# Chapter 4

# Estimation of wing loading and thrust loading - 9

# Lecture 17

## Topics

4.14.11 Variations of THP and BSFC with flight velocity and altitude

**Example 4.20**

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**4.15 Gas turbine engines**

4.15.1 Propulsive efficiency

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### 4.14.11 Variations of THP and BSFC with flight velocity and altitude

As mentioned earlier,THP equals $\eta_p \times \text{BHP}$. Thus, the variations of THP with V and h depends on variations of $\eta_p$ and BHP with V and h. In this context, the following may be recalled.

(i)At a given altitude and RPM, the engine output (BHP) of a piston engine is almost constant with flight velocity.

(ii) BHP decreases with altitude, in this case, as given by Eqs.(4.104) or (4.105).

(iii) The propeller efficiency $\left(\eta_p\right)$ depends on BHP, h, V, n and $\beta$. For a variable pitch propeller $\eta_p$ remains nearly constant over a wide range of flight speeds.

Thus, for a piston engined airplane with variable pitch propeller, the THP vs V curve for a chosen RPM and h remains flat over a wide range of flight speeds. A typical variations of THP with V, at chosen 'RPM(N)' and with 'h' as parameter are shown in Fig.4.16.

From the engine charts the fuel flow rate and BSFC are known at chosen MAP & N. From these values, the BSFC, at the chosen MAP & N, can be calculated using Eq.(4.108). Section 6 in Appendix A of Ref.3.3 presents typical calculations.

Example 4.20 below presents typical calculations for a turboprop airplane with variable pitch propeller.

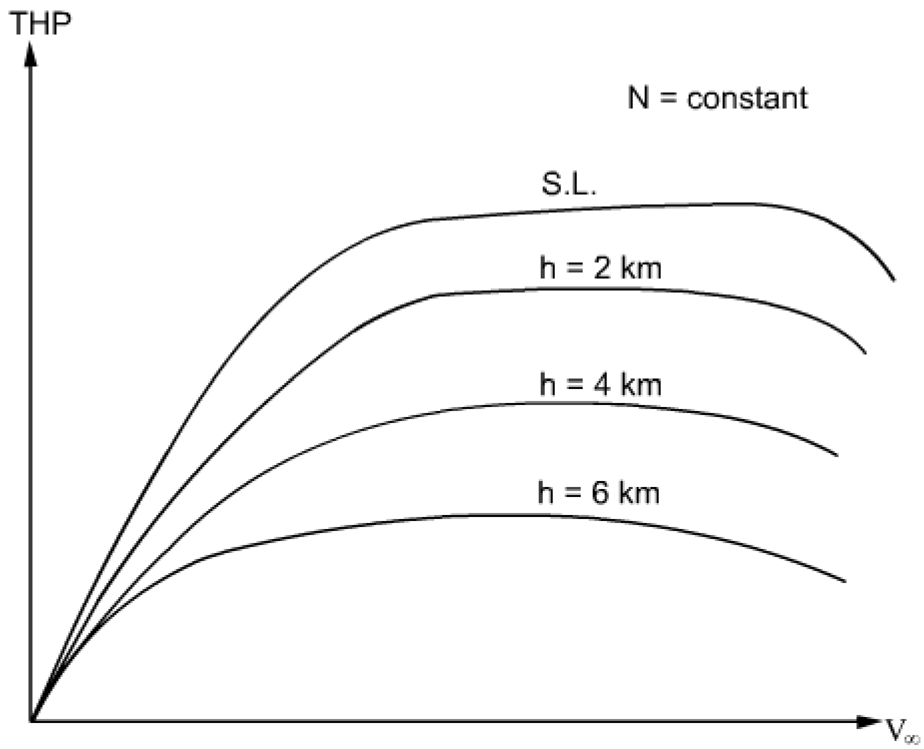



Fig.4.16 Schematic variation of THP with flight speed for piston engined airplane with variable pitch propeller

**Example 4.20**

For the sixty seater turboprop airplane considered in examples 4.11 to 4.16 and 4.19, obtain the THP vs V curves for climb rating at sea level and 15000' (4572 m) altitudes. Assume (a) the total uninstalled take-off power from two engines is 3222 kW, (b) The installed power is 93% of uninstalled power, (c) propeller RPM is 1200 and (d) propeller is four bladed variable pitch propeller of diameter 3.95 m.

**Solution**

(A) Sea level case

Take-off uninstalled BHP = 3222 kW

Take-off installed power = 3222 x 0.93 = 2996 kW

Take-off installed power per engine = 2996/2 = 1498 kW

From chapter 10 of Ref.1.19 the engine has a flat rating at sea level. The ratio of the climb rating to take-off rating is 0.85. The characteristics of four bladed propeller as given in Fig.4.15c are used. Other data are : N = 1200, or n = 20

d = 3.95 m

Further, $C_P$ = P (in watts)/ ($\rho n^2 d^5$) and J = V/nd, V in m/s

The calculations are presented in the following tables.

h = sea level

| V (kts) | 50 | 100 | 150 | 200 | 300 | 350 |
|---|---|---|---|---|---|---|
| V (kmph) | 92.6 | 185.2 | 277.8 | 370.4 | 555.6 | 648.2 |
| $P_{climb}/P_{take\text{-}off}$ | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 | 0.85 |
| BHP (kW) (per engine) | 1273 | 1273 | 1273 | 1273 | 1273 | 1273 |
| $C_P$ | 0.1351 | 0.1351 | 0.1351 | 0.1351 | 0.1351 | 0.1351 |
| J | 0.3256 | 0.6512 | 0.9768 | 1.302 | 1.9536 | 2.279 |
| $C_S = J/C_P^{1/5}$ | 0.4859 | 0.9718 | 1.458 | 1.943 | 2.915 | 3.40 |
| $\eta_p$ (Fig.4.15c) | 0.50 | 0.74 | 0.835 | 0.85 | 0.766 | 0.71 |
| $\beta$ (degrees) | 15 | 16.5 | 22 | 30 | 39 | 43 |
| THP per engine (kW) | 636.5 | 942 | 1063 | 1082 | 975 | 904 |
| THP (2 engines) (kW) | 1273 | 1884 | 2126 | 2164 | 1950 | 1868 |

(B) h = 15000' (4572 m)

At this altitude the power output increases with flight speed. Values of $P_{climb}/P_{Takeoff}$ are obtained from Ref.1.19, chapter 10. The values at V = 300 kts and 350 kts are obtained by extrapolation.

$\rho$ = 0.7708 kg/m$^3$

| V (kts) | 50 | 100 | 150 | 200 | 300 | 350 |
|---|---|---|---|---|---|---|
| V (kmph) | 92.6 | 185.2 | 277.8 | 370.4 | 555.6 | 648.2 |
| $P_{climb}/P_{Take\text{-}off}$ | 0.67 | 0.68 | 0.70 | 0.72 | 0.789 | 0.823 |
| BHP (kW) (per engine) | 1003.7 | 1018.6 | 1048.6 | 1078.6 | 1181.9 | 1232.9 |
| $C_P$ | 0.1693 | 0.1718 | 0.1769 | 0.1819 | 0.1994 | 0.2080 |
| J | 0.3256 | 0.6512 | 0.9768 | 1.302 | 1.9536 | 2.279 |
| $C_S = J/C_P^{1/5}$ | 0.4644 | 0.9262 | 1.3812 | 1.8308 | 2.697 | 3.12 |
| $\eta_p$ | 0.48 | 0.735 | 0.835 | 0.85 | 0.82 | 0.767 |
| $\beta$ (degrees) | 15 | 15 | 20 | 28 | 43 | 45 |
| THP (per engine) | 481.8 | 748.7 | 875.6 | 916.8 | 969.2 | 945.6 |
| THP (2 engines) | 963.6 | 1497.4 | 1751.2 | 1833.6 | 1938.4 | 1891.2 |

Table for Case (C)

h = 25000' (7620 m), ρ = 0.5489

The values at V = 300 kts and 350 kts are obtained by extrapolation

| V (kts) | 50 | 100 | 150 | 200 | 300 | 350 |
|---|---|---|---|---|---|---|
| V (kmph) | 92.6 | 185.2 | 277.8 | 370.4 | 555.6 | 648.2 |
| $P_{climb}$/ $P_{Take\text{-}off}$ | 0.52 | 0.54 | 0.56 | 0.58 | 0.62 | 0.64 |
| BHP (kW) (per engine) | 779 | 809 | 838.9 | 868.8 | 928.8 | 958.7 |
| $C_P$ | 0.1844 | 0.1916 | 0.1986 | 0.2058 | 0.2200 | 0.2270 |
| J | 0.3256 | 0.6512 | 0.9768 | 1.302 | 1.9536 | 2.279 |
| $C_S$ | 0.4566 | 0.9062 | 1.3496 | 1.7862 | 2.6445 | 3.065 |
| $\eta_p$ | 0.48 | 0.73 | 0.83 | 0.85 | 0.82 | 0.77 |
| THP (per engine) | 373.9 | 590.6 | 696.3 | 738.5 | 761.6 | 738.2 |
| THP (2 engines) | 797.8 | 1181.2 | 1392.6 | 1477 | 1523.2 | 1476.4 |

Figure E4.20 presents the variations of THP vs V at sea level, 15000' (4572 m) and 25000' (7620 m).

**Remarks :**

(i) Comparing the variations of THP vs V with altitude as parameter in Figs.4.16 and E 4.20, it is seen that the variations in the two cases show significant have differences. The differences are due to the following reasons.

For a piston-engined airplane the BHP, at chosen RPM and altitude is nearly constant with flight speed, whereas for a turboprop airplane BHP increases with flight speed. (See tables corresponding to 15000 ft and 25000 ft and Fig.4.21). However, the maximum output is limited to sea level rating.

(ii) The rapid decrease in the output with altitude results in lower service ceiling for piston engined airplanes than the turboprop airplanes. Service ceiling is around 4 to 5 km for piston engined airplanes whereas it is 7 to 8 km for turbo-prop airplanes.

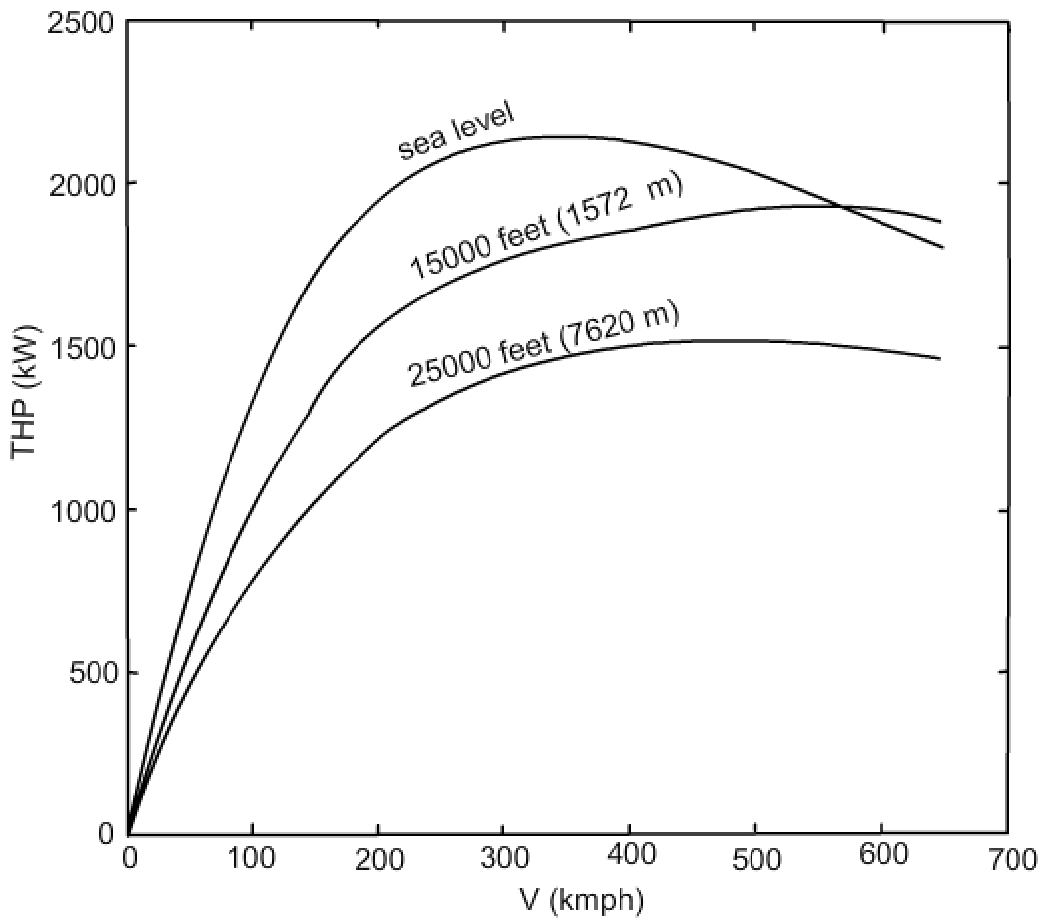



Fig.E4.20 THP vs V at different altitudes for a turboprop engine

(The BHP values at 556 and 648 kmph were obtained by extrapolating curve in Ref.1.19, chapter 10)

### 4.14.12 Loss of propeller efficiency at high speeds

As noted earlier, the propeller blade is like a rotating wing with forward motion. The resultant velocity at the propeller tip ($V_{Rtip}$) would be the highest. It is equal to:

$V_{Rtip} = \{V_\infty{}^2 + (2\pi n R)^2\}^{1/2}$, where R is the radius of the propeller.

When the Mach number corresponding to $V_{Rtip}$ exceeds the critical Mach number for the airfoil used on the propeller, the drag coefficient of the airfoil would increase and the lift coefficient would decrease. Consequently, the efficiency of the propeller would decrease. This loss of efficiency can be delayed to higher flight Mach numbers by use of advanced propellers. These propellers have swept blades and are being used on turboprop airplanes up to flight Mach number of 0.7. Figure 4.17a shows one such propeller placed in a wind tunnel and Fig.4.17b shows another propeller mounted on ATR 72 airplane.

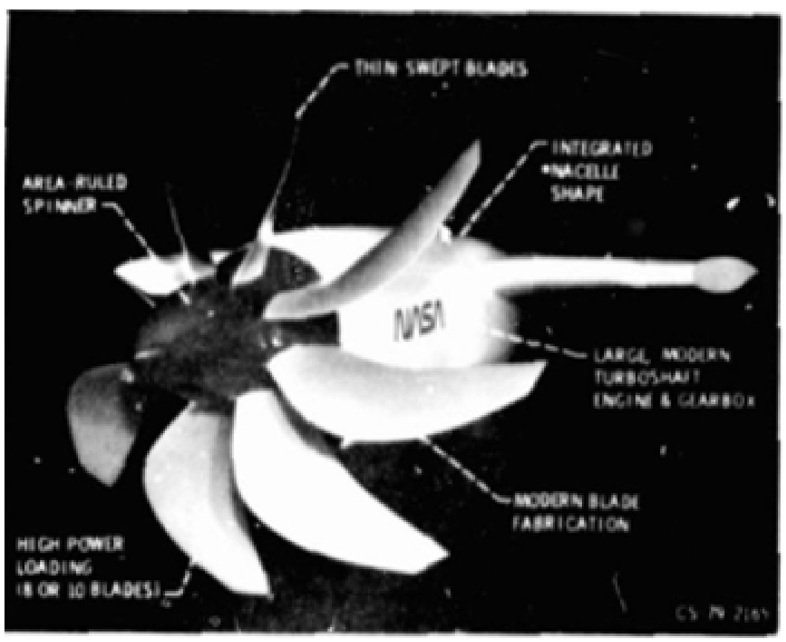
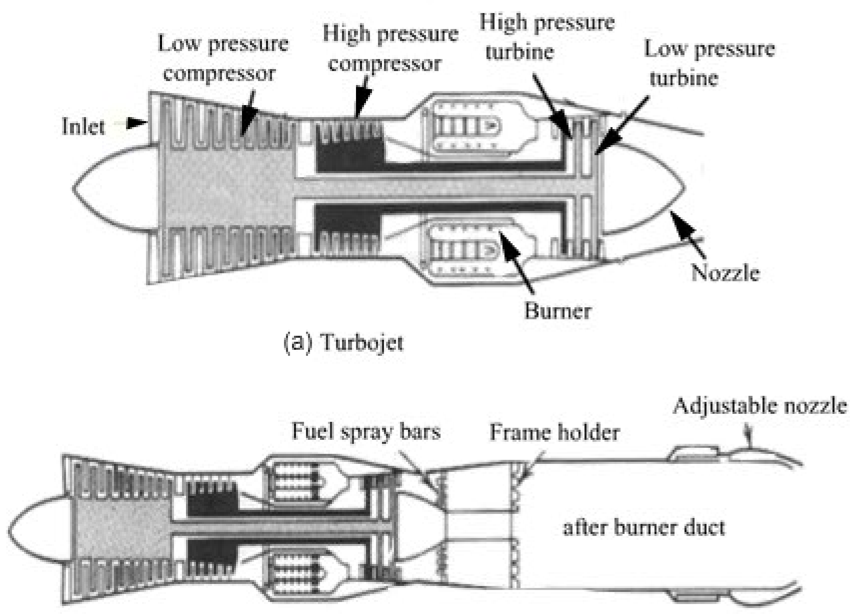



Fig.4.17a Advanced propeller being tested in a wind tunnel
(Adapted from Ref.4.10)


Fig.4.17b Advanced propeller mounted on ATR72 airplane
(Source : www.fspilotshop.com)

## 4.15 Gas turbine engines

A gas turbine engine consists of a diffuser to decelerate the air stream entering the engine, a compressor, a combustion chamber, a turbine and a nozzle (Fig.4.18a). In some turbojet engines, an afterburner is incorporated between the exit of the turbine and the entry of the nozzle (Fig.4.18b).The hot gases leaving the combustion chamber expand partly in the turbine and partly in the nozzle. The need for three variants of gas turbine engines viz. turboprop, turbofan & turbojet can be explained by considering their propulsive efficiencies.




(a) Turbojet

(b) Turbojet with afterburner

Fig.4.18 Turbojet engine

(Source : http://www.aerospaceweb.org)

### 4.15.1 Propulsive efficiency

Propulsive efficiency is the ratio of useful work done by the air stream and the energy supplied to it.

In a gas turbine engine, the velocity of the air stream ( $V_\infty$ ) is augmented to $V_j$, the velocity of the jet stream, thereby imparting kinetic energy at the rate of:

$$(\dot{m}/2)\,[\,V_j^{\,2} - V_\infty^{\,2}] \qquad (4.135)$$

where, $\dot{m}$ is the mass flow rate.

The engine develops a thrust T and hence results in a useful work of $T\,V_\infty$ .

Noting that:

$$T = \dot{m}\;(V_j - V_\infty), \qquad (4.136)$$

the propulsive efficiency ($\eta_{propulsive}$) is:

$$\eta_{propulsive} = \frac{\dot{m}(V_j - V_\infty)(V_\infty)}{\frac{\dot{m}}{2}(V_j^2 - V_\infty^2)} = \frac{2}{1 + \frac{V_j}{V_\infty}} \qquad (4.137)$$

### 4.15.2 Why turboprop, turbofan and turbojet engines?

The overall efficiency of a gas turbine engine is the product of items like cycle efficiency, combustion efficiency, mechanical efficiency and propulsive efficiency. The cycle efficiency depends on the engine cycle and in turn on the maximum temperature / pressure in the engine. The combustion efficiency and mechanical efficiency are generally of the order of 95%. Thus, propulsive efficiency finally decides the overall efficiency of a gas turbine engine as a propulsive system.

**Remark:**

The action of a propeller is also similar to that of a jet engine i.e. it also enhances velocity of the free stream from $V_\infty$ to $V_j$, In this case, $V_j$ is the velocity of the stream far behind the propeller (see subsection 4.14.7). Hence, the propulsive efficiency of a propeller which was called ideal efficiency of propeller, is also given by Eq.(4.137), which is the same as given by Eq.(4.121).

The variation of propulsive efficiency with flight speed provides the reason for use of turboprop, turbofan and turbojet engines in airplanes operating at different range of flight speeds. Consider the variation of propulsive efficiency with flight speed. For this purpose, a subsonic jet engine with convergent nozzle is considered. In this case, the Mach number at the exit, would be unity and the temperature of the exhaust gases would be around 600 K. Under these conditions, $V_j$, the velocity of jet exhaust would be around 500 m/s. Using Eq.(4.137), the values of propulsive efficiency obtained at different flight speeds ($V_\infty$) are given in the Table 4.3.

| $V_\infty$ (m/s) | 100 | 125 | 166.7 | 250 | 333.3 | 400 |
|---|---|---|---|---|---|---|
| $V_j$ / $V_\infty$ | 5 | 4 | 3 | 2 | 1.5 | 1.25 |
| $\eta_p$ % | 33.3 | 40.0 | 50.0 | 66.7 | 80.0 | 88.9 |

Table 4.3 Variation of propulsive efficiency with flight speed for $V_j$ = 500 m/s

**Remarks:**

**i) Turboprop engine**

It is observed from Table 4.3 that $\eta_p$ will be low if a pure jet engine is used at low speeds. An analysis of Eqs.(4.136) and (4.137) points out that for having adequate thrust and high propulsive efficiency at low flight speeds, a small increment in velocity should be given to a large mass of air. This is effectively done by a propeller. Thus for airplanes with flight Mach number less than about 0.5, a turboprop engine is used (Fig.4.19). In this case, the turbine drives the compressor and also the propeller through a gearbox (Fig.4.19). The gear box is needed because the turbine r.p.m. would be around 15000-20000 whereas, the propeller rotates at about 1000 to 3000 r.p.m.

For practical reasons, the expansion of the gases coming out of the combustion chamber is not allowed to take place completely in the turbine and a part of the expansion is carried out in the nozzle. Hence, in a turboprop engine, about 80 to 90% of the total output is produced through the propeller and the rest 20 to 10% as output from the jet coming out of the nozzle.

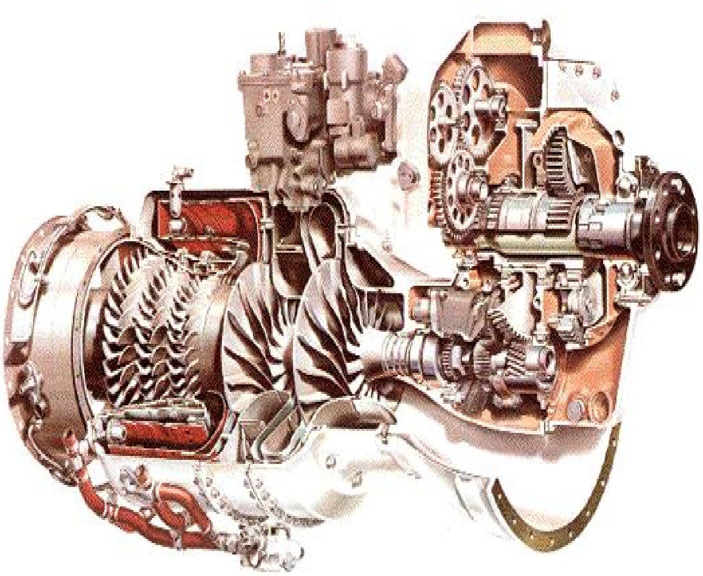


Fig.4.19 Turboprop engine

(Source: www.aircraftenginedesign.com)

**ii) Turbofan engine**

As the flight Mach number increases beyond 0.7, the propeller efficiency decreases rapidly due to the formation of shock waves at the tip of the propeller blade. Hence, for airplanes flying near Mach number of unity, a turbo-fan engine is used (Fig.4.20).In this engine a major portion of the power output (about 60%) is obtained as jet thrust and the rest as thrust from the fan. A fan has a smaller diameter as compared to the propeller and it is generally placed inside a duct. A ducted fan has a higher propulsive efficiency than a propeller.

It is observed in Fig.4.20 that all the air taken in by the fan does not go through the turbine. Incidentally the part of the engine consisting of the compressor, combustion chamber, turbine and nozzle is called 'Gas generator'. The ratio of the mass of the air that passes through the fan to the mass of air that passes through the gas generator is called 'Bypass ratio'.

Early turbofan engines had bypass ratio of 1:1. At present, it is around 6.5:1 and is likely to increase in future.

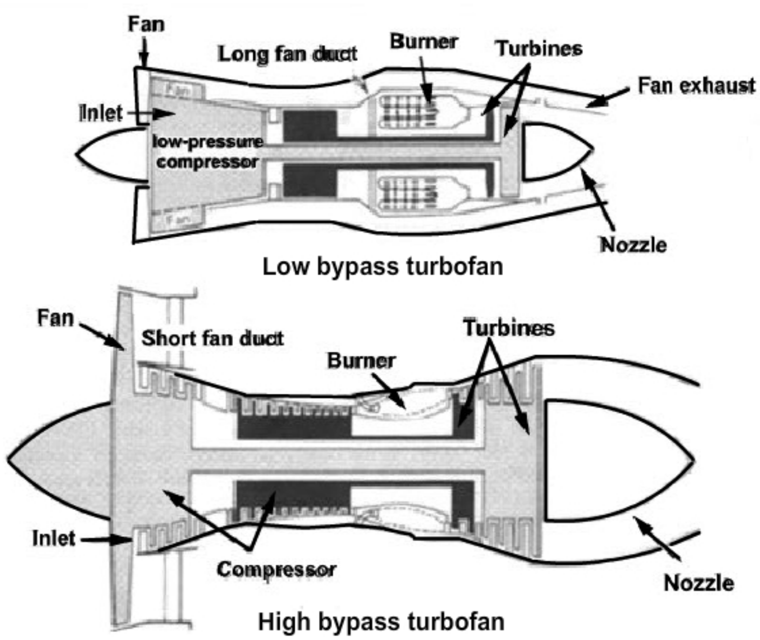



Fig.4.20 Turbofan engine

(Source : http://www.aerospaceweb.org)

**iii) Turbojet engine**

At supersonic Mach numbers, up to three, a turbo-jet engine is used. In this engine entire power output is through the jet thrust.