Chapter 4

Estimation of wing loading and thrust loading - 6

Lecture 14

Topics

4.10.4 Optimization of wing loading from consideration of range for AWEPC

To derive an expression for the optimum wing loading, based on prescribed range, Ref.1.6 starts from the following basic relationship for an airplane with engine-propeller combination, which in standard notation appears as:

$$R = \int_{W_1}^{W_2} \frac{3.6V}{(BSFC)P} dW \;\; ; \;\; P = \frac{TV}{1000\,\eta_{prop}} \;\; ; \;\; T = \frac{C_L}{C_D} W$$

Hence,

$$R = 3600 \int_{W_1}^{W_2} -\frac{(C_L/C_D)\eta_p}{(BSFC)W} \; dW$$

Or $$R = 3600\frac{(C_L/C_D)\eta_p}{(BSFC)}\ln\left(\frac{W_1}{W_2}\right) \tag{4.96}$$

Approximating{ln(W_1/W_2) } as {2(W_1-W_2)/(W_1+W_2)}, yields:

$$R = 3600\ \frac{\eta_p}{BSFC}\frac{C_L}{C_D}\ 2\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$R = 3600\ \frac{\eta_p}{BSFC}\frac{C_L}{C_D}\frac{W_f}{W_{mean}}$$

$$\overline{W}_f = \frac{W_f}{W_{mean}} = \frac{R}{3600\frac{C_L}{C_D}\frac{\eta_p}{BSFC}}$$

$$\frac{C_L}{C_D}\frac{\eta_p}{BSFC} = \frac{p}{q}\frac{\eta_p}{C_D BSFC}$$

For chosen q, η_p and BSFC, the term $\overline{W}_f$ is proportional to p/C_D and

$$\frac{p}{C_D} = \frac{p}{F_1+F_2p+F_3p^2}$$

Or $$\overline{W}_f = \frac{R(BSFC)q}{3600\eta_p}\left[\frac{F_1}{p}+F_2+F_3p\right] \tag{4.96a}$$

$$\frac{d\overline{W}_f}{dp} = 0 \text{ gives } \left(p_{opt}\right)_{range} = \sqrt{\frac{F_1}{F_3}} \tag{4.97}$$

Example 4.14

Consider the 60 seater turboprop airplane of examples 2.1, 3.1 and 4.12. Obtain the optimum wing loading from the consideration of range. Assume that (a) the airplane cruises at a flight speed of 500 kmph at 4.5 km altitude, (b) the range is 1740 km and (c) η_p and BSFC during cruise are 0.85 and 2.7 N/kW – hr respectively.

Obtain the range of the wing loading if an increase by 5 % is permitted above the minimum value of fuel required.

Solution :

From example 4.12 it is noted that :

$$C_D = 0.01319 + 2.635\times10^{-6}\,p + 0.036\frac{p^2}{q^2}$$

Further, the given data are :

R = 1740 km, BSFC = 2.7 N/kW -hr, η_p = 0.85

At 4.5 km, $\rho = 0.7768$, V_{cr} = 500 kmph = 138.9 m/s

$$q = \frac{1}{2}\times0.7768\times138.9^2 = 7493.5\ \text{N/m}^2$$

$$\left(p_{opt}\right)_{range} = \sqrt{\frac{F_1}{F_3}}$$

$$= \sqrt{\frac{0.01319\times7493.5^2}{0.036}} = 4539\ \text{N/m}^2$$

From Eq.(4.97):

$$\left(\bar{W}_f\right)_{min} = \frac{R\times BSFC\times q}{3600\,\eta_p}\left[\frac{2F_1}{p_{opt}}+F_2\right]$$

$$= \frac{1740\times2.7\times7493.5}{3600\times0.85}\left[\frac{2\times0.01319}{4539}+2.635\times10^{-6}\right]$$

$$= 0.09718$$

If $\bar{W}_f$ is allowed to be 5 % greater than $\left(\bar{W}_f\right)_{min}$ then $\bar{W}_f$ = 1.05 $\times$ 0.09718 = 0.102

Consequently,

$$0.102 = \frac{R\times BSFC\times q}{3600\times\eta_p}\left[\frac{F_1}{p}+F_2+F_3p\right]$$

$$\text{Or } 0.102 = \frac{1740\times2.7\times7493.5}{3600\times0.85}\left[\frac{0.01319}{p}+2.635\times10^{-6}+\frac{0.036}{7493.5^2}p\right]$$

$$\text{Or } 8.8659\times10^{-6} = \frac{0.01319}{p}+2.4354\times10^{-6} + 6.4111\times10^{-10}\,p$$

Or $p^2 - 9719\,p + 20573693 = 0$

Or p = 3116, 6604

Answers : $(p_{opt})_{range}$ **= 4539 N/m²**

Range of wing loading for $\bar{W}_f$ being within $\pm 5\,\%$ of $\left(\bar{W}_f\right)_{min}$:

3116, 6604 N/m².

4.10.5 Selection of optimum wing loading for AWEPC

The procedure to select the optimum wing loading of a jet airplane, outlined in section 4.9, can be followed in the present case also. The tabular method illustrated in example 4.10 is adopted here also and is presented in example 4.15.

Example 4.15

Obtain the optimum wing loading for the 60 seater turboprop airplane considered in examples 4.11 to 4.14.

Solution :

The following table can be prepared based on the calculations in examples 4.11 to 4.14.

Performance criterion	p_{opt} (N/m²)	Allowable range of W/S (N/m²)	Example
s_{land}	3399 for s_{land} = 1200 m	3059 – 3739 (s_{land} between 1080 to 1320 m)	4.11
V_{max}	5489 for $(P/W)_{min}$ = 0.01213	3709-8123 (P/W) = 0.0127	4.12
Range	4539 for $(\bar{W}_f)_{min}$ = 0.09718	3116 – 6604 for $\bar{W}_f$ = 0.102	4.14

Table E 4.15 Allowable Wing loadings for different design criteria for AWEPC

Remarks:

(i) From table E 4.15 it is observed that the highest value of the lower limit of the allowable wing loadings is 3709 N/m^2 and the lowest value of upper limits of allowable wing loadings is 3739 N/m^2. The range of wing loading where all criteria are satisfied is seen to be narrow. However, for a passenger airplane the power required for V_{max} and fuel consumption during cruise could be given higher weightage and W/S = 3739 or 3740 N/m^2 can be selected. This value would be further refined in the subsequent stages of design calculations.

(ii) From Table 2.1 it is seen that this value of wing loading (3740 N/m^2 = 381.2 kgf/m^2) is in the range of wing loadings for similar airplanes.

(iii) Check for power required during take-off

As noted in subsection 4.8.1, Loftin (Ref.3.2) gives guidelines regarding take-off balanced field length (BFL) for jet airplanes and FAR 23 take-off field length of general aviation aircraft. Recently, Scholz and Nita (Ref.4.6) have described procedure for preliminary sizing of large propeller driven airplanes. They consider data from current propeller driven transport airplanes and arrive at certain correlations. As regards the take-off balance field length, they suggest a parameter slightly different from TOP defined in Eq.(4.60) and used for jet airplane. They (Ref.4.6) suggest that the quantity $\{(P_{TO}/m_{TO}) / (m_{TO}/S)\}$ should be greater than

$$\frac{k_{TO} \times V \times g}{BFL \times \sigma \times C_{LTO} \times \eta_{pTO}}$$

where,

P_{TO} = engine shaft horse power under sea level static condition in watts.

m_{TO} = take-off mass = W_{TO}/g

k_{TO} = Take-off factor which has a value of 2.25 based on correlation (Fig.6 of Ref.4.6)

V = representative speed during take-off = $V_{TO}/\sqrt{2}$

$$V_{TO} = \left\{\frac{2(W_{TO}/S)}{\rho C_{LTO}}\right\}^{1/2}$$

η_{pTO} = propeller efficiency at the speed 'V'

The following steps illustrate the application of suggestion in Ref.4.6.

(a) Based on data in Table 2.1, the BFL for airplane under consideration can be taken as 1400 m.

(b) Take-off is taken to be at sea level or $\sigma = 1.0$

(c) C_{LTO} is 0.8 C_{Lland}. From subsection 4.10.1, C_{Lland} = 2.7. Hence,

C_{LTO} = 0.8 x 2.7 = 2.16.

(d) At this stage W/S = 3740 N/m^2. Hence,

$$V_{TO} = \sqrt{\frac{2(W/S)}{\rho C_{LTO}}} = \sqrt{\frac{3\times3740}{1.225\times2.16}} = 53.16 \text{ m/s}$$

Consequently, V = V/$\sqrt{2}$ = 53.16/$\sqrt{2}$ = 37.6 m/s

(e) Estimation of η_{pTO} :

Figure 7 of Ref.4.6 presents η_{pTO} vs V with disc loading (DL) as parameter.

$$DL = \frac{P_{TO}}{\rho S_D}$$, P_{TO} is engine output of each engine in kW.

S_D = area of propeller disc

As an initial estimate P_{TO} is taken as power required corresponding to V_{max} consideration (subsection 4.10.2). The sea level static power required for this case is 3130 kW. Taking this to be supplied by two engines, the rating of each engine is 1565 kW. The diameter of propeller is taken as 3.9 m, based on the value for ATR – 72 – 200 (Table 2.1). Hence,

$$DL = \frac{1565}{1.225\times\frac{\pi}{4}3.9^2} = 107$$

From Fig.7 of Ref.4.6, for DL = 107 and V = 37.6 m/s, η_{pTO} is 0.66.

(f) With the above values

$$\frac{K_{TO}\times V\times g}{BFL\times\sigma\times C_{LTO}\times\eta_{PTO}} = \frac{2.25\times37.6\times9.81}{1400\times1\times2.16\times0.66} = 0.4158 \qquad (4.97a)$$

(g)Further with, (A) p_{TO} = 3130 kw = 3130000 w, (B) m_{TO} = 21280 kgf and

(C) W/S = 3740 M/m^2 or $\frac{m_{TO}}{S} = \frac{3740}{9.81}$ = 381.24 kgf/m^2 yields:

$$\frac{p_{TO}/m_{TO}}{m_{TO}/S} = \frac{3130000/21280}{381.24} = 0.3858 \qquad (4.97b)$$

(h) Comparing values in Eqs.(4.97a) and (4.97b) it is observed that the combination of P_{TO} = 3130 kW and W/S = 3740 N/m^2 is slightly inadequate to satisfy the requirement of BFL of 1400 m. The power available should be more or (W/S) should be less or both. In this context, the following may be pointed out. An installed power of 3130 kW with W = 208757 N would give P/W = 0.0150 kW/N. This value is only slightly lower that the value of P/W of 0.0153 for ATR-72-200 (Table 2.1). This airplane has sea level static power of 3222 kW and weight of 21500 kgf. Further the chosen engine should have an output close to those at available engines. Hence, an engine with sea level rating of 1611 kW is tentatively chosen. Then to satisfy the take-off field length requirement, the wing loading is changed to :

$$0.4158 = \frac{3222 \times 1000 / 21280}{\left(\text{W/S in N/m}^2\right)/9.81}$$

$$\text{Or W/S} = \frac{3222000 \times 9.81}{21280 \times 0.4158} = 3572 \text{ N/m}^2 = 364.1 \text{ kgf/m}^2$$

Thus, at this stage of preliminary design a wing loading of 3570 N/m^2 (363.9 kgf/m^2) is chosen. Incidently this value of the wing loading is close to W/S of 352.5 kgf/m^2 for ATR-72-200. This indicates appropriateness of the process of selecting the wing loading.

4.10.6 Determination of engine output required

The wing loading, has been chosen following the procedure described in subsection 4.9 and 4.10.5. Subsequently, the thrust or power required for critical cases like $(V_C)_{max}$, V_{max}, s_{TO} and H_{max}, can be calculated with this value of wing loading. For military airplanes the thrust required for (a) sustained turn (b) specific excess power and (c) acceleration within a certain duration of time, also need to be calculated. Further, the thrust or power required calculated at prescribed flight speed and altitude need to be multiplied by a suitable factor so as to give the sea level static thrust or power. This value prescribes the rating of the engine. If the engine ratings required in various cases mentioned above are significantly different from each other, it indicates a mismatch in specifications. Slight changes in specifications would result in a more efficient design. For example, if the value of (T/W) required for $(V_c)_{max}$ is 0.35, whereas it is only 0.25 from other considerations, then a reduction in $(V_c)_{max}$ specification would bring about a compromise.

Example 4.16

Obtain the engine rating required for the sixty-seater airplane considered in examples 4.11 to 4.15. Assume that the wing loading is 3570 N/m^2.

Solution :

The critical cases are :

(i) V_{max} = 550 kmph = 152.8 m/s at h = 4.5 km.

(ii) $(R/C)_{max}$ = 540 m/min = 9 m/s at sea level.

The given data are as follows

(a) The drag polar is taken as $C_D = 0.02224 + 0.036\,C_L^2$

(b) W = 21280 kgf = 208757 N

(c) S = W/(W/S) = 208757/3570 = 58.48 m^2

(d) ρ at 4.5 km is 0.7768 kg/m^3

(I) Power required for V_{max} = 152.8 m/s at 4.5 km altitude

$$\text{Power required} = P_r = \frac{1}{2000}\times\rho V^3 SC_D$$

$$C_L = \frac{2W}{\rho SV^2} = \frac{2\times3570}{0.7768\times152.8^2} = 0.394$$

$$C_D = 0.02224+0.036\times0.394^2 = 0.02783$$

$$\text{Hence, } P_r = \frac{1}{2000}\times0.7768\times152.8^3\times58.48\times0.02783 = 2255.12\text{ kW}$$

Taking $\eta_p = 0.85$, BHP = 2255.12/0.85 = 2653.1 kW

From example 4.12, the required sea level static power rating corresponding to the above value of P_r would be :

$\frac{2653.1}{0.789} = 3362.6 \text{ kW or } 1681.3 \text{ kW/engine}$

(II) Power required for $(R/C)_{max}$ at sea level, from Eq.(4.91)

$$(P_r)_{R/C} = \frac{TV}{1000} = \frac{WV_c + TV}{1000}$$

The quantity (DV) would be minimum when flight velocity corresponds to $V=V_{mp}$,

V_{mp} = V for minimum power. This occurs at $C_{Lmp} = \sqrt{\frac{3C_{DO}}{K}}$

In the present case : $C_{Lmp} = \sqrt{\frac{3 \times 0.02224}{0.036}} = 1.361$

$C_{Dmp} = 0.02224 + 0.036 \times 1.361^2 = 0.0889$

$$V_{mp} = \sqrt{\frac{2 \times 3570}{1.225 \times 1.361}} = 65.44 \text{ m/s}$$

$$\frac{DV}{1000} = \frac{1}{2000} \times 1.225 \times 65.44^3 \times 58.48 \times 0.0889 = 892.4 \text{ kW}$$

Hence, $(P_r)_{R/C} = 208757 \times \frac{9}{1000} + 892.4 = 2771.2 \text{ kW}$

Taking $\eta_p = 0.85$,

$(BHP)_{R/C}$ = 2771.2/0.85 = 3260.2 kW or 1630 kW/engine.

Keeping these results in view the engine rating of 1611 kw per engine or total of 3222 kw would be adequate to satisfy V_{max}, $(R/C)_{max}$ at s_{TO} requirements.

The above results bring out the following important aspects.

The engine ratings required for V_{max} and $(R/C)_{max}$ are slightly more than the rating of 3222 kW for P & W / 24 fitted on ATR – 72 200. Since, the required ratings are only marginally higher, extends of choosing an engine with higher rating, it may be preferable to choose the thrusted engine and opt for slightly lower V_{max} and $(R/C)_{max}$. It may also be pointed that power output of the engine is the output on the test bed of the engine manufacturer. It is called uninstalled output. The output of the engine as installed on the airplane would be less and is called installed power. According to Ref.1.19, chapter 10, the installed power would be about 93% of the uninstalled power.

In the present case installed power would be 3222 x 0.93 = 2996.5 kW. Required power for V_{max} is 3362.6 kW. Hence V_{max}, instead of 550 kmph, would be roughly 550 x $(2996.5/3362.6)^{1/3}$ = 501.3 kmph which is about 4 % lower than earlier specification. It is interesting to note that Vmax of ATR 72-200 is 526 kmph. The chosen value of $(R/C)_{max}$ of 540 m/min is perhaps on the higher side. The airplane IPTN 250 N has $(R/C)_{max}$ of 564 m/min, but its (P/W) is 0.0201 as compared to (P/W) of 0.0153 for ATR 72-200 (see table 2.1).

The reader can work out and show that with available power of 2996.5 kW and η_p = 0.85, (R/C) would be 7.93 m/s or 476 m/min. This value is higher than the sea level $(R/C)_{max}$ of XAC Y-7-100 airplane (see table 2.1).

4.11 Procedure for selection of wing loading and thrust loading from Ref.1.18

Though the procedure for selecting wing loading and thrust loading described in section 4.2. is recommended, the procedure given in Ref.1.18 is briefly mentioned to complement the information given earlier. According to this procedure, explained in chapter 5 of Ref.1.18, T/W or P/W is chosen from the data collection or from the recommended values. Subsequently, W/S is selected to satisfy various performance requirements.

Reference 1.18, chapter 5, gives the following formulae for P/W and T/W.

$$T/W = aM^c_{max} \tag{4.98}$$

$$P/W = aV^c_{max} \tag{4.99}$$

The values of a and c are given in Table 5.3 & 5.4 of Ref.1.18. After having chosen T/W or P/W, the wing loading is selected from the following considerations.

4.11.1 Wing loading from take-off consideration

The take off distance (S_{tO}) is a function of the take-off parameter (TOP).

$$TOP = \left(\frac{W/S}{\sigma C_{LTO}(T/W)}\right) \text{ or } \left(\frac{W/S}{\sigma C_{LTO}(BHP/W)}\right) \tag{4.100}$$

Figure 5.4 of Ref.1.18 gives plots of TOP vs BFL for jet airplanes.

Among the terms in TOP, the three quantities namely (i) T/W has already been chosen, (ii) the altitude for the take off field and hence σ is known and (iii) C_{LTO} is 0.8 times C_{Lmax} during landing (see subsection 4.3.2 for estimation of C_{Lmax}). Hence, the remaining term i.e. W/S can be calculated.

4.11.2 Wing loading from landing consideration

Equation (5.11) of Ref.5.18 expresses the landing distance (s_{land}) by the following expression.

$$s_{Land} = 80(W/S)\frac{1}{\sigma C_{L_{max}}} + S_a \tag{4.101}$$

where, s_{land} is in feet and W/S is in lb/ft^2 ; S_a depends on the type of airplane. Since, σ and C_{Lmax} are known, W/S can be calculated from Eq.(4.101). Subsequently, W/S based on take-off weight can be calculated (see subsection 4.3.4 for relation between take-off weight and landing weight).

4.11.3 Wing loading from range consideration

Based on the consideration of maximum range, Ref.1.18, chapter 5, recommends the following expressions for the wing loading of a jet airplane.

$$W/S = q\sqrt{\pi A e \frac{C_{DO}}{3}} \quad \text{for jet airplanes}$$

$$= q\sqrt{\pi A e C_{DO}} \quad \text{for airplanes with propellers}$$

$$q = \frac{1}{2}\rho_{cruise} V_{cruise}^2$$

4.11.4 Wing loading from consideration of endurance

Based on the consideration of maximum endurance, Ref.1.18, Chapter 5 recommends the following expressions for optimum wing loading.

$$W/S = q\sqrt{\pi A e C_{D_o}} \quad \text{For jet airplane} \tag{4.102}$$

$$= q\sqrt{3\pi A e C_{D_o}} \quad \text{For airplane with propellers} \tag{4.103}$$

Note that ‘q’ in Eqs.(4.102) and (4.103) corresponds to the altitude and flight velocity during loiter phase of flight.

Remark:

Reference 1.18, chapter 5, mentions that for fighter airplanes, the T/W must satisfy the sustained turn rate consideration. Further, the thrust must be adequate for climb with one engine off as prescribed by regulatory agencies. The wing loading and thrust loading is further optimized after the first cycle of preliminary design.