (From example 6.2):

Area (S_t) = 11.11 m^2 , Span (b_t) = 7.45 m

Root chord (c_{rt}) = 1.86 m, Tip chord (c_{tt}) = 1.12 m

$\lambda_t = 0.6$, Mean aerodynamic chord of h.tail = $\bar{c}_t$ = 1.86 m

Aspect ratio (A_t) = 5, $\Lambda_{l.e.}$ = 11.24°

Distance between aerodynamic centres of wing and h.tail = l_t = 13.31 m

$S_{elevator}$ / S_t = 0.35 ; Elevator with unshielded horn.

Sweep of half chord of line h.tail $(\Lambda_{\frac{1}{2}})_t$ = 5.64°, $\tan\left(\Lambda_{\frac{1}{2}}\right)_t$ = 0.0987

From Ref.5.6, section 3.2, the slope of lift curve $(C_{L\alpha t})$ at M_{cr} is :

$$C_{L\alpha t} = \frac{2\pi A}{2+\sqrt{4+\frac{A^2\beta^2}{\eta^2}\left[1+\frac{\tan^2\Lambda_{\frac{1}{2}}}{\beta^2}\right]}}$$

$$C_{L\alpha t} = \frac{2\pi\times 5}{2+\sqrt{4+\frac{5^2\times 0.902^2}{1}\left(1+\frac{0.0987^2}{0.902^2}\right)}} = 4.515\ \text{rad}^{-1} = 0.0788\ \text{deg}^{-1}$$

(E) Other parameters

$\frac{d\varepsilon}{d\alpha}$:

The value of $l_t/\bar{c}_W$ is 13.31/2.295 = 5.8

This value is rather large. Hence, the following simpler formula is used to calculate $d\varepsilon/d\alpha$ i.e.

$$\frac{d\varepsilon}{d\alpha} = \frac{2C_{L\alpha W}}{\pi A_W} = \frac{2\times 5.703}{\pi\times 12} = 0.307$$

Slope of lift curve of airplane ($C_{L\alpha}$) :

This quantity is given by (subsection 2.7.1 of Ref.3.1):

$$C_{L\alpha} = C_{L\alpha W} + \eta\frac{S_t}{S}C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)$$

$$\eta = \text{tail efficiency} = \left(\frac{1}{2}\rho V_t^2\right)/\left(\frac{1}{2}\rho V_\infty^2\right)$$

The value of η is influenced by : (a) the wake of wing (b) the propeller slipstream and (c) the boundary layer on fuselage. In the present case, a T-tail configuration is used. From Fig. E8.2 it is evident that the tail is not affected by any of the three aforesaid factors. Hence, η is taken as 1.

Consequently,

$$C_{L\alpha} = 5.793 + 1 \times \frac{11.11}{58.48} \times 4.915(1-0.307)$$

$$= 6.387 \text{ rad}^{-1} = 0.1115 \text{ deg}^{-1}$$

(II) Contribution of wing to C_{mo} and $C_{m\alpha}$

$$C_{mow} = C_{mac\,w} + C_{Low}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$

$$C_{Low} = C_{L\alpha W}(i_W - \alpha_{oLW}) = 5.793\frac{(2.9+1.8)}{57.3} = 0.475$$

$$C_{mow} = -0.07 + 0.475\left(\frac{x_{cg}}{\bar{c}} - 0.25\right)$$

$$\varepsilon_0 = \frac{d\varepsilon}{d\alpha}(i_W - \alpha_{oLW}) = 0.307\{2.9-(-1.8)\} = 1.440$$

$$C_{m\alpha w} = C_{L\alpha W}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) = 5.723\left(\frac{x_g}{\bar{c}} - 0.25\right)$$

(III) Contribution of fuselage

The contribution is obtained using the method outline in section 2.5, and sub - sections 2.5.1 to 2.5.4 of Ref.3.1 and illustrated in example 2.4 of the same reference. This contribtuion is expressed as :

$$C_{mf} = C_{mof} + C_{m\alpha f}\,\alpha$$

$$C_{mof} = \frac{k_2 - k_1}{36.5S\bar{c}}\sum_{x=0}^{l_f} w_f^2(\alpha_{0Lf} + i_f)\Delta x \quad (9.44)$$

$$C_{m\alpha f} = \frac{k_2 - k_1}{36.5S\bar{c}}\sum_{x=0}^{l_f} w_f^2 \frac{d\varepsilon}{d\alpha}\Delta x \quad (9.45)$$

Estimation of C_{mof}

To obtain C_{mof} the fuselage is subdivided into nine equal to sections. These are shown in Fig E9.1a.

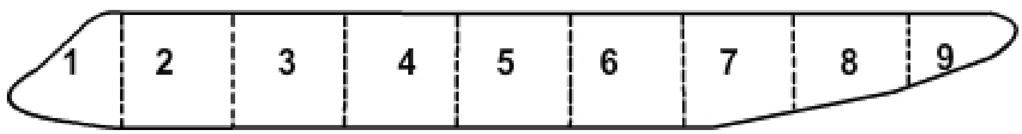

Fig.E9.1a Subdivisions of fuselage for calculation of C_{mof}

In Eq.(9.44), Δx is the length of the subdivision of the fuselage and w_f is the width of the fuselage in the middle of the subdivision (Fig.E9.1b). Table E9.1a present Δx and w_f at various stations along the fuselage.

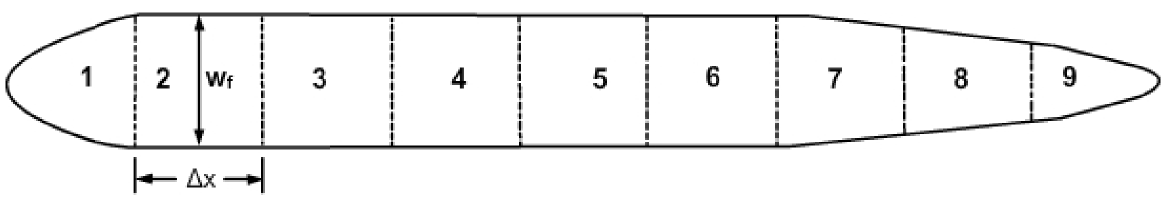

Fig.E9.1b Plan view of fuselage of airplane under design

The quantity α_{oLf} equals $i_w + \alpha_{oLW}$. In the present case α_{oLf} = 2.9 - 1.8 = 1.1°

The quantity i_f is the incidence angle of the camber line with respect to FRL. It is indicated in Fig.E9.1c.

Fig.E9.1c Fuselage camber

It is noted that i_f equals :(a) -11.1^o in the middle of the first subdivision, (b) zero in subdivisions 2 to 6 and (c) -6.5^o in the middle of subdivisions 7,8 and 9.

The quantity (k_2-k_1) depends on the fineness ratio (A_f) of the fuselage

$$A_f = l_f / d_e \,;\; d_e = \sqrt{A_{fmax} / (\pi/4)}$$

A_{fmax} = Area of maximum crosssection of fuselage.

Station	Δx (m)	w_f (m) in the middle of Δx	$\alpha_{oLf} + i_f$	$w_f^2 (\alpha_{oLf} + i_f) \Delta x$
1	2.786	1.73	$1.1-11.1=-10^o$	-83.38
2	2.786	2.88	1.1^o	25.42
3	2.786	2.88	1.1^o	25.42
4	2.786	2.88	1.1^o	25.42
5	2.786	2.88	1.1^o	25.42
6	2.786	2.88	1.1^o	25.42
7	2.786	2.74	$1.1-6.5=-5.4^o$	-112.95
8	2.786	2.01	-5.4	-60.78
9	2.786	1.23	-5.4^o	-22.76
				$\sum$ = -152.77

Table E 9.1a Estimation of C_{mof}

Since, the fuselage is circular in cross section d_e = 2.88 m

Consequently, l_f/d_e = 25.07/2.88 = 8.70

From Fig.2.19 of Ref.3.1, (k_2-k_1) = 0.92

Substituting various values in Eq.(9.44) gives :

$$C_{mof} = \frac{0.92(-152.77)}{36.5 \times 58.48 \times 2.295} = -0.029$$

Estimation of $C_{m\alpha f}$:

The esimation of $C_{m\alpha f}$ is carried out using Eq(9.45). This equation involves $(d\varepsilon/d\alpha)$ which depends on the local value of upwash / downwash along the fuselage. An empirical procedure, generally regarded adequate is as follows.

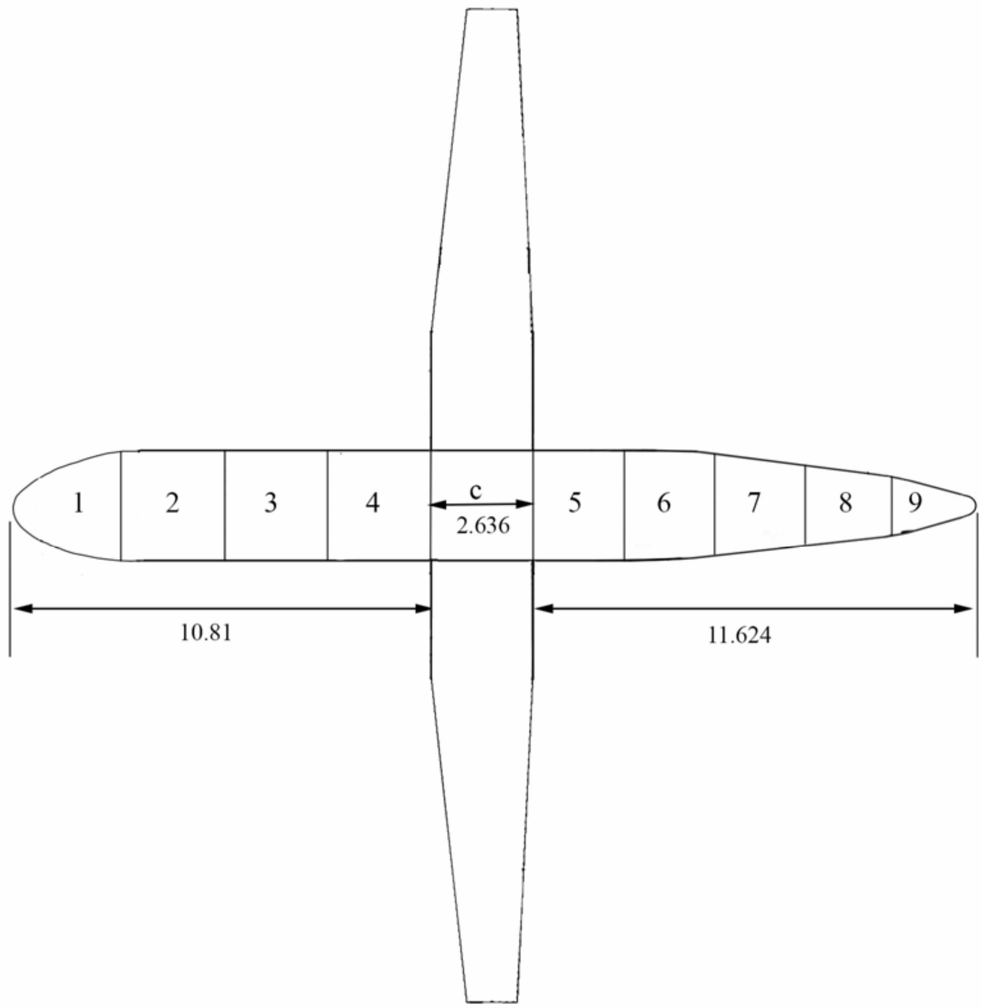

Fig. E9.1d Subdivisions of fuselage for calculating $C_{m\alpha f}$

The fuselage is subdivided as shown in Fig.E9.1d. The portion of the fuselage ahead of the root chord is subdivided into four equi-distant portions each of length 2.705 m (10.81/4) These subdivisions are denoted as 1,2,3 and 4 in Fig.E9.1d. The portion of fuselage aft of the root chord of wing is divided into five equi-distant sections each of length 2.3248 m (11.624/5) and denoted as 5,6,7,8 and 9. The root chord (Fig.E9.1d) has length c = 2.636 m. Thus, the total fuselage length of 25.07 m is divided as: (2.7025 x 4 + 2.636 + 2.3248 x 5). The length l_h in Eq.(9.46) is the distance of the aerodynamic centre of horizontal tail behind the trailing edge of the root chord of wing. It is 11.484 m. Here, the calculations of the quantities needed to obtain $C_{m\alpha f}$ are shown in Table E9.2b. The second column of this table shows Δx which is the length of each subdivision of the fuselage. The third column gives the width of the fuselage in the middle of the subdivision. The fourth column gives the distance x or x_i. For subdivision 4, nearest to the leading edge of the rootchord, this distance is taken equal to the length of this section ahead of the root chord viz. 2.7025 m. For subdivisions 3,2 and 1 of this column, the distance is x_i the distance of the middle point of this subdivision from the leading edge of the root chord. For sub divisions 5 to 9 of this column, the distance x_i is at the distance of the middle point of the subdivision from the trailing edge of the root chord. The fifth column shows $x/\bar{c}$ for other rows. The sixth column is $d\varepsilon/d\alpha$ - the upwash / downwash at the subdivision. For row four the upwash value is based on curve 'b' in Fig.2.23 of Ref.3.1. For rows 3,2 and 1 the upwash value is based on curve 'a' of Fig.2.23 of Ref.3.1.

The rows 5 to 9 of this column show the downwash for the corresponding subdivisions. The value of $d\varepsilon/d\alpha$ at these subdivisions, behind the root chord, is given by:

$$\frac{d\varepsilon}{d\alpha} = \frac{x_i}{l_h}\left[1-\left(\frac{d\varepsilon}{d\alpha}\right)_{tail}\right] \tag{9.46}$$

It may be pointed out that $d\varepsilon/d\alpha$, at tail is 0.307 for this airplane. Using values of x_i and l_h the values of downwash are tabulated in column 6 of table E9.1b. The last column shows values of $w_f^2\left(d\varepsilon/d\alpha\right)\Delta x$. The sum, $\sum w_f^2\left(d\varepsilon/d\alpha\right)\Delta x$ is 115.77.

In section 2.5.4 of Ref.3.1, it is pointed out that the values of $\left(d\varepsilon/d\alpha\right)$ in Fig.2.23 of Ref.3.1 and that by Eq.(9.46), are for $C_{L\alpha W}$ = 0.0785 deg^{-1}.

Since, $C_{L\alpha W}$ is 0.1011 / degree^{-1}, the actual value of the sum is :

115.77 x (0.1011 / 0.0785) = 149.1

Finally,

$$C_{m\alpha f} = \frac{k_2-k_1}{36.5S\bar{c}}\sum_{x=0}^{l_f} w_f^2 \frac{d\varepsilon}{d\alpha}\Delta x = \frac{0.92\times149.1\times57.3}{36.5\times58.48\times2.295} = 1.604 \text{ rad}^{-1}$$

Station	Δx m	w_f	x_i or x	($x_i$or x) /c	$\frac{d\varepsilon}{d\alpha}$	$w_f^2 \frac{d\varepsilon}{d\alpha} \Delta x$
1	2.7025	2.30	9.459	3.588	1.043	14.91
2	2.7025	2.88	6.756	2.563	1.043	23.38
3	2.7025	2.88	4.054	1.538	1.087	24.36
4	2.7025	2.88	2.7025	1.025	1.74	39.00
5	2.3248	2.88	1.162	0.441	0.070	1.35
6	2.3248	2.88	3.487	1.323	0.210	4.049
7	2.3248	2.38	5.812	2.205	0.351	4.623
8	2.3248	1.51	8.137	3.087	0.491	2.602
9	2.3248	1.01	10.462	3.969	0.631	1.496
						$\sum$115.77

Table E 9.1.b Estimation of $C_{m\alpha f}$