

Chapter 3

Weight estimation - 2

Lecture 7

Topics

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3.5 Estimation of fuel fraction (W_f/W_0)

The weight of fuel needed depends on the following.

- I. Fuel required for mission.
- II. Fuel required as reserve.
- III. Trapped fuel which cannot be pumped out.

The fuel required for the mission depends on the following factors.

- a) Mission to be flown.
- b) Aerodynamics of the airplane viz. (L / D) ratio.
- c) SFC of the engine.

3.5.1 Mission profile

a) **Simple mission:** For a transport airplane the mission profile would generally consist of (a) warm up and take off, (b) climb, (c) cruise, (d) descent, (e) loiter and (f) landing(Fig.3.6). Sometimes the airplane may be required to go to alternate airport if the permission to land is refused. Allowance also has to be made for head winds encountered en-route.

Remarks:

(i) For a military airplane the flight profile could consist of (a) warm up and take-off, (b) climb, (c) cruise to target area, (d) performing mission in the target area, (e) cruise back towards the base, (f) descent, (g) loiter and (h) land. In the target area the airplane may carry out reconnaissance, or drop bombs or engage in combat.

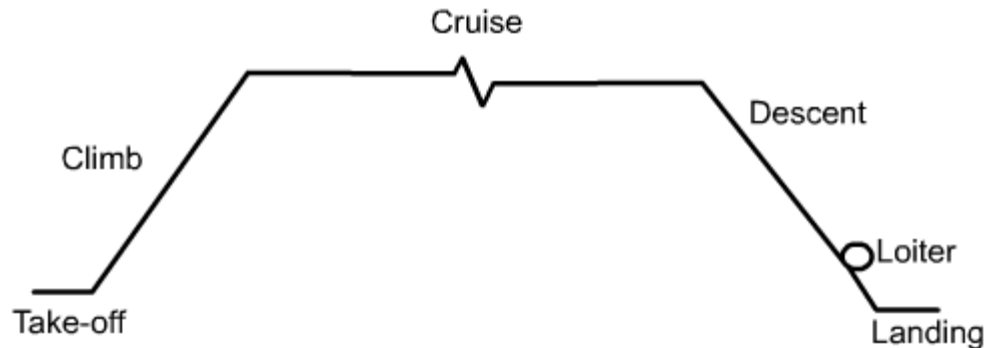


Fig.3.6 Typical mission profile of a transport airplane

As additional examples of the mission profiles the following three cases can be cited.

- (a) A trainer airplane, after reaching the specified area, may perform various maneuvers and return to the base.
- (b) An airplane on a humanitarian mission may go to the desired destination, drop food and relief supplies and return to the base.
- (c) In some advanced countries the doctors from cities fly to the remote areas, examine the patients and fly back.

ii) The various segments of the mission can be grouped into the following five categories.

- (a) Warm up, taxiing and take-off.
- (b) Climb to cruise altitude.
- (c) Cruise according to a specified flight plan. This item is covered under the topic of "Range" in "Performance analysis".
- (d) Loiter over a certain area for a specified period of time. This item is covered under the topic "Endurance" in "Performance analysis".
- (e) Descent and landing.

3.5.2 Weight fractions for various segments of mission

The fuel required in a particular phase of the mission depends on (a) the weight of the airplane at the start of that phase and (b) the distance covered or the duration of time for the phase. Keeping these in view, the approach to estimate fuel fraction for chosen mission profile is, as follows.

- i) Let the mission consist of 'n' phases.
- ii) The fuel fractions for the phase 'i' is denoted as W_i / W_{i-1} .
- iii) Let W_0 be the weight at the start of the flight (say warm up) and W_n be the weight at the end of last phase (say landing). Then, W_n/W_0 is expressed as:

$$\frac{W_n}{W_0} = \frac{W_1}{W_0} \times \frac{W_2}{W_1} \times \dots \times \frac{W_{n-1}}{W_{n-2}} \times \frac{W_n}{W_{n-1}} \quad (3.26)$$

- iv) The fuel fractions (W_i/W_{i-1}) for all phases are estimated and (W_n/W_0) is calculated from Eq.(3.26).

Subsequently, the fuel fraction (W_f/W_0) is deduced as:

$$\frac{W_f}{W_0} = K_{tf} \left(1 - \frac{W_n}{W_0} \right); \text{ where } K_{tf} \text{ is factor allowing for trapped fuel.} \quad (3.26a)$$

3.5.3 Fuel fraction for warm up, taxing and take-off (W_1 / W_0)

Reference 1.15, chapter 7 and Ref.1.18, chapter 3 give rough guidelines for fuel consumed during these phases of flight. Reference 1.12, Vol.I chapter 2 gives breakup of fuel used in warm-up, taxing and take-off for different types of airplanes. Based on this data, the rough guidelines are as follows.

For home built and single engined piston airplanes W_1/W_0 is 0.99. For twin engined turboprops, jet transports (both civil and military), flying boats and supersonic airplanes W_1/W_0 is 0.98. For military trainers and fighters W_1/W_0 is 0.97.

3.5.4 Fuel fraction for climb (W_2 / W_1)

The following guidelines are given based on the data in Ref.1.12, Vol.I, chapter 2. The low speed airplanes including the twin-engined airplanes and flying boat cruise at moderate altitude (say 4 to 6 km) and hence W_2/W_1 is taken as 0.99. The military and civil transport jets cruise around 11 km altitude and W_2/W_1 is

taken as 0.98. The fighter airplanes have very powerful engines and attain supersonic Mach number at the end of the climb. In this case, W_2/W_1 is between 0.9 to 0.96. Similarly, the supersonic transport airplanes which cruise at high altitudes (15 to 18 km), W_2/W_1 is around 0.9. Reference 1.15, chapter 7 gives more elaborate procedure to estimate W_2/W_1 and is followed for fighter and supersonic cruise airplanes.

3.5.5 Fuel fraction during cruise – outline of approach

Equations (3.8) and (3.10) present the Breguet formulae for range of airplanes with engine-propeller combination and with jet engine respectively. Consult books on performance analysis (e.g. section 7.4.2 of Ref.3.3) for the derivation of these equations. However, it may be pointed out that while deriving these formulae it is assumed that the following quantities remain constant during the flight.

- (a) Lift coefficient.
- (b) Specific fuel consumption (BSFC or TSFC).
- (c) Propeller efficiency for airplanes with engine-propeller combination and
- (d) Flight altitude.

Equations for range can also be derived when the flight velocity remains constant instead of the lift coefficient.

References 1.15, chapter 7 and 1.18, chapter 17 consider a slightly different but simpler approach. The derivation is as follows.

In a flight at velocity V (in m/s), the distance dR (in km) covered when a quantity of fuel dW_f (in N) is consumed in time dt , is given as :

$$dR = dW_f \times (\text{km} / \text{N of fuel}) \quad (3.27)$$

Now, in a time interval dt , the distance covered in km is $3.6 V dt$, where V is the flight speed in m/s; the factor 3.6 is to convert velocity to kmph. Note dt is in hrs. Further, for jet engined airplanes the fuel consumed, dW_f , in the time interval ' dt ' is :

$$dW_f = \text{TSFC} \times T \times dt$$

where T is in N, TSFC is N/N-hr or hr^{-1} and dt is in hrs.

$$\text{Hence, (km / N of fuel)} = \frac{3.6V \times dt}{\text{TFSC} \times T \times dt}$$

Substituting this in Eq. (3.27) gives :

$$dR = dW_f \frac{3.6V \times dt}{\text{TFSC} \times T \times dt} = \frac{3.6V}{\text{TFSC} \times T} dW_f \quad (3.28)$$

Noting that, $T = W \frac{C_D}{C_L} = \frac{W}{(L/D)}$ and $dW_f = -dW$, gives :

$$dR = \frac{-3.6 \times V \times (L/D)}{\text{TFSC} \times W} dW \quad (3.29)$$

Assuming V , TSFC and (L/D) to be constant and taking W_{i-1} and W_i as the weights of the airplane at the beginning and the end of the cruise, and integrating Eq.(3.29), yields :

$$R = \frac{-3.6 \times V \times (L/D)}{\text{TFSC}} \ln \left(\frac{W_{i-1}}{W_i} \right) \quad (3.30)$$

$$\text{Or } \frac{W_i}{W_{i-1}} = \exp \left\{ \frac{-R \times \text{TSFC}}{3.6 \times V \times (L/D)} \right\}; \quad V \text{ in m/s.} \quad (3.31)$$

For an airplane with engine-propeller combination, the fuel consumed in the time interval 'dt' is :

$$\text{BSFC} \times \text{BHP} \times dt = \text{BSFC} \times \frac{\text{THP}}{\eta_p} \times dt = \frac{\text{BSFC} \times T \times V \times dt}{\eta_p \times 1000}$$

$$\text{Hence, (km / N of fuel)} = \frac{3.6 \times V \times dt}{\text{BSFC} \times \frac{TV}{\eta_p \times 1000} \times dt}$$

Substituting this in Eq.(3.27) yields:

$$dR = \frac{3.6 \times V \times dt \times dW_f}{\text{BSFC} \times \frac{TV}{\eta_p \times 1000} \times dt} = \frac{3600 \times \eta_p}{\text{BSFC} \times T} dW_f \quad (3.32)$$

Noting that, $T = W \frac{C_D}{C_L} = \frac{W}{L/D}$ and $dW_f = -dW$ yields:

$$dR = \frac{-3600 \times \eta_p \times (L/D)}{\text{BSFC}} \frac{dW}{W} \quad (3.33)$$

Assuming η_p , BSFC and L/D to be constant and integrating Eq.(3.33) gives:

$$R = -\frac{3600 \times \eta_p}{\text{BSFC}} (L/D) \ln\left(\frac{W_i}{W_{i-1}}\right)$$

$$\text{Or } \frac{W_i}{W_{i-1}} = \exp\left\{\frac{-R \times \text{BSFC}}{3600\eta_p(L/D)}\right\} \quad (3.34)$$

Remarks:

i) While deriving Eq.(3.30) both V and C_L are assumed to be constant, during cruise. However, weight of the airplane decreases during the flight and to satisfy $L = W = (1/2)\rho V^2 S C_L$ the values of ρ should decrease as weight decreases. The consequence is, the altitude of the airplane should increase as the flight progresses. This is called 'Cruise climb'. However, the change in the altitude is small and the flight can be regarded as level flight.

ii) To evaluate the fuel fraction using Eq.(3.31) requires values of TSFC, V and (L/D). When Eq.(3.34) is used, the values of BSFC, (L/D) and η_p are needed.

iii) The following may be pointed out.

(a) For airplanes with engine-propeller combination, Eq.(3.34) shows that the fuel required would be minimum when the flight takes place at a C_L corresponding to $(L/D)_{\max}$.

(b) For jet engined airplanes, Eq.(3.31) shows that for minimizing the fuel required, the product $V(C_L/C_D)$ should be maximum. Since, V is proportional to $1/(C_L^{1/2})$, the quantity (VC_L/C_D) is maximised when C_L corresponds to

$(C_L^{1/2}/C_D)_{\max}$. Assuming parabolic polar i.e. $C_D = C_{D0} + KC_L^2$, it is shown that the value of C_L corresponding to $(C_L^{1/2}/C_D)_{\max}$ is $\sqrt{C_{D0}/3K}$. The value of (L/D) for this value of C_L is $0.866 (L/D)_{\max}$.

3.5.6 Fuel fraction during loiter – outline of approach

Equations (3.9) and (3.11) present the Breguet formulae for endurance of airplanes with engine-propeller combination and with jet engine, respectively. Books on performance analysis (e.g. section 7.4.2 of Ref.3.3) be consulted for

derivation of these equations. As mentioned earlier, the derivations of these formulae assume that the following quantities remain constant during the flight.

- (a) Lift coefficient
- (b) Specific fuel consumption (BSFC or TSFC)
- (c) Propeller efficiency for airplanes with engine-propeller combination and
- (d) Flight altitude.

Equation for endurance can also be derived when the flight velocity remains constant instead of the lift coefficient.

Reference 1.18 chapter 3 considers a slightly different but simpler approach. The derivation is as follows.

In a flight at velocity V , the time elapse dE (in hr) when a quantity of fuel dW_f (in N) is consumed is given by :

$$dE = dW_f \times \left(\frac{\text{hr}}{\text{N of fuel}} \right) = \left(\frac{dW_f}{\text{N of fuel/hr.}} \right)$$

For a jet engined airplane, (N of fuel/ hr) = TSFC \times T

$$\text{Hence, } dE = \frac{dW_f}{\text{TSFC} \times T}$$

Noting that, $T = W(C_D/C_L) = W/(L/D)$ and $dW_f = - dW$ gives:

$$dE = - dW \left(\frac{(L/D)}{\text{TSFC} \times W} \right) \quad (3.35)$$

Let (a) (L/D) and TSFC be assumed to remain constant during the flight,

(b) W_{i-1} and W_i the weights of the airplane at the beginning and end of flight.

On integrating Eq.(3.35), gives

$$E = \left(\frac{L/D}{\text{TSFC}} \right) \ln \left(\frac{W_{i-1}}{W_i} \right) \quad (3.36)$$

$$\text{Or } \frac{W_i}{W_{i-1}} = \exp \left\{ \frac{- E \times \text{TSFC}}{(L/D)} \right\} \quad (3.37)$$

For an airplane with engine-propeller combination, the quantity (N of fuel/ hr) is :

$$(\text{N of fuel/ hr}) = \text{BSFC} \times \text{BHP} = \text{BSFC} \times \frac{T \times V}{1000 \times \eta_p}$$

$$\begin{aligned} \text{Consequently, } dE &= \frac{dW_f}{\text{BSFC} \times \frac{TV}{1000\eta_p}} = \frac{dW_f \times 1000 \times \eta_p}{\text{BSFC} \times W \times \frac{C_D}{C_L} \times V} \\ &= \frac{-1000 \times \eta_p \times (L/D) dW}{\text{BSFC} \times V \times W} \end{aligned} \quad (3.38)$$

Assuming η_p , BSFC, (L/D) and V to be constant during flight and integrating Eq.(3.38) yields:

$$E = \frac{-1000 \times \eta_p (L/D)}{\text{BSFC} \times V} \ln \left(\frac{W_i}{W_{i-1}} \right) \quad (3.39)$$

$$\text{Or } \frac{W_{i-1}}{W_i} = \exp \left\{ \frac{-E \times \text{BSFC} \times V}{1000 \times \eta_p \times (L/D)} \right\} \quad (3.40)$$

Remarks:

(i) As mentioned earlier, in a flight with both (L/D) and V as constant, the flight altitude of airplane would increase as the weight of airplane decreases due to consumption of fuel. This is called cruise-climb. However, change in flight altitude is small and flight can be regarded as level flight.

(ii) To evaluate the fuel fraction for loiter of specified duration, the values of BSFC, V , η_p and (L/D) are required for airplanes with engine-propeller combination and those of TSFC and (L/D) for the jet airplanes.

(iii) Equation (3.37) shows that for a jet airplane the fuel required for a specified endurance, would be minimum when the flight takes place at C_L corresponding to $(L/D)_{\max}$.

Equation (3.40) shows that for an airplane with engine-propeller combination the fuel required for given endurance would be minimum, when $V/(L/D)$ is minimum. Since, V is proportional to $1/(C_L^{1/2})$, this implies that

$(C_D / C_L^{3/2})$ should be minimum or $(C_L^{3/2} / C_D)$ should be maximum, for fuel required to be minimum. For a parabolic polar ($C_D = C_{D0} + KC_L^2$), it can be shown that value of C_L corresponding to $(C_L^{3/2} / C_D)_{\max}$ is $(3C_{D0}/K)^{1/2}$. The value of (L/D) corresponding to this value or C_L is $0.866 (L/D)_{\max}$.

3.5.7 Estimation of $(L/D)_{\max}$ – outline of approach

Raymer (Reference 1.18, chapter 3) finds that $(L/D)_{\max}$ for a chosen type of airplane depends on the wetted aspect ratio (A_{wet}) defined as :

$$A_{\text{wet}} = \frac{b^2}{S_{\text{wet}}} \quad (3.41)$$

where, b = wing span and S_{wet} = wetted area of the airplane. Further S_{wet} , for a chosen type of airplane, is a multiple of S the wing area. Using Figs.3.5 and 3.6 of Ref.1.18 one can get a rough estimate of $(L/D)_{\max}$.

However, keeping in view the need for drag polar for optimization of wing loading in chapter 4, the expressions for drag polar ($C_D = C_{D0} + KC_L^2$) are deduced which appear to be adequate at this stage of preliminary design. The general expressions for C_{D0} and K obtained in Ref.1.15 chapter 6 are used for this purpose. Though Ref.1.15 gives general expression for C_{D0} and K for both subsonic and supersonic airplanes, here the attention is focussed on subsonic airplanes.

Reference 1.15 chapter 6 gives the following expressions for C_{D0} and K for subsonic airplanes with wings of moderate to high aspect ratio ($A > 5$).

$$C_{D0} = 0.005 \tau R_w T_f S^{-0.1} \left(1 - \frac{2C_{lf}}{R_w} \right) \times \left[1 - 0.2M + 0.12 \left\{ \frac{M(\cos \Lambda_{1/4})^{1/2}}{A_f - (t/c)} \right\}^{20} \right] \quad (3.42)$$

$$K = \left\{ \frac{1}{\pi A} (1 + 0.12M^6) \right\} \left\{ 1 + \frac{0.142 + f(\lambda) A (10 \frac{t}{c})^{0.33}}{\left(\cos \Lambda_{1/4} \right)^2} \right\} + \frac{0.1(3N_e + 1)}{(4 + A)^{0.8}} \quad (3.43)$$

where, A = wing aspect ratio

M = Mach number

S = wing area

t/c = wing thickness ratio

λ = taper ratio of wing

$\Lambda_{1/4}$ = quarter chord sweep of wing

N_e = number of engines, if any, located on top surface of wing

A_f = airfoil factor. A value of 0.93 is suggested for special airfoils and 0.75 for older (NACA) airfoils.

C_{lf} = function of airfoil chord over which the flow is laminar.

$$R_w = S_{wet} / S$$

T_f = a factor which is unity for a very streamlined shape and takes into account the increase in C_D due to departure from streamlined shape.

τ = A factor which gives correction for wing thickness ratio and is given by :

$$\tau = \left[\left(\frac{R_w - 2}{R_w} \right) + \frac{1.9}{R_w} \left\{ 1 + 0.526 \left(\frac{t/c}{0.25} \right)^3 \right\} \right] \quad (3.44)$$

$f(\lambda)$ = factor which takes into account effect of wing taper ratio.

$$= 0.005 [1 + 1.5(\lambda - 0.6)^2] \quad (3.45)$$

Once C_{D0} and K are obtained, $(L/D)_{max}$ is given by :

$$(L/D)_{max} = \frac{1}{2\sqrt{C_{D0} K}} \quad (3.46)$$

Three typical cases viz. a high subsonic jet airplane, a turboprop airplane and a low speed piston-engined airplane are considered. Based on Eqs.(3.42) and (3.43) drag polars are deduced for these three cases. Suggestions are also given to obtain drag polars of similar airplanes.

3.5.8 Drag polar of typical high subsonic jet airplane

A typical high subsonic jet has the following features:

(a) $M = 0.8$, (b) $A = 9$, (c) an advanced supercritical airfoil with t/c of 0.14 ,
(d) taper ratio of 0.25, (e) sweep $(\Lambda_{1/4})$ of 30° , (f) $N_e = 0$, (g) due to high Reynolds number the flow can be treated as turbulent almost from the leading edge or $C_{lf} = 0$.

From chapter 6 of Ref.1.15 the wetted area of such airplanes is 5.5 times the wing planform area or $R_w = 5.5$. Further, T_f is 1.1.

Hence, in the present case, with the above chosen parameters, the following values are obtained.

$$(I) 1 - \frac{2C_{lf}}{R_w} = 1.0 .$$

$$(II) A_f - (t/c) = 0.93 - 0.14 = 0.79 .$$

$$(III) \tau = \left(\frac{5.5-2}{5.5} \right) + \frac{1.9}{5.5} \left\{ 1 + 0.526 \left(\frac{0.14}{0.25} \right)^3 \right\} = 1.013 .$$

Remark:

For this category of airplanes the effect of t/c on τ can be obtained by substituting different values of (t/c) in Eq.(3.44). The following values for τ are obtained.

t/c	0.1	0.14	0.18
τ	0.9926	1.013	1.049

$$(IV) f(\lambda) = 0.005 [1 + 1.5 (0.25 - 0.6)^2] = 0.00592 .$$

$$(V) \left[\frac{M(\cos \Lambda_1)^{1/2}}{A_f - (t/c)} \right]^{20} = \left[\frac{0.8 \times (0.866)^{1/2}}{0.93 - 0.14} \right]^{20} = 0.305 .$$

$$(VI) 1 - 0.2M + 0.12 \left[\frac{M(\cos \Lambda_1)^{1/2}}{A_f - (t/c)} \right]^{20} = 1 - 0.2 \times 0.8 + 0.12 \times 0.305 = 0.8766 .$$

Consequently,

$$\begin{aligned} C_{D0} &= 0.005 \times \tau \times 5.5 \times 1.1 \times 1 \times 0.8766 S^{-0.1} \\ &= 0.02652 \tau S^{-0.1} \end{aligned} \quad (3.47)$$

The effect of swept angle on C_{D0} is of secondary nature. However, it is observed that C_{D0} should be reduced by about 0.4% for each increase in sweep by 1° . The effect of aspect ratio on C_{D0} is negligible.

Thus, for $\Lambda_1 = 30^\circ$, $A = 9$ and $t/c = 0.14$

$$C_{D0} = 0.02686 S^{-0.1} \quad (3.48)$$

From Eq.(3.48) the variation of C_{D0} with S is as follows.

$S(m^2)$	50	100	500
C_{D0}	0.0182	0.0169	0.0144

Remark:

For Boeing 747 with $S = 511 \text{ m}^2$, and $\Lambda_{1/4} = 38.5^\circ$, these parameters would give C_{D0} of 0.014. This value is close to the actual value for the airplane. See also Appendix C in Ref. 3.1

Value of K :

The terms in Eq.(3.43) are now evaluated for typical high subsonic jet airplane with features mentioned under items (a) to (g) at the beginning of this subsection. The following quantities are obtained.

$$(I) 1 + 0.12 M^6 = 1 + 0.12 (0.80)^6 = 1.0315$$

$$(II) \frac{1 + 0.142 + f(\lambda) A \left(10 \frac{t}{c}\right)^{0.33}}{\cos^2 \Lambda_{1/4}} = 1 + \frac{0.142 + 0.0592 \times 9 \times (1.4)^{0.33}}{\cos^2 30^\circ}$$

$$= 1 + \frac{0.2015}{\cos^2 30^\circ} = 1.2687$$

$$(III) \frac{0.1(3N_e + 1)}{(4 + A)^{0.8}} = \frac{0.1(0 + 1)}{(4 + 9)^{0.8}} = 0.0128$$

Hence, Eq.(3.43) yields:

$$K = \frac{1}{\pi \times 9} \{1.0315 \times (1.2678 + 0.0128)\} = 0.0468$$

$$\text{Expressing } K = \frac{1}{\pi A e}, \text{ gives } e = 0.757$$

Remarks:

(i) Angle of sweep does have significant effect on K. For $M = 0.8$, $t/c = 0.14$, $\lambda = 0.25$, $N_e = 0$ and $C_{lf} = 0$, Eq.(3.43) gives:

$$K = \frac{1.0315}{\pi A} \left\{ 1 + \frac{0.2015}{\cos^2 \Lambda_{1/4}} + 0.0128 \right\} = \frac{1}{\pi A} \left\{ 1.0447 + \frac{0.2078}{\cos^2 \Lambda_{1/4}} \right\} \quad (3.49)$$

For an airplane with $S = 100 \text{ m}^2$, $\Lambda_{1/4} = 30^\circ$, Eqs.(3.48) and (3.49) give the following drag polar.

$$C_D = 0.0169 + 0.0468 C_L^2$$

$$\text{Hence, } (L/D)_{\max} = \frac{1}{2\sqrt{C_{D0}K}} = \frac{1}{\sqrt{0.0169 \times 0.0468}} = 17.8$$

This value of $(L/D)_{\max}$ is typical of such airplanes(Ref.1.18, Chapter 3).

i) Boeing 787-Dreamliner being brought out by Boeing has very smooth surface, $(t/c)_{\text{average}} = 11\%$, $\Lambda_{1/4} = 32^\circ$, $S = 331 \text{ m}^2$ and aspect ratio of 10.4 with winglets at wing tips. It is estimated to have a C_{D0} of 0.0128 and K of 0.04 resulting in $(L/D)_{\max}$ of 22.

It may be added that the effect of winglets on reducing induced drag can be estimated approximately by adding half the height of winglet to the wingspan (Ref.1.15, chapter 5, see also section 3.2.21 of Ref.3.3)