Chapter 4

Estimation of wing loading and thrust loading - 3

Lectures 11

Topics

4.6 Selection of wing loading based on rate of climb $(R/C)_{max}$

The specifications of an airplane generally include the rate of climb at sea-level. In this subsection, the wing loading is optimised such that the thrust required is minimum for the specified rate of climb $(R/C)_{max}$ or $(V_c)_{max}$.

Recall that $(V_c) = V\,(T_a - D)\,/\,W$, T_a = thrust available (4.37)

Or $V_c = V\,(\bar{t}_{R/C} - C_D \frac{1}{2}\rho V^2 \frac{S}{W})$; $\bar{t}_{R/C} = T_a / W$

$$= V\,(\bar{t}_{R/C} - C_D \frac{1}{2}\rho \frac{V^2}{p}) \tag{4.38}$$

Or $$\left(\bar{t}_{R/C}\right)_V = \left(\frac{V_c}{V}\right) + \frac{1}{2}\rho\,\frac{V^2}{p}C_D = \frac{V_c}{V} + q\,\frac{C_D}{p}\,;\; q = \frac{1}{2}\rho V^2 \tag{4.39}$$

Remark:

In a climb L = W cos γ where, γ is the angle of climb (see section 3.2.2). Taking L approximately equal to W, for estimation of drag in climb, is a reasonable approximation for the following reasons.

(A) When γ is small, cos $\gamma \approx 1$ and

(B) When γ is large, the lift dependent drag is only a small fraction of the thrust required. Hence, error in estimation of rate of climb is small. See section 6.4.1 of Ref.3.3 for further details.

Substituting, $C_D = F_1 + F_2\, p + F_3\, p^2$ in Eq.(4.39) gives :

$$\bar{t}_{R/C} = \left(\frac{V_c}{V}\right) + q\left(\frac{F_1}{p} + F_2 + F_3 p\right) \tag{4.40}$$

Equation (4.40) shows that $\bar{t}_{R/C}$ depends on both p and flight velocity(V).

Hence, the combination of p & V which will result in $(\bar{t}_{R/C})_{min}$ is given by:

$$\frac{\partial(\bar{t}_{R/C})}{\partial p} = 0 \text{ and } \frac{\partial(\bar{t}_{R/C})}{\partial V} = 0 \tag{4.41}$$

Eq.(4.40) can be rewritten as :

$$\bar{t}_{R/C} = \frac{V_c}{V} + \frac{F_1}{p}\frac{1}{2}\rho V^2 + \frac{1}{2}\rho V^2 F_2 + \frac{Kp}{\frac{1}{2}\rho V^2} \tag{4.42}$$

Taking derivatives of $\bar{t}_{R/C}$ with p and V and equating them to zero yields:

$$\frac{\partial(\bar{t}_{R/C})}{\partial p} = -\frac{F_1}{p^2}\frac{1}{2}\rho V^2 + \frac{K}{\frac{1}{2}\rho V^2} = 0 \tag{4.43}$$

$$\frac{\partial(\bar{t}_{R/C})}{\partial V} = -\frac{V_c}{V^2} + \frac{F_1}{p}\rho V + \rho V F - \frac{4Kp}{\rho V^3} = 0 \tag{4.44}$$

From Eq.(4.43), the optimum value of wing loading (p_{opt}) is:

$$p_{opt} = \frac{1}{2}\rho V^2 \sqrt{F_1/K} \tag{4.45}$$

Substituting for p_{opt} is Eq.(4.44) gives V_{opt} i.e.

$$-\frac{V_c}{V_{opt}^2} + \frac{\rho V_{opt} F_1}{\frac{1}{2}\rho V_{opt}^2 \sqrt{F_1/K}} + \rho V_{opt} F_2 - \frac{4K}{\rho V_{opt}^3}\frac{1}{2}\rho V^2 \sqrt{F_1/K} = 0$$

$$\text{Or } -\frac{V_c}{V_{opt}^2} + \frac{2\sqrt{F_1 K}}{V_{opt}} + \rho V_{opt} F_2 - \frac{2\sqrt{F_1 K}}{V_{opt}} = 0$$

$$\text{Or } V_{opt} = \left(V_c/\rho F_2\right)^{1/3} \tag{4.46}$$

Substituting for V_{opt} in Eq.(4.45) gives:

$$\left(p_{opt}\right)_{R/C} = \frac{1}{2}\rho\left(\frac{V_c}{\rho F_2}\right)^{2/3}\sqrt{F_1/K} \qquad (4.47)$$

Substituting for V as V_{opt} and p as $(p_{opt})_{R/C}$ in Eq.(4.42) yields $(\bar{t}_{R/C})_{min}$ as :

$$\left(\bar{t}_{R/C}\right)_{rmin} = \frac{V_c}{V_{opt}} + \frac{1}{2}\rho V_{opt}^2\left\{\frac{2F_1}{\left(p_{opt}\right)_{R/C}} + F_2\right\} \qquad (4.47a)$$

For the case considered in examples 4.1, 4.2 and 4.3, let the maximum rate of climb be 700 m/min or 11.667 m/s at sea level. Hence,

ρ = 1.225 kg/m^3, F_1 = 0.00884, F_2 = 1.447 x 10^{-6} m^2/N, K = 0.0444

Substituting in Eqs.(4.46), (4.47) and (4.47a) yields:

$$V_{opt} = \left(\frac{11.667}{1.225\times1.447\times10^{-6}}\right)^{1/3} = 187.41\ \text{m/s}$$

$$\left(p_{opt}\right)_{R/C} = \frac{1}{2}\times1.225\times187.41^2\sqrt{0.00884/0.0444} = 9599\ \text{N/m}^2$$

and $\left(\bar{t}_{R/C}\right)_{min} = \frac{11.667}{187.41} + \frac{1}{2}\times1.225\times187.41^2\left\{\frac{2\times0.00884}{9599} + 1.447\times10^{-6}\right\} = 0.1330$

Remarks:

(i) In Reference 4.4, chapter 4, the mathematical aspects of obtaining maximum/minimum of a function of many variables are discussed. In the present case, with two variables, the conditions for $\left(\bar{t}_{R/C}\right)$ to be minimum, are :

$$\left.\begin{aligned} &\text{(A)}\ \frac{\partial^2 t_{R/C}}{\partial p^2}\ \text{and}\ \frac{\partial^2 t_{R/C}}{\partial V^2}\ \text{are positive} \\ &\quad\ \text{and} \\ &\text{(B)}\ \frac{\bar{t}_{R/C}}{\partial p\,\partial V} < \frac{\partial^2(\bar{t}_{R/C})}{\partial p^2}\times\frac{\partial^2\left(\bar{t}_{R/C}\right)}{\partial V^2} \end{aligned}\right\} \qquad (4.48)$$

The reader may verify that these conditions are satisfied at V_{opt} and p_{opt}.

(ii) To get a feel of the variation of $\bar{t}_{R/C}$ with p and V, the case described above is considered again. It is noted that, V_C = 11.667 m/s, ρ = 1.225 kg/m^3,

F_1 = 0.00884, F_2 = 1.447 x 10^{-6} m^2/N and K = 0.0444. The variations of the values of $\bar{t}_{R/C}$, obtained by varying p & V in Eq.(4.40), are shown in Fig.4.7a. The values of V were selected between 80 to 200 m/s and those of p between 1000 to 15000. The minimum value is not easily discernible. However, the curves are nearly flat in the neighbourhood of $(\bar{t}_{R/C})_{min}$. This indicates that by allowing a small increase over $(\bar{t}_{R/C})_{min}$, a wide choice of wing loading would be available in which $(\bar{t}_{R/C})$ would be close to $(\bar{t}_{R/C})_{min}$.

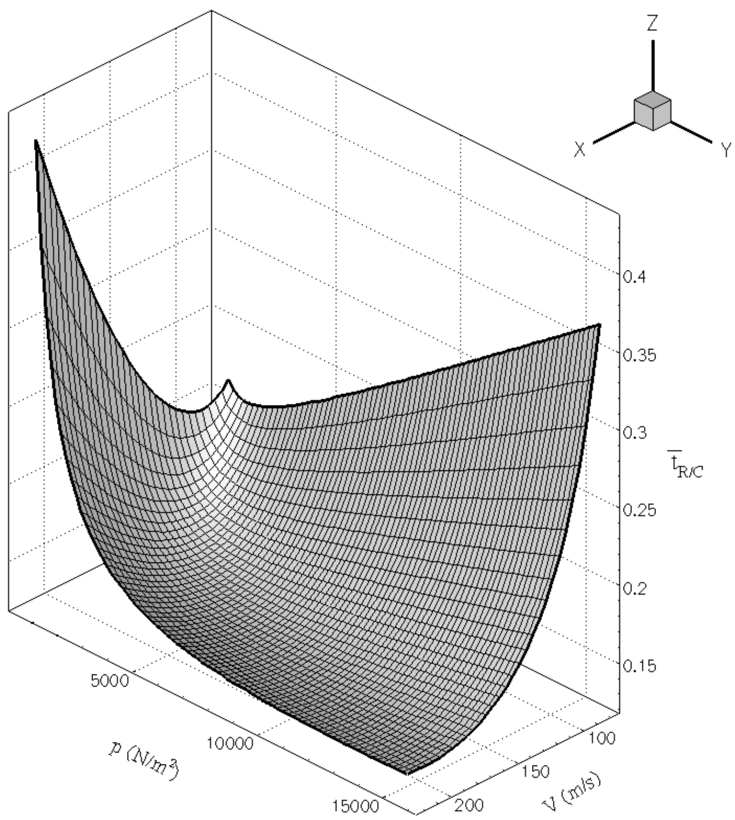

Fig.4.7a Variations of $\bar{t}_{R/C}$ with p and V for a jet airplane

(iii) In Ref.1.6, chapter IV a different approach to obtain p_{opt} and V_{opt} is suggested. The steps are as follows.

(A)Equation (4.39) shows that $\bar{t}_{R/C}$ depends on p & V. As a first step choose a value of 'V'. Then, in Eq.(4.40), V and hence, q have prescribed values.

Consequently, the optimum value of p for chosen value of V is obtained by differentiating Eq.(4.40) with p and equating to zero, i.e.

$$\frac{\partial \bar{t}_{R/C}}{\partial p} = 0 + q\left(-\frac{F_1}{(p_{R/C})_V^2} + 0 + F_3\right) = 0$$

$$\text{Or } (p_{R/C})_V = \sqrt{F_1/F_3} \tag{4.49}$$

(B) In view of the first term in Eq.(4.40), the value of $\left(\bar{t}_{R/C}\right)$ obtained with this value of $(p_{R/C})_V$ will depend on V. Let it be denoted by $\left(\bar{t}_{R/C}\right)_{min V}$ i.e.

$$(t_{R/C})_{min V} = \frac{V_c}{V} + q\left(\frac{2F_1}{(p_{R/C})_V} + F_2\right) \tag{4.50}$$

Obtain $\left(\bar{t}_{R/C}\right)_{min V}$ at different values of 'V' viz. V_1, V_2, …….. V_n. Figure 4.7b shows the plots of $\bar{t}_{R/C}$ vs p with 'V' as parameter for the case shown in Fig.4.7a.

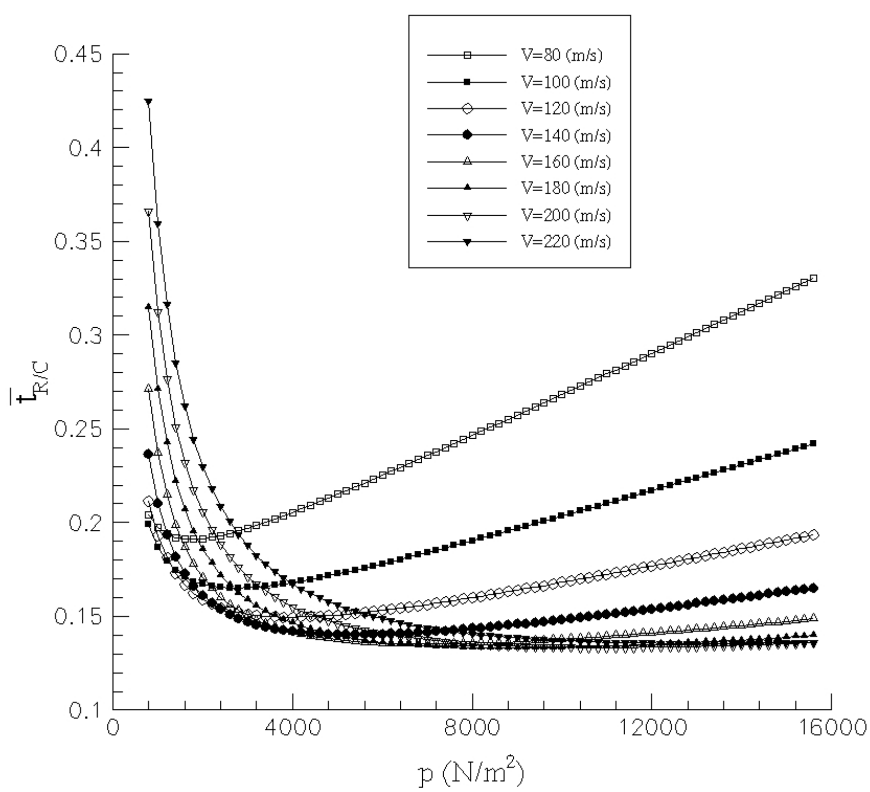

Fig.4.7b Variations of $\left(\bar{t}_{R/C}\right)$ vs p with V as parameter

(C)Join the points corresponding to $\left(\bar{t}_{R/C}\right)_{min\,V}$ on these curves. Minimum of these values gives $\left(\bar{t}_{R/C}\right)_{min}$. The corresponding p_{opt} and V_{opt} can be obtained by interpolating nearby values. To avoid cluttering in Fig.4.7b, the minimum values are presented in Table 4.2 and plotted in Fig.4.7c. It is seen that $\left(\bar{t}_{R/C}\right)_{min}$ of 0.1330 occurs for 'V' between 185 to 190 m/s and 'p' between 9354 and 9866 which are close to the values obtained by the exact analysis.

V (m/s)	q(N/m^2)	$(p_{R/C})_V$ N/m^2 (Eq.4.49)	$(\bar{t}_{R/C})_{min\,V}$ (Eq.4.50)
80	3920	1740	0.1914
100	6125	2733	0.1652
120	8820	3935	0.1496
140	12005	5357	0.1403
150	13781	6149	0.1374
160	15680	6996	0.1353
170	17701	7898	0.1339
180	19845	8855	0.1332
185	20963	9354	0.1330
190	22111	9866	0.1330
200	24500	10932	0.1334
220	29645	13228	0.1356
240	35280	15742	0.1393

Table 4.2 Variation of $\left(\bar{t}_{R/C}\right)_{min\,V}$ with V

Thus, the procedure suggested in Ref.1.6, chapter 4 appears to be valid in the present case.

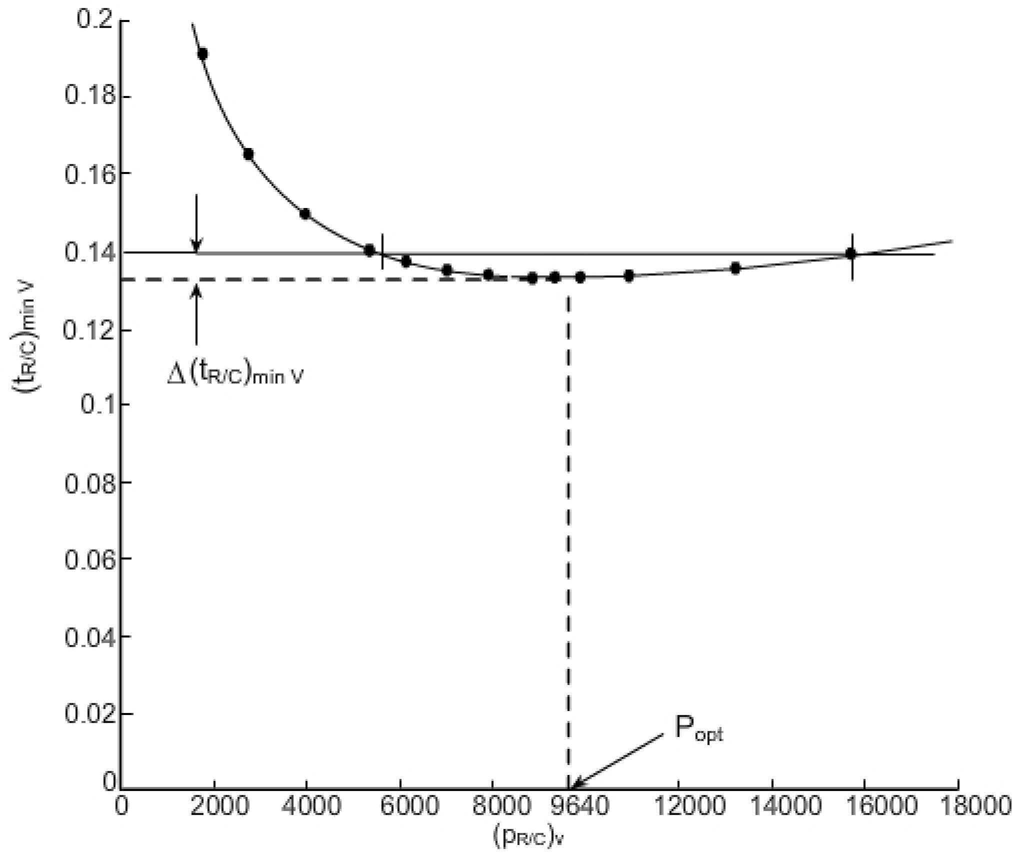

Fig.4.7c Variation of $(t_{R/C})_{minV}$ with $(p_{R/C})_v$

Figure 4.7c shows the variation of $(t_{R/C})_{minV}$ vs $(p_{R/C})_v$. The following can be observed.

(a) The minimum of $(t_{R/C})_{minV}$ i.e. $\left(\bar{t}_{R/C}\right)_{min}$ is 0.1330. It occurs for V between 185 to 190 m/s for W/S around 9600.

(b) The curve is very flat near the minimum. If a 5 % increase is permitted over $\left(\bar{t}_{R/C}\right)_{min}$ then $\left(\bar{t}_{R/C}\right)$ can be 0.133 x 1.05 = 0.1397. From Table 4.2 it is observed that, with this allowance, the wing loading can be in the range of 5660 to 15710 N/m^2 with $\left(\bar{t}_{R/C}\right)$ within 5 % of $\left(\bar{t}_{R/C}\right)_{min}$.

4.6.1 Effect of variation of thrust available with flight speed on $(p_{opt})_{R/C}$

The following three points may be noted.

(i) The derivation of Eq.(4.42) is based on Eq.(4.37). While carrying out the intermediate steps it is tacitly assumed that T_a is constant with flight velocity V.

(ii) In the case of high bypass turbofan engines, the thrust available at sea level is not constant with flight speed (V). It decreases significantly with V. Further, the rating of a jet engine is the thrust output at sea level at V = 0.

(iii) The final goal of the present optimisation, $(p_{opt})_{R/C}$, is to obtain a wing loading which will minimize the engine output, with engine setting adjusted for climb rating.

In view of these three facts and to get the minimum engine rating, required to satisfy the $(R/C)_{max}$ specification, the values of $\left(\bar{t}_{R/C}\right)_{min\,V}$ at different values of V, obtained above, should be multiplied by the ratio of the sea level static thrust (T_{sls}) to the thrust available at chosen V with climb setting. This ratio is denoted here as $\left\{T_{sls}/(T_v)_{climb}\right\}$. A different curve is obtained when this correction is applied and the resulting curve would give a different $(p_{opt})_{R/C}$. The details are explained with the help of example 4.6.

Example 4.6

For the jet airplane considered in examples 4.2 and 4.3, obtain the optimum wing loading when the maximum rate of climb is prescribed as 700 m/min at sea level. It is further given that the proposed engine may have a bypass ratio of 6.5. For such an engine the ratio of the sea level static thrust to the climb thrust $(T_{sls})/(T_V)_{climb}$ at different flight speed is approximately given in the table below.

Flight speed V (m/s)	80	100	120	140	150	160	170	180	190	200
$(T_{sls})/(T_V)_{climb}$	1.515	1.613	1.686	1.764	1.808	1.851	1.897	1.949	2.001	2.053

T_{sls} = Sea level static thrust ; $(T_V)_{climb}$ = Thrust at a flight speed of 'V'

Remark: This variation of $(T_{sls})/(T_V)_{climb}$ is based on the curves presented in Ref.1.14, chapter 9.

Solution:

The data from preliminary three-view drawing stage are

W = 60,000 kgf, W/S = 5500 N/m^2.

$$C_D = 0.00884 + 1.447\times10^{-6}\,p + 0.0444\,\frac{p^2}{q^2}$$

From Eqs.(4.49) and (4.50):

$$(p_{R/C})_V = (F_1/F_3)^{1/2}$$

$$\left(\bar{t}_{R/C}\right)_V = \frac{V_C}{V} + q\left\{\frac{F_1}{(p_{R/C})_V} + F_2 + F_3\,(p_{R/C})_V\right\} = \frac{V_c}{V} + q\left\{\frac{2F_1}{(p_{R/C})_V} + F_2\right\}$$

V_c = 700 m/min = 11.67 m/s

$$(p_{R/C})_V = (F_1/F_3)^{1/2} = q\left(\frac{0.00884}{0.0444}\right)^{1/2} = 0.4462q$$

$$\left(\bar{t}_{R/C}\right)_V = \frac{11.67}{V} + \frac{q}{(p_{R/C})_V}\left\{2F_1 + F_2\,(p_{R/C})_V\right\}$$

The calculations are presented in Table E4.6. The columns 1 to 4 in this table are the same as in Table 4.2.

V (m/s)	q (N/m^2)	$(p_{R/C})_V$ N/m^2	$(\bar{t}_{R/C})_V$	$T_{sls}/(T_v)_{climb}$	$(\bar{t}_{sl})_{R/C}$
80	3920	1740	0.1914	1.515	0.2899
100	6125	2733	0.1652	1.613	0.2664
120	8820	3935	0.1496	1.686	0.2522
140	12005	5357	0.1403	1.764	0.2475
150	13781	6149	0.1374	1.808	0.2484
160	15680	6996	0.1353	1.851	0.2504
170	17701	7898	0.1339	1.897	0.2540
180	19845	8855	0.1332	1.949	0.2596
190	22111	9866	0.1330	2.001	0.2662
200	24500	10932	0.1334	2.053	0.2739

Table E4.6 Optimization of Wing loading from$(R/C)_{max}$ consideration

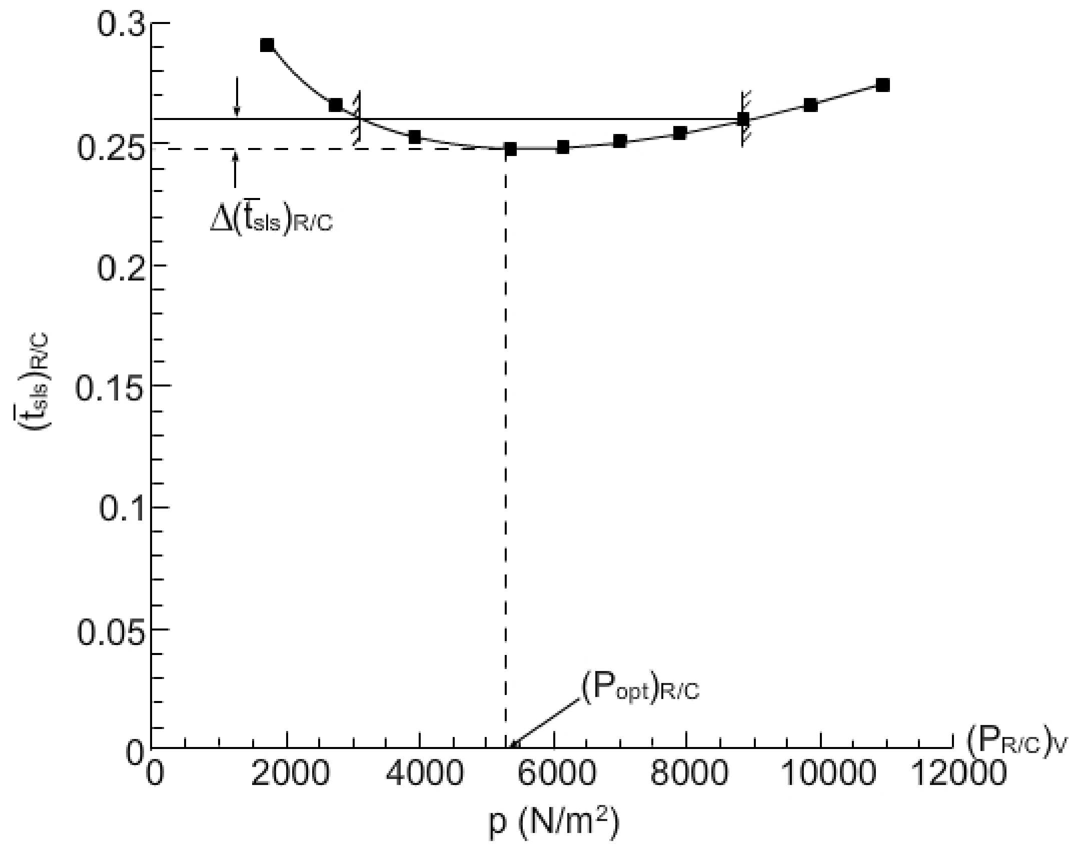

Fig.4.7d Variation of $(\bar{t}_{sls})_{R/C}$ with $(p_{R/C})_V$

The first column in Table E 4.6 gives the chosen values of velocity. The second column gives the dynamic pressure (q). The third column gives the optimum wing loading (p_{opt}) obtained from Eq.(4.49). The fourth column gives the corresponding thrust loading from Eq.(4.50). It shows that the minimum thrust loading of around 0.1330 occurs for wing loading of around 9866 N/m^2. To take into account the variation of available thrust with velocity, the fifth column gives the ratio of the sea level static thrust to the climb thrust available at chosen V. The sixth column is the product of the values in columns 4 and 5 and in turn gives the thrust loading required in terms of sea level static thrust. Figure 4.7d shows the variation of $(\bar{t}_{sls})_{R/C}$ with $(p_{R/C})_V$. From Fig.4.7d it is observed that, the lowest value of $(\bar{t}_{sl})_{R/C}$ or $(\bar{t}_{slsmin})_{R/C}$ of around 0.2475 occurs around W/S of 5350 N/m^2.

If a 5% increase if permitted over $\left(\overline{t}_{slsmin}\right)_{R/C}$, then $\left(\overline{t}_{sls}\right)_{R/C}$ can be 1.05 x 0.2475 = 0.2499. With this allowance, it is observed, from Fig.4.7d, that a wing loading in the range of 3120 to 8855 would give near optimum results.

Remarks:

(i) In the foregoing discussion the wing loading has been optimised and the required thrust loading has been obtained for $(R/C)_{max}$ at a specified altitude. However, the thrust loading for an actual airplane has also to satisfy FAR climb requirements. The requirements specify the climb gradient (G_T) under various conditions like flap setting, landing gear position, one engine inoperative etc. The climb gradient is defined as

$$G_T = (T-D)/W \tag{4.51}$$

Website of FAA can be referred to for details. Brief information is available in Appendix F of Ref.1.18.

Consult chapter 3 of Ref.1.12 part I for explanation of terms like initial climb segment requirement, second segment climb requirement, enroute climb requirement and balked landing requirement.

(ii) In example 4.6 the prescribed value of $(R/C)_{max}$ at sea level is taken as 700 m/min. However, this value would depend on the type of airplane. For example, a short range airplane would have a higher value of $(R/C)_{max}$ as it needs to attain the cruising altitude in a short duration. It may be added that, according to Ref.1.2, a short range transport is an airplane which has a range of less than 1200 nautical miles (2224 km).

4.7 Selection of wing loading based on range (R)

To derive an expression for the optimum wing loading, based on prescribed range, Ref.1.6 starts from the following basic relationship for a jet airplane which, with standard notations, appears as (see Eqs.(3.28) and (3.29) in subsection 3.5.5).

$$R = 3.6\int_{W_1}^{W_2} \frac{-V}{TSFC \times T} dW = 3.6\int_{W_1}^{W_2} \frac{-(L/D)V}{TSFC \times W} dW \tag{4.52}$$

Assuming, as in section 3.5.5,

$$\frac{L/D}{TSFC}V = \frac{L/D}{TSFC}\sqrt{\frac{2p}{\rho_0\,\sigma\,C_L}} \approx \text{constant ,yields:} \tag{4.53}$$

$$R = \frac{3.6}{TSFC}\sqrt{\frac{2}{\rho_0}}\,\frac{\sqrt{C_L}}{C_D}\sqrt{\frac{p}{\sigma}}\,\ln\left(\frac{W_1}{W_2}\right) \tag{4.54}$$

Approximating $\ln\left(\frac{W_1}{W_2}\right) = 2\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$, yields:

$$R = \frac{3.6}{TSFC}\sqrt{\frac{2}{\rho_0}}\,\frac{\sqrt{C_L}}{C_D}\sqrt{\frac{p}{\sigma}}\;2\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \tag{4.55}$$

$$= \frac{3.6}{TSFC}\sqrt{\frac{2}{\rho_0}}\,\frac{\sqrt{C_L}}{C_D}\sqrt{\frac{p}{\sigma}}\left(\frac{W_f}{W_{mean}}\right);\; W_{mean} = \frac{W_1 + W_2}{2};\; W_f = W_1 - W_2 \tag{4.56}$$

Noting, $C_L = \dfrac{2W}{\rho V^2 S} = \dfrac{p}{q}$ yields:

$$\overline{W}_f = \frac{R}{3.6}\sqrt{\frac{\rho_0}{2}}\,\frac{C_D}{\sqrt{C_L}}\,TSFC\sqrt{\frac{\sigma}{p}}\;;\; \overline{W}_f = \frac{W_f}{W_{mean}}$$

$$= \frac{R}{3.6}\sqrt{\frac{\rho_0}{2}}\,\frac{C_D}{\sqrt{p/q}}\,TSFC\sqrt{\frac{\sigma}{p}}$$

$$= \frac{R}{3.6}\sqrt{\frac{\rho_0}{2}}\;TSFC\,\sqrt{\sigma q}\,\frac{C_D}{p}$$

$$= \frac{R}{3.6}\sqrt{\frac{\rho_0}{2}}\;TSFC\,\sqrt{\sigma q}\,\left(\frac{F_1}{p} + F_2 + F_3\,p\right) \tag{4.57}$$

For optimum wing loading from range consideration,

$$\left(\frac{d\,\overline{W}_f}{dp}\right) = 0$$

This gives $(p)_{Rmax} = (F_1 / F_3)^{1/2}$; note F_3 involves dynamic pressure (q) (4.58)

For chosen h_{cr} and V_{cr} which determine the value of q, the optimum wing loading from range considerations is given by :

$$p_{Rmax} = q\,(\pi A e F_1)^{1/2} \tag{4.59}$$

Allowing 5% increase in $\overline{W}_f$, gives a range of wing loadings which would give near optimum results (Fig.4.8).

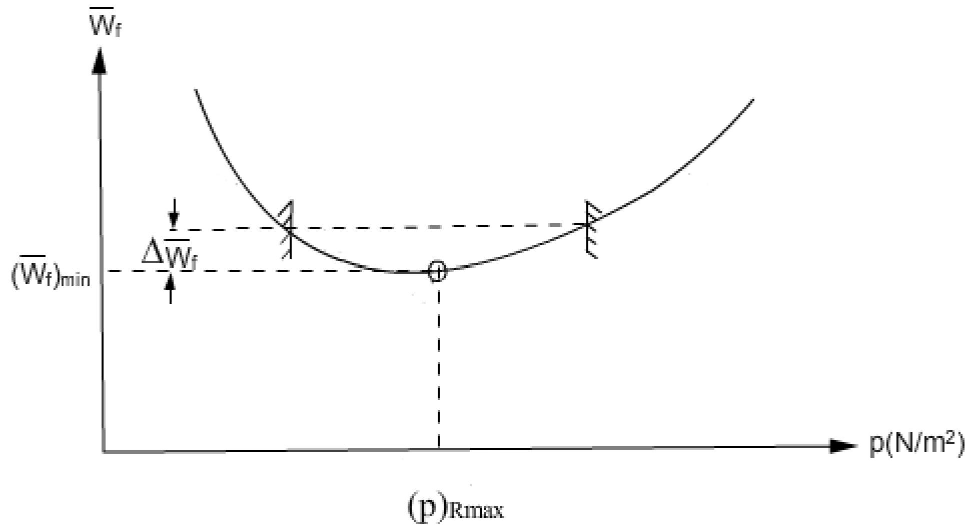

Fig.4.8 Fuel required for range vs wing loading

Example 4.7

For the jet airplane considered in examples 4.2 and 4.3, obtain the optimum wing loading from cruise consideration. It is prescribed that (a) the cruise takes place at M = 0.8 and h_{cr} = 11 km and (b) the range is 4000 km. Assume that TSFC is 0.6 hr^{-1}. Obtain $(\overline{W}_f)_{min}$ and the range of wing loadings when an increase by 5% is permitted above $(\overline{W}_f)_{min}$.

Solution:

The data, based on preliminary three-view drawing stage, are as follows:

W = 60,000 kgf, W/S = 5500 N/m^2

$$C_D = 0.00884 + 1.447\times10^{-6}\,p + 0.0444\,\frac{p^2}{q^2}$$

Prescribed conditions are:

M_{cr} = 0.8, h_{cr} = 11 km

Hence, speed of sound (a) = 295.1 m/s and density (ρ) = 0.364 kg/m^2

Consequently,

V_{cr} = 0.8 x 295.1 = 236.1 m/s

$q = 0.5\times\rho\times V^2 = 0.5\times0.364\times236.1^2 = 10145.3$ N/m^2

$F_3 = 0.0444/(10145.3)^2 = 4.314\times10^{-10}$ m^4/N^2

From Eq.(4.58):

$$p_{Rmax} = \sqrt{F_1/F_3} = \sqrt{\frac{0.00884}{4.314\times10^{-10}}} = 4526.9 \text{ N/m}^2$$

From Eq.(4.57)

$$\left(\bar{W}_f\right)_{min} = \frac{R}{3.6}\sqrt{\frac{\rho}{2}}\,TSFC\sqrt{q}\left(\frac{F_1}{p}+F_2+F_3p\right)$$

$$= \frac{4000}{3.6}\sqrt{\frac{0.364}{2}}\times0.6\times\sqrt{10145.3}\times\left(\frac{2\times0.00884}{4526.9}+1.447\times10^{-6}\right)$$

$$= 0.1533$$

Allowing 5 % increase over $\left(\bar{W}_f\right)_{min}$ gives:

$$\bar{W}_f = 0.1533\times1.05 = 0.161$$

From Eq.(4.57):

$$0.161 = \frac{4000}{3.6}\sqrt{\frac{0.364}{2}}\times0.6\sqrt{10145.3}\times\left(\frac{0.00884}{p}+1.447\times10^{-6}+4.314\times10^{-10}\,p\right)$$

which gives,

p = 3133 and 6540 N/m^2

Hence, wing loading in the range of 3133 to 6540 N/m^2 would give near optimum results.

Answers : $\left(\bar{W}_f\right)_{min}$ = 0.1553 ; Range of wing loading where, $\bar{W}_f$ is within 5% of $\left(\bar{W}_f\right)_{min}$: p = 3133 to 6540 N/m^2

Remarks :

(i) Equation (4.56) shows that the range(R) is proportional to $\frac{1}{TSFC\sqrt{\sigma}}$. From Fig.4.22 it is observed that at a chosen V_{cr} or M_{cr}, TSFC decreases slightly with altitude. Further density ratio (σ) is known to decrease with altitude. As a consequence of these, the airplanes with jet engines cruise at altitudes close to the service ceiling. However, when the distance between the starting airport and the destination is short, the airplane would generally cruise at 6-8 km altitude.

(ii) Reference 1.18, chapter 5, considers the range (R) as given by Eq.(3.30). As explained in subsection 3.5.5, the maximum range of a jet airplane occurs in a flight with C_L corresponding to $\left(C_L^{1/2}/C_D\right)_{max}$. Based on this consideration the optimum wing loading from the cruise consideration is obtained in Ref.1.18, as : $q\sqrt{\pi A e C_{Do}/3}$ comparing this result with that in Eq.(4.58) it is observed that this expression would give an optimum wing loading which is much lower than the wing loadings of airplanes in this category. This again brings out the advantage of the alternate break up of drag polar.