Chapter 4

Estimation of wing loading and thrust loading - 5

Lecture 13

Topics

4.9 Selection of the optimum wing loading for a jet airplane

The discussion in sections 4.4 to 4.8 was essentially directed towards a jet airplane. The procedure for selecting wing loading for airplanes with engine propeller combination is different in some aspects and would be covered in section 4.10. In the present section the considerations mentioned earlier (sections 4.4 to 4.8) are taken into account and a final wing loading is arrived at. The steps are as follows.

1. The curves corresponding to s_{land}, $\bar{t}_{Vmax}$, $\bar{t}_{Hmax}$, $\bar{t}_{R/Cmax}$, $\bar{W}_f$ and BFL for take-off are plotted in the same graph (Fig.4.10)

2.Permissible variations around each of the optima are allowed and the upper and lower limits of allowable wing loading (p) are obtained from various considerations (Fig.4.10).

3.In some situations, a range of p may be available, where all the criteria are satisfied (Fig.4.10). In this case, a wing loading, in this range, is selected depending on the criteria which the designer may like to give more importance. As mentioned in section 4.8.2, a higher value of p is beneficial to reduce the wing weight.

4.If a range of wing loading, where all the criteria are satisfied, is not available, the final wing loading is chosen based on the most important consideration for the design.

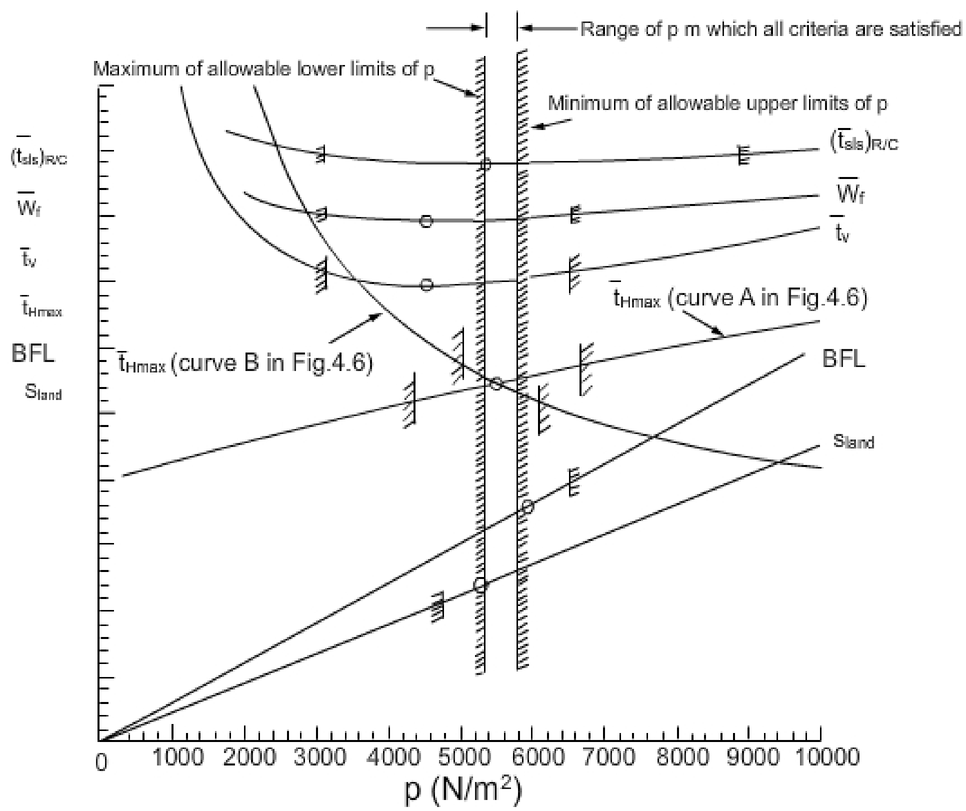

Fig.4.10 Selection of wing loading

Example 4.10

Obtain the optimum wing loading for the airplane considered in examples 4.1 to 4.9.

Solution :

The results as shown in Fig.4.10 can be plotted. However, the following tabular method is adopted.

Performance Criterion	p_{opt} (N/m^2)	Allowable range of W/S (N/m^2)	Example
s_{land}	5276 for s_{land} = 1425 m	4748 – 5803 (s_{land} between 1282.5 to 1567.5m)	4.1 Remark (iii)
V_{cr}	4527 for $(\bar{t}_V = 0.0543)$	3135 – 6536 $(\bar{t}_V = 0.057)$	4.4
H_{max}	5500 for $\bar{t}_{Hmax}$ = 0.0546	5028 – 6084 ($\bar{t}_{Hmax}$ = 0.0519 to 0.0573)	4.5
$(R/C)_{max}$	5350 for $(\bar{t}_{sls})_{R/C}$ = 0.2475	3120 – 8855 $(\bar{t}_{sls})_{R/C}$ = 0.2595	4.6
$(\bar{W}_f)_{min}$	4527 for $\bar{W}_f$ = 0.1533	3133 – 6540 $\bar{W}_f$ = 0.161	4.7
BFL for T.0. with T/W = 0.3	5924 for BFL = 2150 m	5322 – 6516 (BFL = 1935 to 2365)	4.8
Sensitivity to turbulance		≥ 4650	4.9

Table E 4.10 Allowable range of wing loadings for different design criteria

The range of wing loadings in which all criteria are satisfied can be obtained by obtaining (a) the highest value of the lower limit of the allowable wing loadings among all cases and (b) the lowest value of the upper limits of the allowable wing loading among all cases from Table E4.10. It is noticed that the highest value of the lower limits in all cases is 5322 N/m^2 for BFL criterion and the lowest value of the upper limits is 5803 N/m^2 for s_{land} criterion. Thus, when W/S is between 5322 to 5803 N/m^2, all criteria are satisfied.

Remarks:

(i) The value of 5322 is greater than the minimum required from the criterion of sensitivity to turbulence. Thus, this criterion is also satisfied.

(ii) As mentioned in section 4.8.2 a higher wing loading reduces structural weight of wing. This suggests that W/S = 5803 may be a better choice.

(iii) If fuel consumption is overriding criterion, then Table E4.10 indicates that W/S = 5322 N/m^2, the lower limit of the allowable range of wing loading, may be a better choice.

(iv) The final choice of wing loading will be arrived at after repeating the preliminary design cycle.

(v) The value of wing loading arrived at this stage is within the range of wing loadings for 150 seater high subsonic jet airplane viz. 5200 to 6200 N/m^2.

(vi)In section 3.8 of Appendix 10.2 the wing loading optimised from s_{land}, V_{max}, $(R/C)_{max}$, $\bar{W}_f$ and H_{max} considerations is modified to take into account the take-off requirement.

4.10 Optimization of wing loading for airplane with engine propeller combination (AWEPC)

In sections 4.4 to 4.8 the optimization of wing loading for jet engined airplane was considered. In this section the distinguishing features of the optimization of wing loading for an airplane with engine propeller combination are pointed out. In this section, the short form AWEPC is used for "Airplane with Engine-Propeller Combination ".

4.10.1 Optimization of wing loading from consideration of landing distance (s_{land}) for AWEPC

Depending on its weight, an AWEPC may be governed by FAR 23 or FAR 25. The relationships between s_{land} and approach speed (V_A) are given by Eqs.(4.2) and (4.2a) for airplane governed by FAR 23 and 25 respectively. Following the steps in section 4.3.1 the relationships given below are obtained :

For FAR 25: $p_{land} = 0.8563\ \rho_0\, \sigma\, C_{Lmax}\, s_{land}$ (4.87)

For FAR 23: $p_{land} = 0.8453\, \rho_0\, \sigma\, C_{Lmax}\, s_{land}$ (4.88)

Remarks:

(i) The airplanes with engine-propeller combination may operate from smaller landing fields than the jet airplanes and would have smaller value of s_{land}.

(ii)These airplanes may not have complex high lift devices and as such have a lower C_{Lmax}.

Example 4.11

Consider the 60 seater airplane considered in example 2.1. Obtain the wing loading from landing consideration. Assume that s_{land} = 1200 m, C_{Lmax} = 2.7 (double slotted flap) and $\sigma = 1.0$.

Solution:

This type of airplane would be governed by FAR 25. Hence,

$p_{land} = 0.8563\ \rho_0\, \sigma\, C_{Lmax}\, s_{land}$

Or p_{land} = 0.8563 x 1.225 x 1 x 2.7 x 1200 = 3399 N/m^2

Allowing 10% variation in s_{land} i.e (1200 ± 120)m gives p_{land} between 3059 to 3739 N/m^2.

Taking $W_{land} = W_{TO}$ the permissible limits on p_{land} are 3059 to 3739 N/m^2.

Answer : Optimum wing loading from landing consideration is 3399 N/m^2.

4.10.2 Optimization of wing loading from consideration of prescribed flight velocity (V_p) for AWEPC

From section 3.2.1 the power required (P_v) for AWEPC at a flight velocity (V_p) is given by:

$$P_V = \frac{1}{2}\frac{\rho V_p^3 S C_D}{1000\eta_p}$$

Or $$\frac{P_V}{W} = \frac{1}{2}\frac{\rho V_p^3}{1000\eta_p}\frac{C_D}{p}$$

Writing $C_D = F_1 + F_2p + F_3 p^2$ yields:

$$\frac{P_V}{W} = \frac{\rho V_p^3}{2000\eta_p}\left(\frac{F_1}{p} + F_2 + F_3 p\right) \tag{4.89}$$

Hence, $\left(p_{opt}\right)_V = \sqrt{F_1/F_3}$ (4.90)

Example 4.12

Consider the 60 seater turboprop airplane of examples 2.1 and 3.1.

(a) Obtain the drag polar and its alternate representation.

(b) Obtain the optimum wing loading from the consideration of V_{max} which is 10% higher than V_{cr} but still at h_{cr} of 4.5 km

(c) Obtain the range of wing loading if an increase by 5 % is permitted above the minimum value of power required.

Solution :

The data available are as follows.

The gross weight, as revised after example 3.1, is 21280 kgf = 208757 N

W/S = 350 kgf/m^2 = 3434 N/m^2

Hence, wing area (S) = 208757/3434 = 60.79 m^2

Wing parameters : A = 12, $\Lambda = 0$, $\lambda = 0.5$, t/c = 18 %

Fuselage width where wing is located = 2.8 m

Empennage : $S_{ht}/S = 0.21$, $S_{vt}/S = 0.20$

(a) From Eq.(3.58), for a turboprop airplane, for preliminary design purpose :

$$C_D \approx 0.03354\, S^{-0.1} + \frac{1.356}{\pi A} C_L^2$$

Noting that S = 60.79 m^2, and A = 12, gives the drag polar as :

$$C_D \approx 0.03354(60.79)^{-0.1} + \frac{1.356}{\pi \times 12} C_L^2 = 0.02224 + 0.036\, C_L^2$$

From section 3.5.9, $R_w = (S_{wet})_{airplane}/S = 5$

Hence, $C_{fe} = \dfrac{0.02224}{5} = 0.004448$

Wing span = b = $\sqrt{AS} = \sqrt{12\times 60.79} = 27.00$ m

Further, $S = \dfrac{b}{2}(c_r + c_t) = \dfrac{b}{2}c_r(1+\lambda)$

Hence, $c_r = \dfrac{60.79\times 2}{27\times 1.5} = 3.00$ m and $c_t = 1.5$ m

Consequently, the equation for the local chord is :

$$c = c_r - \frac{c_r - c_t}{b/2}y = 3.60 - 0.1111\,y$$

Half of the width of the fuselage, where wing is located = 2.8/2 = 1.4 m

Hence, semispan of exposed wing = $\dfrac{27}{2} - 1.40 = 12.1$ m

Root chord of exposed wing = 3.0 – 0.1111 x 1.4 = 2.844 m

Tip chord of exposed wing = 1.5 m

Hence, $S_{exposed\ wing}$ = 12.1 (2.844 + 1.5) = 52.56 m^2

$(S_{wet})_W = 2 \times S_{exposedWing}\{1 + 1.2\ (t/c)\}$

$= 2 \times 52.56 \times \{1 + 1.2 \times 0.18\} = 127.83\ m^2$

$$K_t = 1 + \frac{S_{ht}}{S} + \frac{S_{vt}}{S} = 1 + 0.21 + 0.20 = 1.41$$

Hence,

$$F_1 = K_t\,C_{fe} \times \frac{(S_{wet})_w}{S} = 1.41\times 0.004448 \times \frac{127.83}{60.78} = 0.01319$$

$$F_2 = \frac{C_{Do} - F_1}{(W/S)_{old}} = \frac{0.02224 - 0.01319}{3434} = 2.635\times 10^{-6}\ m^2/N$$

$$F_3 = \frac{K}{q^2}$$

Hence, the alternate representation of drag polar is :

$$C_D = 0.01319 + 2.635 \times 10^{-6}\ p + \frac{0.036}{q^2}p^2$$

(b) To obtain the optimum wing loading from the consideration of V_{max} at h_{cr} the steps are as follows.

V_{max} = 1.1×500 = 550 kmph = 152.8 m/s

h_{cr} = 4.5 km, ρ_{cr} = 0.7768 kg/m^3

Hence, $q_{max} = \frac{1}{2}\rho V_{max}^2 = \frac{1}{2}\times 0.7768\times 152.8^2 = 9068.3\ \text{N/m}^2$

Hence, $F_3 = \frac{0.036}{9068.3^2} = 4.378\times 10^{-10}\ \text{m}^4/\text{N}^2$

From Eq.(4.90)

$$\left(p_{opt}\right)_v = \sqrt{F_1/F_3} = \sqrt{0.01319/\left(4.378\times 10^{-10}\right)} = 5489\ \text{N/m}^2$$

From Eq.(4.89)

$$\frac{P}{W} = \frac{\rho V_p^3}{2000\eta_p}\left(\frac{F_1}{p}+F_2+F_3 p\right)$$

Taking $\eta_p = 0.85$ (section 3.5.13)

$$\left(\frac{P}{W}\right)_{min} = \frac{0.7768\times 152.83}{2000\times 0.85}\left\{\frac{2\times 0.01319}{5489}+2.635\times 10^{-6}\right\}$$

$$= 0.01213\ \text{kW/N}$$

(c) Allowing P/W to be 5 % higher than $(P/W)_{min}$ gives :

P/W = 0.01213 x 1.05 = 0.01274

Hence,

$$0.01274 = \frac{0.7768\times 152.83}{2000\times 0.85}\left\{\frac{0.01319}{p}+2.635\times 10^{-6}+4.378\times 10^{-10}p\right\}$$

Or $7.815\times 10^{-6} = \frac{0.01319}{p}+2.635\times 10^{-6}+4.378\times 10^{-10}p$

Or $4.378 \times 10^{-10} p^2 - 5.18 \times 10^{-6} p + 0.01319 = 0$

Or $p^2 - 11831.9 p + 3.0128 \times 10^7 = 0$

Or p = 3709,8123 N/m^2

Hence, the range of p in which (P/W) is within 5 % of $(P/W)_{min}$ is :

3709 to 8123 N/m^2.

Answers:

(a) Alternate form of drag polar : $C_D = 0.01319 + 2.635 \times 10^{-6}\ p + \frac{0.036}{q^2} p^2$

(b) Optimum wing loading from V_{max} consideration : $5489\ N/m^2$

(c) Range of wing loading when P/W is within 5 % of $(P/W)_{min}$:
3709 to 8123 N/m^2.

Remarks :

(i) $(BHP)_{reqd}$ at h_{cr} & V_{max} is :

0.01213 x 208757 = 2532.2 kW

(ii) To convert this $(BHP)_{reqd}$ to sea level static power, the above value is multiplied by the ratio of the power available under sea level static condition to the power available at V_{max} and h_{cr} for an engine with rating close to that for airplane under design.

The data on characteristics of turboprop engines appear to be available in the following cases.

(a) Reference 3.4, chapter 6, contains information on variations with flight speed for three settings namely take-off power, climb power and cruise power, for PW/120 engine with sea level static, take-off rating of 2000 HP (1491 KW). However, value for climb power at h = 4.5 km and V = 550 kmph can not be obtained accurately by extrapolation of available data.

(b) Reference 1.18, Appendix E, contains data on an engine with sea level static power of 6500 HP (4847 kW).

(c) Reference 1.20, Appendix J, contains data on Allison 501-MT (T56-A-5) engine with sea level static rating of 5000 HP (3729 kW).

(d) Reference 1.19, chapter 10, contains non-dimensional plots of $\{SHP/(SHP)_{sea\ level\ take\text{-}off}\}$ vs altitude with flight speed as parameter. The data are available for climb power and cruise power settings.

The data from Ref.1.19, chapter 10 is used for the present purpose of obtaining the ratio of "$(P_{vmax,hcr} / P_{sea\ level\ static})$". It is assumed that the climb rating would be appropriate for V_{max} case and the cruise rating for cruising flight.

The curves for climb power setting are available at various altitudes for flight speeds upto 300 kts (556 kmph).

The value of $(P/P_{sealevel\ static})_{climb}$ is read at h = 15000 ft (which is close to h_{cruise} of 4.5 km) at V = 200 kts. It is found to be 0.72.The values of $(P/P_{sea\ level\ static})_{cruise}$ are read at V = 300 and 200 kts at h = 15000 ft. These values are 0.69 and 0.63 respectively. The ratio of these two values is 0.69/0.63 = 1.095.

Hence, it is estimated that :

$\left(P/P_{sealevelstatic}\right)$ at h = 4.5 km and 550 kmph is :

0.72 x 1.095 = 0.789

Thus,engine rating corresponding to 2532.29 kW at h_{cr} and V_{max} would be about : 2532.2/0.789 = 3209 kW.

Remark :

It may be recalled that the airplane under design is a twin engine design and BHP output of each engine from V_{max} consideration would be (3209/2) or 1605 kW.

4.10.3 Optimization of wing loading from consideration of prescribed $(R/C)_{max}$ for AWEPC

For an airplane with engine-propeller combination, Eq.(3.4) gives the expression for (R/C) or V_c as :

$$V_c = \frac{T\,V - D\,V}{W}$$

$$\text{Or } V_c = \frac{1000\eta_p P}{W} - \frac{1}{2}\frac{\rho V^3 S C_D}{W} \tag{4.91}$$

$$\text{Or } 1000\eta_p \frac{P}{W} = V_c + \frac{1}{2}\rho V^3 \frac{C_D}{(W/S)} = V_c + \frac{1}{2}\ \rho\ V^3 \frac{C_D}{p} \tag{4.92}$$

For a chosen V, the optimum wing loading is given by:

$$\frac{d(P/W)}{dp} = 0$$

Or $$\frac{1}{2}\rho V^3\left[\frac{d}{dp}\{(F_1/p)+F_2+F_3\,p\}\right]=0$$

Or $$\left(p_{opt}\right)_{R/C} = \sqrt{\frac{F_1}{F_3}} = q\sqrt{F_1\,\pi\,Ae} \qquad (4.93)$$

$$\left(\frac{P}{W}\right)_{R/C} = \frac{V_c}{1000\eta_p} + \frac{1}{2}\frac{\rho V^3}{1000\eta_p}\left(\frac{F_1}{\left(p_{opt}\right)_{R/C}} + F_2 + F_3\left(p_{opt}\right)_{R/C}\right) \qquad (4.94)$$

The second term on right hand side of Eq.(4.94) involves ρV^3 and $(p_{opt})_{R/C}$ involves $q(F_1\,\pi\,Ae)^{1/2}$. This term can be rewritten as:

$$\frac{1}{2}\frac{\rho V^3}{1000\eta_p}\left\{\frac{2F_1}{q\sqrt{F_1\,\pi\,Ae}}+F_2\right\}$$

If η_p is assumed constant then, $(P/W)_{R/C}$ would be:

$$\left(\frac{P}{W}\right)_{R/C} = \frac{V_c}{1000\eta_p} + \frac{1}{1000\eta_p}\left(\frac{2\sqrt{F_1}V}{\sqrt{\pi Ae}} + F_2\frac{1}{2}\rho V^3\right) \qquad (4.95)$$

From Eq.(4.95) it is observed that the first term on the right hand side is a constant.The second term is small when V is low. Further, from Eq.(4.93), $(p_{opt})_{R/C}$ depends on 'q' and in turn on 'V'. Thus, a low wing loading appears beneficial from $(R/C)_{max}$ consideration. Example 4.13, given below, points out additional features of the optimisation in this case.

Example 4.13

Consider the 60 seater turboprop airplane of examples 2.1, 3.1 and 4.12. Obtain the optimum Wing loading from $(R/C)_{max}$ consideration. Assume that

(a) the airplane is required to have a rate of climb of 540 m/min at sea level and

(b) the airplane has a variable pitch propeller and can give $\eta_p = 0.85$.

Solution :

From example 4.12 it is noted that :

$$C_D = 0.01319 + 2.635 \times 10^{-6}\, p + \frac{0.036}{q^2} p^2$$

Or $F_1 = 0.01319$, $F_2 = 2.635 \times 10^{-6}$

$V_c = R/C = 540$ m/min = 9 m/s

From Eq.(4.93):

$$\left(P_{opt}\right)_{R/C} = \sqrt{\frac{F_1}{F_3}} = q\sqrt{F_1 \pi Ae}; \text{ Here, } \pi Ae = 1/0.036 = 27.78$$

$$= \frac{1}{2}\rho V^2 \sqrt{F_1 \pi Ae}$$

From Eq.(4.95):

$$(P/W)_{R/C} = \frac{V_c}{1000\, \eta_p} + \frac{1}{1000\, \eta_p}\left[\frac{2\sqrt{F_1}V}{\sqrt{\pi Ae}} + F_2 \frac{1}{2}\rho V^3\right]$$

Substituting various quantities yields :

$$\left(P_{opt}\right)_{R/C} = \frac{1}{2} \times 1.225 \times \sqrt{0.01319 \times 27.78}\, V^2 = 0.3708\, V^2$$

$$(P/W)_{R/C} = \frac{9}{1000 \times 0.85} + \frac{1}{1000 \times 0.85}\left[\frac{2\sqrt{0.01319}}{\sqrt{27.78}} V + 2.635 \times 10^{-6} \times \frac{1}{2} \times 1.225\, V^3\right]$$

$$= 0.0106 + \frac{1}{850}\left[0.04358\, V + 1.614 \times 10^{-6}\; V^3\right]$$

Results given in the Table below are obtained for different values of flight velocity (V).

It is observed that the values of $(P/W)_{R/C}$ are lower, for lower values of flight velocity. However, the values of $(p_{opt})_{R/C}$ are also low as compared to the optimum values corresponding to those from considerations of s_{land} and V_{max}. It is suggested that the wing loading be chosen from considerations of s_{land} and $V_{max.}$

It is suggested that the wing loading be chosen from considerations other than $(R/C)_{max}$. Subsequently, the value of $(P/W)_{R/C}$ be worked out for the chosen value of wing loading.

V (m/s)	$(p_{opt})_{R/C}$ (N/m^2)	$(p/W)_{R/C}$ (kW/N)
60	1335	0.01409
65	1567	0.01445
70	1817	0.01484
75	2086	0.01525
80	2373	0.01567
85	2679	0.01612
90	3003	0.01660
95	3346	0.01710
100	3708	0.01763