Chapter 9

Cross-checks on design of tail surfaces - 4

Lecture 37

Topics

Example 9.1 (part II)

Example 9.1 (part II)

As mentioned at the beginning of example 9.1 (part I), the subdivisions part I and part II are for convenience. Part II considers the contributions of nacelle power and horizontal tail and the analysis of longitudinal static stability and control.

(IV) Contributions of nacelle and power

The contribution of nacelle is generally neglected.

The value of $(C_{mo})_p$ is taken as zero. Exact estimation of $(C_{m\alpha})_p$ is difficult. Reference 1.18, chapter 16 gives the following guidelines.

For propeller driven airplanes:

$$\Delta\left(\frac{dC_m}{dC_L}\right)_p = 0.02 \text{ x (distance of propeller ahead of c.g.} / \bar{c}_w)$$

$$\text{Or } \left(\Delta C_{m\alpha}\right)_p = \Delta\left(\frac{dC_m}{dC_L}\right)_p \times C_{L\alpha}$$

In the present case, from example 8.1, the location of propeller is 1.58 m ahead of the leading edge of wing or 1.58 + 0.811 = 2.391 m ahead of c.g. Hence,

$$\left(\Delta C_{m\alpha}\right)_p = 0.02\times\frac{2.391}{2.295}\times 6.387 = 0.133 \text{ rad}^{-1}$$

(V) Contribution of h.tail

$$\left(C_{m\alpha t}\right)_{\text{stick-fixed}} = -V_H \eta C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)$$
$$= -1.1\times1\times4.515\left(1-0.307\right)$$
$$= -3.444\ \text{rad}^{-1}$$

$$C_{m\delta} = -V_H \eta C_{L\alpha t}\,\tau\ ,\quad \tau = \frac{C_{L\delta e}}{C_{L\alpha t}}$$

From Fig.2.32 of Ref.3.1, for $S_e/S_t = 0.35$, the value of τ is 0.58.

Hence, $C_{m\delta} = -\ 1.1 \times 1 \times 4.515 \times 0.58 = -\ 2.881\ \text{rad}^{-1}$

$$C_{mot} = -V_H \eta C_{L\alpha t}\left(i_t - \varepsilon_0\right)$$
$$= -1.1\times1\times4.515\times\frac{\left(i_t-1.44\right)}{57.3}$$
$$= -0.0867\left(i_t-1.44\right)$$

Note : The value of i_t is obtained in the next step

(VI) Estimation of incidence of h.tail (i_t)

Subsection 6.3.5 deals with horizontal tail incidence (i_t). This angle is chosen such that during the cruise, the lift required from tail, to make the airplane pitching moment zero, is produced without elevator deflection.

To obtain i_t, the value of C_{mcg} in set to zero with: (a) $V = V_{cr}$, $h = h_{cr}$ or $C_L = C_{Lcr}$ and (b) c.g. location as with design gross weight i.e. $\left(x_{cg}/\bar{c}\right) = 0.25$ in the present case.

$$C_{mcg} = \left(C_{mo}\right)_w + C_{m\alpha w}\,\alpha + \left(C_{mo}\right)_f + C_{m\alpha f}\,\alpha$$
$$+\left(C_{mo}\right)_{np} + \left(C_{m\alpha}\right)_{np}\alpha + \left(C_{mo}\right)_t + \left(C_{m\alpha}\right)_t\alpha$$

Substituting various values

$$C_{mcg} = -\ 0.07 + 0.475\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{L\alpha w}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right)$$
$$-0.029 + 1.604\,\alpha + 0.133\,\alpha - 0.0867\left(i_t - 1.44 + \tau_{tab}\,\delta_t\right)$$
$$-\ 3.44\,\alpha + 2.881\ \delta_e \tag{9.47}$$

Noting that :(a) the wing is set such that in cruise $\alpha = 0$ (b) in cruise $\delta_e = 0$

and tab deflection (δ_t) is also zero, Eq.(9.47) becomes :

$$0 = -\ 0.07 + 0 + 0 - 0.029 + 0 + 0$$
$$-\ 0.0867\ (i_t - 1.44) + 0 + 0$$

Or $i_t = 0.3^o$

The value of i_t obtained is realistic. However, in view of the approximations made at various stages of calculations, a value of i_t = 0 can be taken at this stage of preliminary design. The final value of i_t would be obtained after wind tunnel tests on a model of the proposed airplane or after performing CFD calculations.

(VII) Stick-fixed neutral point (x_{NP})

From Eq.(9.28):

$$\frac{x_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\left\{(C_{m\alpha})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)\right\}$$

Substituting various values yields:

$$\frac{x_{NP}}{\bar{c}} = 0.25 - \frac{1}{5.793}\{1.604+0.133-3.441\}$$
$$0.25+0.294 = 0.544$$

$(C_{m\alpha})_{stick\text{-}fixed}$ when c.g. is at 0.25 $\bar{c}$ is :

$$(C_{m\alpha})_{stick\text{-}fixed\ c.g.\,at\,0.25\bar{c}} = 0+1.604+0.133-3.441 = -1.704\ rad^{-1}$$

(VIII) Stick-free neutral point (x'_{NP})

From Eq.(9.29)

$$\frac{x'_{NP}}{\bar{c}} = \frac{x_{ac}}{\bar{c}} - \frac{1}{C_{L\alpha w}}\left\{(C_{m\alpha})_{f,n,p} - V_H\,\eta\,C_{L\alpha t}\left(1-\frac{d\varepsilon}{d\alpha}\right)\left(1-\tau\frac{C_{h\alpha t}}{C_{h\delta e}}\right)\right\}$$

To obtain x'_{NP} the values of $C_{h\alpha t}$ and $C_{h\delta e}$ need to be known. The values of these two quantities depend on the elevator area and the type of aerodynamic balancing.for the elevator.

From example 6.2:

(a) S_e/S_t = 0.35 and (b) the elevator has unshielded horn balance as shown in Fig. E6.2a.

According to Ref.9.2, chapter 12, the values of $C_{h\alpha t}$ and $C_{h\delta e}$ for a horn balance depend on the ratio M_{horn} / M_{flap}

where, $M_{horn} = S_{horn} \times l_{horn}$

$$M_{flap} = S_{flap} \times l_f$$

The quantities S_{norm}, S_{flap}, l_{horn} and l_f are shown in Fig.E 9.1e.

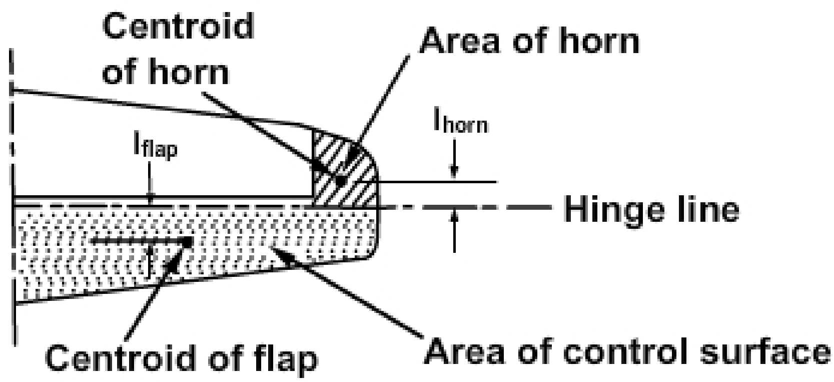

Fig. E9.1e Area and moment arms of flap and horn

From Fig.E6.2a the following values are estimated.

$S_{flap} = 3.152 \text{ m}^2$

$S_{horn} = 0.488 \text{ m}^2$

$l_{horn} = 0.3485 \text{ m}$

$l_f = 0.2115 \text{ m}$

Consequently,

$$\frac{S_{horn} \times l_{horn}}{S_{flap} \times l_f} = \frac{0.488 \times 0.3485}{3.152 \times 0.2115} = 0.255$$

From Ref.9.2, chapter 12, the values of $C_{h\alpha t}$ and $C_{h\delta e}$ are :

$C_{h\alpha t} = -0.0025$ and $C_{h\delta e} = -0.0075$

Hence, from Eq.(9.19):

$$(C_{m\alpha})_{stick\text{-}free} = (C_{m\alpha})_{stick\text{-}fixed} \times \left(1 - \tau \frac{C_{h\alpha t}}{C_{h\delta e}}\right)$$

$$\left(C_{m\alpha t}\right)_{\text{stick-free}} = -3.441\left(1\text{-}0.58\times\frac{0.0025}{0.0075}\right) = \text{-}2.776$$

When ($x_{cg}/\bar{c}$) is 0.25, from Eq.(9.25):

$$\left(C_{m\alpha}\right)_{\text{stick-free}} = 0 + 1.604 + 0.133 - 2.776 = -\,1.039$$

$$\frac{x'_{NP}}{\bar{c}} = 0.25\text{-}\frac{1}{5.793}\left\{1.604+0.133\text{-}2.776\right\}$$

$$= 0.429$$

Remark :

From example 8.1, the aft most c.g. location is at $0.407\bar{c}$ which is ahead of x'_{NP}.

Thus, the cross-check that aft most c.g. location is ahead of the neutral point stick-free is satisfied.

If it is desired that $\left(x'_{NP}/\bar{c}\right)$ be at 0.407 + 0.05 i.e. 0.457, then the area of h.tail would need to be suitably increased.

(IX) Elevator required during landing with c.g. at the foremost location

The foremost c.g. location is taken, from example 8.1, at $0.224\bar{c}$.

To examine this case Eq.(9.32) is used.

The following additional quantities need to be obtained.

(a) $\left(\Delta C_{max}\right)_{flap}$

(b) $C_{L\alpha Wg}, C_{L\alpha tg}$ under landing condition and with correction for proximity of ground.

(c) $\left(d\varepsilon/d\alpha\right)$ in proximity of ground

(d) angle of attack of airplane (α) during landing

(A) $\left(\Delta C_{mac}\right)_{flap}$

From example 4.11; it is known that a double slotted flap has been chosen for the airplane under design.

From chapter 1-9 of Ref.1.5 :

$$\left(\Delta C_{mac}\right)_{flap} = \left(\Delta C_{mac}\right)_{airfoil}\frac{S_f}{S}\frac{\bar{c}_f}{\bar{c}} \qquad (9.49)$$

where, S_f = area of wing which contains the flap

$\overline{c}_f$ = mean aerodynamic chord of the wing which contains the flap

It is assumed that in landing configuration, the flap deflection is 40°. From chapter 8 of Ref.5.1, $(\Delta C_{mac})_{flap}$ for this case is – 0.6.

From the wing planform in Fig.E5.1, the area of wing which contains the flap is estimated as 39.65 m^2.

The mean aerodynamic chord of wing which contains the flap is estimated as 2.691 m. Consequently, from Eq.(9.49) :

$$(\Delta C_{mac})_{flap} = -0.6 \times \frac{39.65}{58.48} \times \frac{2.691}{2.295} = -0.477$$

(B) Angle of attack (α) at landing

The angle of attack at landing is calculated at V = V_{TD}, with landing taking place at sea level,

V_{TD} = Touch down velocity. V_{TD} = 1.15 V_s

V_S is calculated assuming C_{Lmax} = 3.0 (example 4.11)

C_L at V_{TD} is = C_{LTD} = 3 / 1.15^2 = 2.268

Now, from Fig.5.5e, C_{Lmax} of the airfoil is 2.0. Hence, $(\Delta C_{Lmax})_{flap} = 3 - 2 = 1$

It may be pointed out that as the airplane comes into land, the flaps are deployed to give C_{Lmax} but the angle of attack is such that lift coefficient is C_{LTD}.

Hence, C_L due to angle attack is 2.268 – 1 = 1.268

At this stage, $C_{L\alpha w}$ is obtained at V = V_{TD} and correction applied for proximity of ground ; W_{land} is taken as W_0 for simplicity.

$$V_{TD} = \sqrt{\frac{2W}{\rho S C_{LTD}}} = \sqrt{\frac{2 \times 208757}{1.225 \times 58.48 \times 2.268}} = 50.68 \text{ m/s}$$

(C) Mach number at touch down (M_{TD}) is :

M_{TD} = 50.68/340.29 = 0.149

Hence, $\beta = \sqrt{1-0.149^2} = 0.989$

$$(C_{L\alpha w})_{M=M_{TD}} = \frac{2\pi \times 12}{2+\sqrt{4+\frac{12^2 \times 0.989^2}{1}\left(1+\frac{0.03454^2}{0.989^2}\right)}} = 5.369 \text{ rad}^{-1} = 0.0937 \text{ deg}^{-1}$$

Correction to $C_{L\alpha w}$ for proximity of ground in obtained as follows.

From Fig.E 8.2, the height of wing above ground (d_g) is 0.7 + 2.88 = 3.58 m

Hence, $d_g/(b/2)$ = 3.58 / (26.49/2) = 0.27

From Ref.4.7, chapter 5, for A = 12 and d_g /(b/2) of 0.27, $\left(C_{L\alpha wg} / C_{L\alpha w}\right) = 1.054$

Hence, $C_{L\alpha Wg} = 1.054 \times 0.0937 = 0.0987\ \text{deg}^{-1} \quad = 5.659\ \text{rad}^{-1}$

Hence, α_w at touchdown ; $\dfrac{1.268}{0.0987} - 1.8 = 11.05^\circ$. Note: $\alpha_{oLw} = -1.8^\circ$

Angle of attack of airplane = $\alpha = \alpha_{cr} - i_w = 11.05 - 2.9 = 8.15^\circ$

Slope of lift curve for h.tail at M_{TD} :

$$(C_{L\alpha t})_{MT} = \frac{2\times\pi\times5}{2+\sqrt{4+\dfrac{5^2\times0.989^2}{1}\left(1+\dfrac{0.0987^2}{0.989^2}\right)}} = 4.27 / \text{rad}^{-1} = 0.0745\ \text{deg}^{-1}$$

From Fig. E8.2 the h.tail is 8.18 m above the ground. The semispan (b/2) of tail is 3.725 m.

Hence, $\left\{d_g/(b12)\right\}_{tail} = 8.18/3.725 = 2.20$

From Ref.4.7, chapter 5, $C_{L\alpha tg} / C_{L\alpha t} = 1.006$

Hence, $C_{L\alpha tg}$ = 1.006 x 4.27 = 4.3 rad^{-1} = 0.075 deg^{-1}

Downwash in proximity of ground:

From Ref.1.5, chapter 1-9,

$$\left(\frac{d\varepsilon}{d\alpha}\right)_g \approx 0.5\left(\frac{d\varepsilon}{d\alpha}\right), \text{and } (\varepsilon_0)_g \approx 0.5\ \varepsilon$$

Hence, $$\left(\frac{d\varepsilon}{d\alpha}\right)_g = 0.5 \times 0.307 = 0.1535$$

$$(\varepsilon_0)_g = 0.5\times1.44 = 0.72$$

Substituting various values in Eq.(9.32) yields ; note c.g. at 0.224 $\bar{c}$ and $i_t = 0^\circ$, $\delta_{tab} = 0$

$$C_{mcg} = -0.07 - 0.477 + 5.659\left(\frac{2.9+1.8}{57.3}\right)(0.224-0.25) + 5.659(0.224-0.25)\frac{8.15}{57.3}$$

$$-0.029 + 1.604\times\frac{8.15}{57.3} + 0.133\times\frac{8.15}{57.3}$$

$$-1.1\times1\times\frac{4.3}{57.3}\left\{0-0.72+8.15(1-0.1535)+0.58\delta_e\right\}$$

Putting C_{mcg} = 0 for trim, gives the required value of δ_e

$$0 = -\,0.07 - 0.477 - 0.0121 - 0.021(-0.029 + 0.228 + 0.019 - 0.0825\{6.17\}$$

$$-\,0.0825 \times 0.58\,\delta_e$$

Or $\delta_e = -18.2^\circ$

Remarks :

(i) The elevator deflection required to trim the airplane in landing condition with c.g. at most forward location, is less than the desired value of $\left|25^\circ\right|$. Thus, this criterion is also satisfied.

(ii) It may be pointed out that while carrying out calculations for the 60 seater airplane, guidelines have been taken from existing airplanes. The fact that satisfactory results have been obtained at various stages of preliminary design, validates the present approach to preliminary design. In subsequent stages of design, various parameters of the airplane need to be optimised and an improved configuration obtained.