Chapter 9

Cross-checks on design of tail surfaces

(Lectures 34 to 37)

Keywords : **Cross-checks for design of tail surfaces; location of stick-free neutral point ; elevator required for trim at C_{Lmax} near ground and nose wheel lift-off ; desirable level of $C_{n\beta}$; rudder control in crosswind, one engine inoperative cases and spin recovery ; desirable value of $C'_{l\beta}$; choice of dihedral angle; aileron design.**

Topics

Chapter 9

Cross-checks on design of tail surfaces - 1

Lecture 34

Topics

9.1. Introduction

The final design of horizontal and vertical tails requires calculation of the dynamic stability and response of the airplane in cases like: (a) different flight conditions (b) different weights and c.g. locations and (c) possible variations in configuration.

Appendix 'C' of Ref.3.1 presents the procedure to calculate the stability derivatives and solving the characteristic equations for longitudinal and lateral stability. This is an elaborate task especially for the student design projects. On the other hand, the design bureau in an airplane factory has computer packages to evaluate the stability derivatives and to carry out stability and response calculations for a given configuration of the airplane.

However, the approach in the present course material is to arrive at a configuration, without the use of packages, which is reasonably close to the actual airplane. Keeping this in view, the topics dealt with in this chapter are based on the following two observations.

(i) For conventional subsonic airplanes, if the airplane has adequate level of static stability, then it would have reasonable dynamic stability.

(ii) If the areas of the control surfaces are adequate to trim the airplane (i.e. bring the moments about the three airplane axes to zero) in certain critical conditions, then the airplane would have reasonable level of controllability.

Following this approach and taking guidance from Ref.1.5, chapter 1-9 and Ref.1.24, chapters 6 and 12, the configuration of the airplane arrived at so far (i.e. through chapters 2 to 8), is checked for the following cases.

(I) Longitudinal static stability and control

(a) At the rear-most location of c.g., the airplane should be at least neutrally stable for stick-free condition.

(b) At the foremost c.g. location, the elevator must be able to provide control (i.e. trim), at C_{Lmax} in landing configuration.

(c) There should be adequate control for nose wheel lift-off at $V = 0.85\ V_{T0}$

(II) Directional static stability and control

(a) The vertical tail should provide desirable level of directional stability.

(b) Rudder should be powerful enough to provide directional control in (a) cross wind take-off and landing, (b) one engine inoperative condition for multi-engined airplanes, (c) adverse yaw during roll and (d) spin recovery.

(III) Adequacy of dihedral effect and aileron area

(a) The dihedral angle of wing should be such as to give adequate level of dihedral effect.

(b) The aileron area should give desired rate of roll.

Remarks:

i) As menioned in section 6.3 the configurations of h.tail and v.tail have been decided. Also the parameters like aspect ratio, taper ratio, sweep and airfoil section have been tentatively decided based on the data on similar airplanes. In this chapter, the areas of h.tail, v.tail and control surfaces are cross-checked in the light of the aforesaid criteria.

ii) It is assumed that the reader has already undergone a course in airplane stability and control. Reference 3.1; Ref.1.18, chapter 16 and Ref.1.24, chapters 6 and 12 may be consulted to revise the background.

9.2. Cross-checks for longitudinal static stability and control

The following equations are reproduced from Ref.3.1, chapters 2 and 3. Standard notations, are used.

$$C_{mcg} = (C_{mcg})_w + (C_{mcg})_f + (C_{mcg})_n + (C_{mcg})_p + (C_{mcg})_t \quad (9.1)$$

$$= C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e \quad (9.2)$$

$$C_{m\alpha} = (C_{m\alpha})_w + (C_{m\alpha})_f + (C_{m\alpha})_n + (C_{m\alpha})_p + (C_{m\alpha})_t \quad (9.3)$$

$$C_{mo} = (C_{mo})_w + (C_{mo})_{f,n,p} + (C_{mo})_t \quad (9.4)$$

It may be noted that,

C_{mcg} = pitching moment coefficient about c.g. of airplane.

α = angle of attack of airplane

δ_e = deflection of elevator

C_{mo} = Pitching moment coefficient at $\alpha = 0$

$$C_{m\alpha} = \left(dC_{mcg}/d\alpha\right)$$

$$C_{m\delta e} = \left(dC_{mcg}/d\delta_e\right)$$

The suffixes w, f, n, p and t refer to the contribution due to wing, fuselage, nacelle, power and h.tail respectively.

The wing contribution is :

$$C_{mcgw} = C_{macw} + C_{Lw}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \quad (9.5)$$

$$C_{Lw} = C_{L\alpha w}(\alpha + i_w - \alpha_{0Lw}) \quad (9.6)$$

$$= C_{L0w} + C_{L\alpha w}\,\alpha \;;\; C_{L0w} = C_{L\alpha w}(i_w - \alpha_{0Lw}) \quad (9.7)$$

$$\left(C_{mcg}\right)_w = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{L\alpha w}\,\alpha\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \quad (9.8)$$

$$C_{m0w} = C_{macw} + C_{L0w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \quad (9.9)$$

$$(C_{m\alpha})_w = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) \quad (9.10)$$

Note :

C_{LW} = lift coefficient of wing

C_{macw} = pitching moment coefficient of wing

$C_{L\alpha w} = (dC_L \,/\, d\alpha)_w$

i_w, α_{oLW} = wing setting(or incidence) and angle of zero lift respectively

x_{cg}, x_{ac} = location of c.g. of airplane and aerodynamic centre of wing respectively from the leading edge of mean aerodynamic chord of wing

$\bar{c}$ = mean aerodynamic chord of wing

The contribution of h.tail is :

$$C_{mcgt} = -V_H \,\eta\, C_{Lt} \tag{9.11}$$

$$C_{Lt} = C_{L\alpha t}\,\alpha_t + C_{L\delta e}\,\delta_e + C_{L\delta t}\,\delta_t \tag{9.12}$$

$$\alpha_t = \alpha - \varepsilon + i_t = \alpha_w - i_w - \varepsilon + i_t \tag{9.13}$$

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha;\ \varepsilon_0 = \frac{d\varepsilon}{d\alpha}(i_w - \alpha_{0Lw}) \tag{9.14}$$

$$C_{Lt} = C_{L\alpha t}\left\{i_t - \varepsilon_0 + \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right\} + C_{L\delta e}\,\delta_e + C_{L\delta t}\,\delta_t \tag{9.15}$$

$$C_{mcgt} = C_{mot} - V_H \,\eta\, C_{L\alpha t}\left[\alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \tau\,\delta_e\right] \tag{9.16a}$$

$$C_{mot} = -V_H \eta\, C_{L\alpha t}[i_t - \varepsilon_0 + \tau_{tab}\,\delta_t] \tag{9.16b}$$

$$\tau = \frac{\partial C_{Lt}}{\partial \delta_e} \Big/ \frac{\partial C_{Lt}}{\partial \alpha};\ \tau_{tab} = \frac{\partial C_{Lt}}{\partial \delta_t} \Big/ \frac{\partial C_{Lt}}{\partial \alpha} \tag{9.17}$$

$$(C_{m\alpha t})_{stick-fixed} = -V_H \,\eta\, C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{9.18}$$

$$(C_{m\alpha t})_{stick-free} = C'_{m\alpha t} = -V_H \,\eta\, C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \tau\frac{C_{h\alpha t}}{C_{h\delta e}}\right) \tag{9.19}$$

Note :

$V_H = C_{ht} = l_{ht}S_{ht} \,/\, \bar{c}_w S_w$ tail volume ratio of h.tail

$$\eta = \frac{1}{2}\rho V_t^2 \Big/ \frac{1}{2}\rho V_\infty^2 ;$$

V_∞ = free stream velocity or flight speed;

V_t = velocity at h.tail

ρ = atmospheric density (refer subsection 2.4.3 of Ref.3.1 for reasons as to why V_t is not the same as V_∞)

$\alpha_t, \delta_e, \delta_t$ = angle of attack of h.tail, elevator deflection and tab deflection respectively.

i_t, ε = tail setting (or incidence) and down wash due to wing at tail (refer subsection 2.4.2 of Ref.3.1)

$C_{L\alpha t} = (dC_L / d\alpha)_t$, $C_{L\delta e} = (dC_L / d\delta_e)_t$

The quantities ε_0 , τ & τ_{tab} are defined in equations above.

The contributions of fuselage, nacelle and power are expressed as :

$$(C_m)_{f,n,p} = (C_{mo})_{f,n,p} + (C_{m\alpha})_{f,n,p}\,\alpha \qquad (9.20)$$

Substituting expressions for various quantities in Eqs.(9.1), (9.2) and (9.3) and (9.4) yields :

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e \qquad (9.21)$$

$$C_{m0} = C_{macw} + C_{L0W}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m0})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\{i_t - \varepsilon_0 + \tau_{tab}\,\delta_t\} \qquad (9.22)$$

$$C_{m\delta e} = -V_H\,\eta\,C_{L\alpha t}\,\tau \qquad (9.23)$$

$$(C_{m\alpha})_{\text{stick fixed}} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (9.24)$$

$$(C_{m\alpha})_{\text{stick-free}} = C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \tau\frac{C_{h\alpha t}}{C_{h\delta e}}\right) \qquad (9.25)$$

Note :

C_h = Hinge moment coefficient

$$C_{h\alpha t} = \partial C_h / \partial \alpha_t,\; C_{h\delta e} = \partial C_h / \partial \delta_e \qquad (9.26)$$

9.2.1 Neutral point

It is known that the c.g. of the airplane moves during the flight (chapter 8). Further, the contribution of wing to $C_{m\alpha}$ depends sensitively on the location of the c.g. as it is proportional to $\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$. When the c.g. moves aft, x_{cg} increases and the wing contribution becomes less and less negative or more and more positive. There is a c.g. location at which $(C_{m\alpha})_{stick\text{-}fixed}$ becomes zero. This location of c.g. is called the stick-fixed neutral point. In this case, the airplane is neutrally stable. The location of the neutral point can be obtained by putting $C_{m\alpha} = 0$.

From Eqs.(9.24) and (9.25) it is noted that the contribution of tail to $(C_{m\alpha})$ changes when the stick is fixed or free. It may be recalled, from course on stability analysis, that in stick-free case, the elevator is free to move about its hinge.

Substituting $(C_{m\alpha})_{stick\text{-}fixed} = 0$ in Eq.(9.24), gives the stick fixed neutral point. It is denoted by x_{NP}. Hence,

$$0 = C_{L\alpha w}\left(\frac{x_{NP}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + (C_{m\alpha})_{f,n,p} - V_H \eta C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \qquad (9.27)$$

$$\text{Hence, } \frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\left\{(C_{m\alpha})_{f,n,p} - V_H \eta C_{L\alpha t}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right\} \qquad (9.28)$$

Similarly, equating $(C_{m\alpha})_{stick\text{-}free}$ to zero gives stick-free neutral point x'_{NP} as:

$$\frac{x'_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{(C_{m\alpha})_{f,n,p}}{C_{L\alpha w}} - V_H \eta \frac{C_{L\alpha t}}{C_{L\alpha w}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \tau \frac{C_{h\alpha t}}{C_{h\delta e}}\right) \qquad (9.29)$$

Remark:

The quantity 'τ' in Eq.(9.29) is positive. The quantities $C_{h\alpha t}$ and $C_{h\delta e}$ in Eq.(9.29) are both generally negative. As a consequence $x'_{NP}/\bar{c}$ is lower than $x_{NP}/\bar{c}$ or the neutral point stick-free is ahead of neutral point stick-fixed.

9.2.2 Cross-check I – Stick-free neutral point at or beyond aft most c.g. location

From Eq. (9.29) it is noted that wing, fuselage, nacelle, power and h.tail contribute to longitudinal static stability $(C_{m\alpha})_{stick\text{-}free}$. However, the geometric parameters of the wing, fuselage, nacelle and power have already been decided from considerations other than stability. The geometric parameters of h.tail, especially its area (S_t) is still under the control of the designer. By varying the area of h.tail the location of the neutral point can be controlled.

As mentioned in section 9.1, the area of the horizontal tail should be such that the stick-free neutral is beyond the rear most location of c.g. or $(x_{cg})_{aft\ most}$. As a cross-check, the location of the neutral point stick-free is calculated using Eq(9.29). If it is not beyond $(x_{cg})_{aft\ most}$ then the area of the h.tail needs to be increased.

Determination of x'_{NP} for the sixty seater turboprop airplane under design, is carried out in example 9.1.

9.2.3 Longitudinal control

The airplane is said to be trimmed at a given flight speed and altitude when the moments are made zero by suitable deflections of control surfaces. For the longitudinal motion, the trim or $C_{mcg} = 0$ is achieved by suitable deflection of the elevator. The convention regarding the elevator deflection is that a downward deflection of the elevator is taken as positive.

The elevator angle for trim (δ_{trim})can be obtained from Eq.(9.2)

$$C_{mcg} = C_{m0} + C_{m\alpha}\,\alpha + C_{m\delta e}\,\delta_e$$

For trim, $C_{mcg} = 0$.

Hence,

$$0 = C_{m0} + C_{m\alpha}\,\alpha_{trim} + C_{m\delta e}\,\delta_{trim}$$

Or $$\delta_{trim} = \frac{-1}{C_{m\delta e}}\left[C_{m0} + C_{m\alpha}\,\alpha_{trim}\right] \qquad (9.30)$$

Remarks:

(i) From Eq.(9.23), it is noted that the quantity $C_{m\delta e}$ occuring in Eq.(9.30) is negative. Further, for a stable airplane $C_{m\alpha}$ is negative. Hence, Eq.(9.30) points out that, as the airplane becomes more stable (i.e. $C_{m\alpha}$ becomes more negative), the elevator deflection needed for trim, also becomes more negative.

(ii) The maximum negative deflection allowable for elevator is – 25^o.

(iii) From Eq.(9.24), $C_{m\alpha}$ becomes more negative as c.g. of the airplane moves forward. In view of remark (ii) there is a forward position of c.g. beyond which the maximum negative value of elevator deflection will not be able to trim the airplane at C_{Lmax}.

(iv) When the airplane comes in to land, the following changes take place.

(A) The airplane has high value of C_L.

(B) The flaps are deflected and the value of C_{mac} becomes more negative.

(C) The expression for $C_{m\alpha}$ involves $C_{L\alpha w}$, $C_{L\alpha t}$ and $(d\varepsilon/d\alpha)$. The proximity of ground : (a) increases $C_{L\alpha w}$ appreciably, (b) increases $C_{L\alpha t}$ by negligible amount and (c) decreases $d\varepsilon/d\alpha$. All these three factors tend to make $C_{m\alpha}$ more negative than when the airplane is in free flight away from ground.

Hence, the ability of elevator to trim the airplane in landing configuration with c.g. in the most forward position is a critical case for adequacy of the elevator.