Chapter 3

Weight estimation

(Lectures 6,7 and 8)

Keywords: Dependence of airplane performance on airplane parameters and atmospheric characteristics; estimation of empty-weight fraction; estimation of fuel fraction; guidelines for drag polar and SFC for subsonic airplanes; iterative procedure for take-off weight calculation; trade-off studies.

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3.1 Introduction

An accurate estimate of the weight of the airplane is required for the design of the airplane. This is arrived at in various stages. In the last chapter, the procedure to obtain the first estimate of the gross weight was indicated. This was based on the ratio of the payload to the gross weight of similar airplanes. This estimate of the gross weight is refined in this chapter, by estimating (a) the fuel fraction i.e. weight of fuel required for the proposed mission of the airplane, divided by gross weight and (b) empty weight fraction i.e. empty weight of airplane divided the gross weight.

Remark :

An estimate of the fuel required for various phases of the mission of the airplane requires a knowledge of the following items.

a) Methods to estimate the performance of the airplane and

b)The dependence performance on airplane parameters and atmospheric characteristics.

The next section is intended to briefly recapitulate these two items.

3.2 Dependence of airplane performance on airplane parameters and atmospheric characteristics

The airplane performance parameters like maximum speed, maximum rate of climb, ceiling, range, endurance, rate of turn, take-off distance and landing distance, depend on weight of airplane (W) wing area (S), drag polar, thrust / power available, fuel weight etc. This dependence is examined in the following subsections.

3.2.1. Steady level flight – maximum flight speed (V_{max})

Figure 3.1 shows the forces acting on an airplane and the velocity vector in the steady level flight. The equations of motion, in standard notations, for this flight are:

$T - D = 0$

$L - W = 0$

Noting that, $L = 1/2\ (\rho V^2 S C_L)$, and $D = ½\ (\rho V^2 S C_D)$,

The thrust required $= T_r = D = W\ (C_D / C_L)$

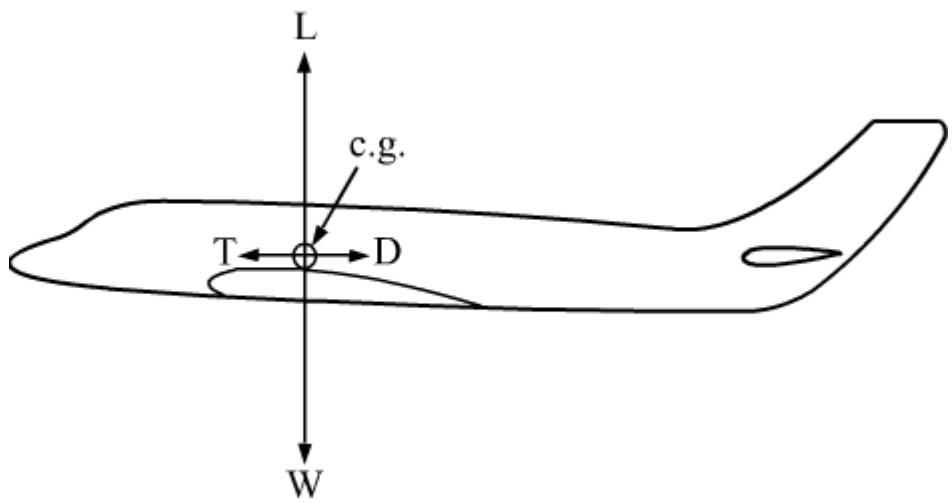

Fig.3.1 Steady level flight

Further, if the drag polar is parabolic i.e. $C_D = C_{D0} + KC_L^2$, then :

$$T_r = D = (1/2)\,\rho V^2 S C_D = (1/2)\,\rho V^2 S\,(C_{D0} + KC_L^2) \qquad (3.1)$$

$$\text{Or} \quad T_r = \frac{1}{2}\rho V^2 S\left[C_{D0} + K\left(\frac{W}{\frac{1}{2}\rho V^2 S}\right)^2\right] = \frac{1}{2}\rho V^2 S C_{D0} + \frac{2KW^2}{\rho S V^2} \qquad (3.2)$$

$$\text{The power required} = P_r = \frac{T_r V}{1000} = \frac{1}{2000}\rho V^3 S C_{D0} + \frac{K W^2}{500\rho V S} \qquad (3.2a)$$

Thus, T_r or P_r depend on W/S, ρ and the drag polar which is characterised by C_{DO} and K. Equations (3.2) and (3.2a) can be expressed as

T_r or P_r = f (W/S, ρ, drag polar).

Further, at maximum speed (V_{max}), (a)the thrust required (T_r) equals, the thrust available (T_a) and (b) the power required (P_r) equals the power available (P_a) , Hence, V_{max} of a jet airplane is dependent on W, T/W, ρ and drag polar i.e.

$$V_{max} = f\,\{W,\ W/S,\ T_a/W,\ \rho,\ \text{drag polar}\} \qquad (3.3)$$

The V_{max} of an airplane with engine-propeller combination is dependent on W, W/P, ρ and drag polar i.e.

$$V_{max} = f\,\{W,\ W/S,\ W/P_a,\ \rho,\ \text{drag polar}\} \qquad (3.3a)$$

3.2.2. Steady Climb – maximum rate of climb $(R/C)_{max}$

Figure 3.2 shows the forces on an airplane and the velocity vector in a steady climb.

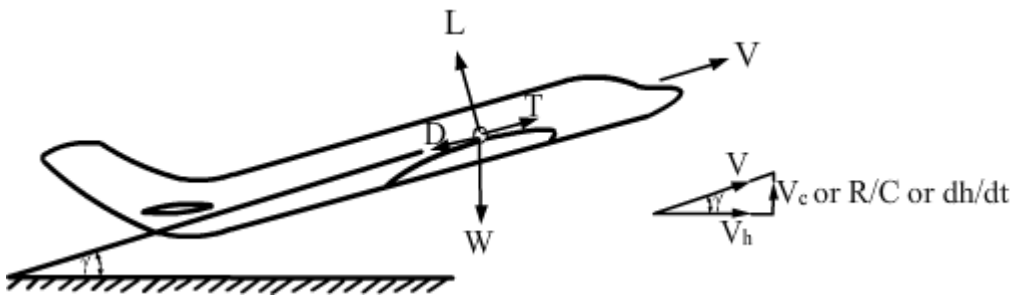

Fig.3.2 Steady Climb

The equations of motion are :

$T - D - W \sin\gamma = 0$

$L - W \cos\gamma = 0$

Hence,

$R / C = V \sin\gamma = V (T - D) / W$

If the drag in climb is approximated as equal to drag in level flight (D_L), then

$$R / C = (TV - D_L V)/ W = 1000(P_a - P_{rL}) / W \qquad (3.4)$$

where, P_{rL} is the power required in level flight at a flight velocity V and P_a is the power available at the same speed.

Hence, R/C is proportional to excess power. For a piston engined airplane, $V_{(R/C)max}$ is approximately equal to V_{mp}; where V_{mp} is the speed corresponding to minimum power in level flight. For a jet airplane, the ratio of $V_{(R/C)max}$ to V_{md} is greater than one and depends on the thrust to weight ratio(T/W); V_{md} is the speed corresponding to the minimum drag in level flight.

The expressions for D_L and P_{rL} are given in the previous subsection. Further, $(R/C)_{max}$ is generally prescribed at sea level and hence ρ in Eq.(3.2) and (3.2a) is equal to that at sea level. Keeping these factors in view the dependence of $(R/C)_{max}$ for a jet airplane can be expressed as :

$$(R/C)_{max} = f(W, W/S, T_a/W, \text{drag polar}) \qquad (3.5)$$

For an airplane with engine propeller combination

The expression is $(R/C)_{max} = f(W, W/S, W/P_a$, drag polar) (3.5a)

3.2.3 Absolute ceiling (H_{max}):

From the engine characteristics, it is known that the thrust horse power available (THP_a) and the thrust available (T_a) decrease with altitude. Further, at a chosen altitude the thrust horse power required (THP_r) and the thrust required (T_r) are minimum at flight speeds which are decided by the drag polar of the airplane. Keeping these in view it can be stated that (i) for an airplane with engine propeller combination, at absolute ceiling or H_{max}, the power available (THP_a) equals the minimum power required in level flight (P_{rmin}) and (ii) for an airplane with jet engine, at H_{max}, the thrust available (T_a) equals the minimum thrust required (T_{rmin}) in level flight. i.e.

At H_{max}, $(THP_a) = (P_r)_{min}$ or $(T_a) = (T_r)_{min}$

From performance analysis, it is known that,

$(T_r)_{min}$ and $(P_r)_{min}$ in level flight occur respectively at C_L corresponding to C_{Lmd} and C_{Lmp}. If the drag polar is parabolic, i.e. $C_D = C_{Do} + KC_L^2$, then:

$$C_{Lmd} = (C_{D0}/K)^{1/2} \quad , \quad C_{Dmd} = 2C_{Do} \text{ and } (C_D/C_L)_{min} = 2\sqrt{C_{Do}K}$$

$$C_{Lmp} = (3C_{D0}/K)^{1/2} \quad , \quad C_{Dmp} = 4C_{Do} \text{ and } \left(C_D/C_L^{3/2}\right)_{min} = \left(\frac{256}{27}C_{Do}K^3\right)^{1/4}$$

Hence,

$$T_{rmin} = W(C_D/C_L)_{min} = 2W\sqrt{C_{Do}K} \tag{3.6}$$

$$P_{rmin} = \frac{1}{1000}\left(\frac{2W^3}{\rho S}\right)^{\frac{1}{2}}\left(\frac{256}{27}C_{Do}K^3\right)^{1/4} \tag{3.7}$$

Hence, H_{max} depends on the drag polar, W / S and variation of engine output with altitude.

3.2.4 Range and endurance for airplanes with engine-propeller combination and with jet engine

Based on the performance analysis the Breguet formulae for range and endurance for airplanes with engine-propeller combination or jet engine, in standard notation, are given below. The range is in km and the endurance is in hours. For derivation of these formulae refer to section 7.4.2 of Ref.3.3.

(a) For an airplane with engine-propeller combination the range (R_{EP}) and endurance (E_{EP}) are:

$$R_{EP} = \frac{8289.3}{BSFC}\frac{\eta_p}{C_D / C_L}\log_{10}(W_1 / W_2) \tag{3.8}$$

$$E_{EP} = \frac{1565.2\,\eta_p}{BSFC(C_D / C_L^{3/2})}\left(\frac{\sigma S}{W_1}\right)^{1/2}\left[\left(\frac{W_1}{W_2}\right)^{\frac{1}{2}} - 1\right] \tag{3.9}$$

where , η_p = propeller efficiency; BSFC = specific fuel consumption is N/kW -hour with BHP in kW; σ = density ratio = $(\rho/\rho_{sealevel})$; W_1 and W_2 are respectively the weights of the airplane at the start and the end of the cruise.

(b) For a Jet-engined airplane the range (R_{jet}) and the endurance (E_{jet}) are given by:

$$R_{jet} = \frac{9.2}{TSFC\,(C_D / C_L^{\frac{1}{2}})}\left(\frac{W_1}{\sigma S}\right)^{\frac{1}{2}}\left[1 - \left(\frac{W_2}{W_1}\right)^{\frac{1}{2}}\right] \tag{3.10}$$

$$E_{jet} = \frac{2.303}{TSFC\,(C_D / C_L)}\log_{10}\left(\frac{W_1}{W_2}\right) \tag{3.11}$$

where, TSFC = Specific fuel consumption in N/N -hr or hr^{-1}.

From Eqs.(3.8) to (3.11) it can be concluded that :

$$R_{EP} = f\left\{BSFC, \eta_p, (C_L / C_D)_{max}, \frac{W_1}{W_2}\right\} \tag{3.12}$$

$$R_{jet} = f\left\{TSFC, (C_L^{1/2} / C_D)_{max}, \frac{W}{S}, \sigma, \frac{W_1}{W_2}\right\} \tag{3.13}$$

$$E_{EP} = f\left\{BSFC, \eta_p, (C_L^{3/2}/C_D)_{max}, \frac{W}{S}, \sigma, \frac{W_1}{W_2}\right\} \quad (3.14)$$

$$E_{jet} = f\left\{TSFC, (C_L/C_D)_{max}, \frac{W_1}{W_2}\right\} \quad (3.15)$$

3.2.5. Turning – minimum radius of turn (r_{min}) and maximum rate of turn ($\dot{\psi}_{max}$)

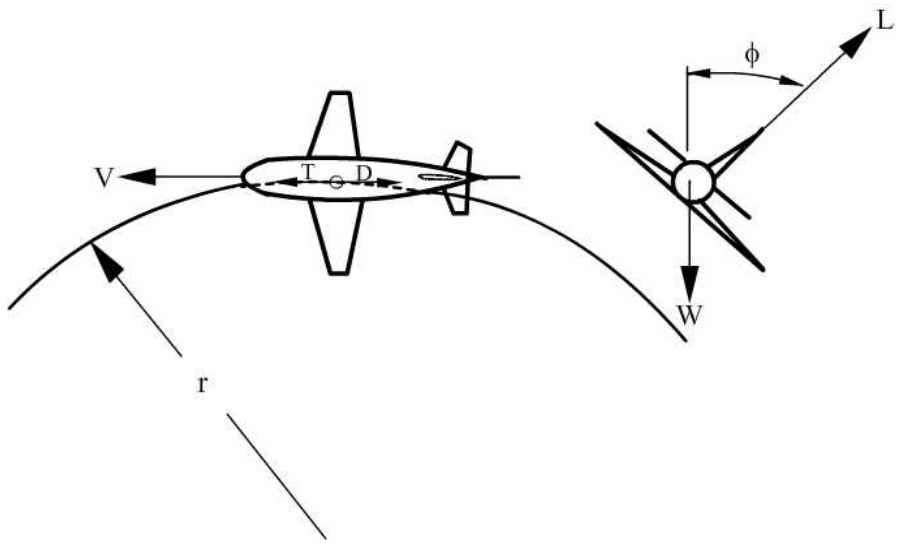

Fig.3.3 Turning flight

Fig. 3.3 shows the forces on an airplane and the velocity vector in a turn.

The Equations of motion in a steady, level, co-ordinated-turn are:

T-D =0

L cosϕ = W

L sinϕ = (W/g) (V^2/r)

where, ϕ = angle of bank and r = radius of turn

Hence,

$$r = V^2 / (g \tan \phi) \text{ and} \quad (3.16)$$

$$\text{rate of turn} = \dot{\psi} = (g \tan \phi) / V \quad (3.17)$$

From the above equations it may be noted that (a) the lift required in turn is greater than the lift required in level flight ($L_{turn} > L_{level}$) (b) the thrust required in

turn is greater than that required in level flight ($T_{rturn} > T_{rlevel}$) and (c) the load factor (n = L / W) is more than unity. We note that an airplane (a) is designed for a prescribed value of n_{max}, (b) has a value of C_{Lmax} depending on its wing design and (c) has a certain value of $(THP_a)_{max}$ or $(T_a)_{max}$ depending on the engine installed. Thus, a turn is limited by C_{Lmax}, n_{max} and the available thrust or power.

Consequently, for a jet airplane,

$$\dot{\psi} = f(T_a/W\ n_{max}, \text{drag polar}, C_{Lmax}) \quad (3.18)$$

$$r_{min} = f\ (T_a/W, n_{max}, \text{drag polar}, C_{Lmax}) \quad (3.19)$$

For an airplane with engine-propeller combination

$$\dot{\psi}_{max} = f(W/THP_a, n_{max}, \text{drag polar}, C_{Lmax}) \quad (3.18a)$$

$$r_{min} = f(W/THP_a, n_{max}, \text{drag polar}, C_{Lmax}) \quad (3.19a)$$

3.2.6. Take off distance (s_{t0})

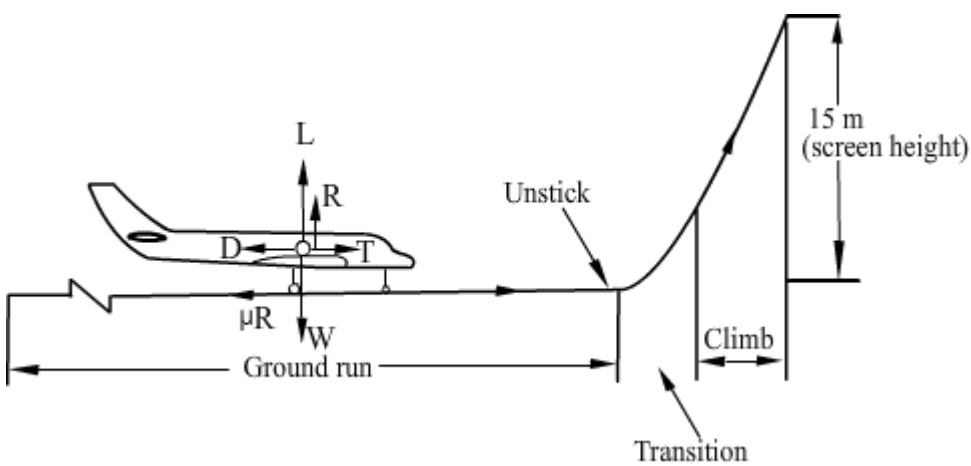

Fig.3.4 Phases of take off flight

Figure 3.4 shows the phases of take-off flight. It also shows the forces on the airplane during the ground run.

The equation of motion during the ground run is:

$$T - D - \mu R = (W / g)\ a$$

Ground reaction = R = W – L, where 'μ' is the coefficient of friction between the ground and the tyres and 'a' is the acceleration.

Hence,

$$\text{ground run} = s_1 = \int_0^{V_{t.o}} \frac{VdV}{a} = \frac{W}{g}\int_0^{V_{t.o}} \frac{V\,dV}{T - D - \mu(W - L)} \qquad (3.20)$$

$$V_{t.o} = k\sqrt{\frac{2W}{\rho S C_{Lmax}}}$$

where, k = 1.1 to 1.3. Hence, higher the value of $V_{t.o}$, longer is the take off run. Consequently, for reducing the take off run, low W/S, high C_{Lmax} and high T/W (or P/W) are suggested. The take-off distance (s_{to}) is proportional to take-off run (s_1).

Hence, for a jet airplane,

$$s_{to} = f\left(T_a/W, C_{Lmax}, \text{polar}, W/S, \mu\right) \qquad (3.21)$$

For an airplane with engine-propeller combination,

$$s_{to} = f\,(W/P_a, C_{Lmax}, \text{polar}, W/S, \mu) \qquad (3.21a)$$

It may be noted that the take-off distance is generally prescribed at sea level and hence 'ρ' is not included in Eqs (3.21) and (3.21a).

3.2.7 Landing distance(s_{land})

Figure 3.5 shows the phases of landing flight. The estimation of landing distance (s_{land}) is more complicated than that of s_{t0}. However, it depends on the square of stalling speed in landing configuration (V_s) and the type of braking system.

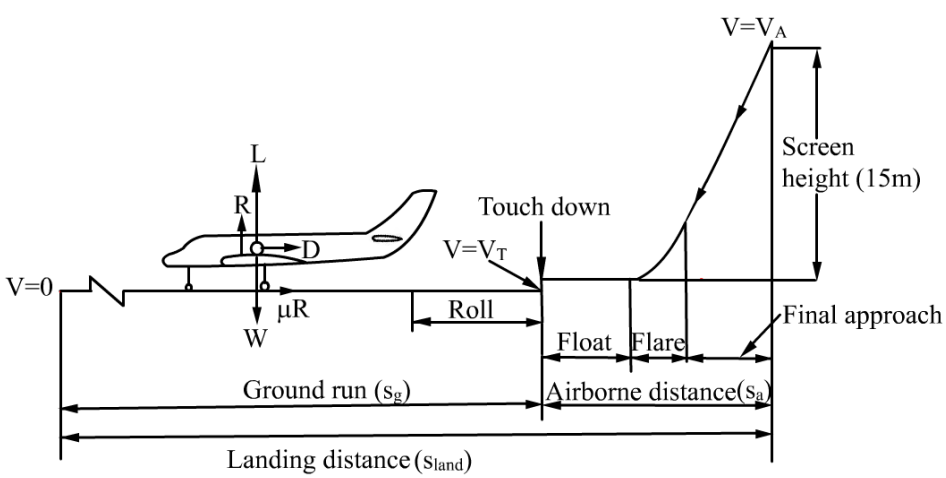

Fig.3.5 Phases of landing flight

The stalling speed is given by :

$$V_S^2 = \frac{2W}{\rho S C_{Lmax}}$$

Thus, for reducing the landing distance requires (a) low wing loading (W/S), (b) high value C_{Lmax} and (c) good braking system i.e.

$s_{land} = f(C_{Lmax}, W/S, \text{braking system})$ (3.22)

Remark:

From the discussion in sub-sections 3.2.1 to 3.2.7 it is observed that the performance of the airplane viz. maximum speed (V_{max}), maximum rate of climb $(R/C)_{max}$, ceiling (H_{max}), maximum range (R_{max}), maximum endurance (E_{max}), minimum radius of turn (r_{min}), maximum rate of turn ($\dot{\psi}_{max}$), take-off distance (s_{to}) and landing distance (s_{land}) are dependent on (i) airplane parameters like weight (W_0), Wing loading (W/S), maximum lift coefficient (C_{Lmax}), drag polar characterized by C_{D0} and K, and the fuel fraction (W_{fuel}/W_0) (ii) engine parameters like thrust loading (T/W) or power loading (W/P), specific fuel consumption (TSFC or BSFC) and propeller efficiency (iii) flight altitude which decides the atmospheric density (ρ) and speed of sound (a) and (iv) other parameters like maximum allowable load factor, type of braking system etc.

In this chapter and in chapters 4 to 9 the airplane parameters are determined based on various considerations. At the end of these steps a configuration, of the airplane under design, is obtained. This configuration will be much better than which was obtained in chapter 2. However, in actual practice the whole process will have to be carried out several times to arrive at the optimum configuration.

3.3 Weight Estimation – outline of approach

A good estimate of the gross weight (W_0) is necessary for further progress in the design process. Different approaches to estimate W_0 are presented in Refs.1.5, 1.6, 1.9, 1.12 and 1.18. Here the approach of Ref.1.18 is followed. In the procedure given in chapter 3 of Ref.1.18, the gross weight (W_0) is expressed as the sum of (a) the weight of the crew (W_{crew}), (b) the weight of payload ($W_{payload}$),

(c) the weight of fuel required for the mission (W_f) and (d) the empty weight (W_e) i.e.

$$W_0 = W_{crew} + W_{payload} + W_f + W_e \tag{3.23}$$

Remarks:

(i)The payload $\left(W_{payload}\right)$ is the weight for which the airplane is designed. For a passenger airplane $W_{payload}$ would be the weight of the passengers plus the baggage. For a cargo airplane $W_{payload}$ would be the weight of the intended cargo. For a trainer airplane $W_{payload}$ would be the weight of the trainee plus the instructor. For special purpose airplanes like agricultural airplane $W_{payload}$ would be the weight of the fertilizer etc. For a fighter airplane $W_{payload}$ would be the weight of the missiles, guns and ammunition. For a bomber airplane $W_{payload}$ would be the weight of bombs and associated equipment.

(ii)The crew members are : (a) the flight crew, (b) cabin crew in passenger airplanes and special crew in airplanes like reconnaissance/patrol or for scientific measurements.

(iii)In passenger airplanes the number of cabin crew is: (a) one cabin crew for about 30 passengers in economy class and (b) one cabin crew for about 15 passengers in first class (Ref. 1.9, chapter 3). Presently the number of flight crew would be two for commercial airplanes. On long range airplanes this number could be more to provide rest period for the pilot.

(iv)As regards the weights of the passengers and baggage are concerned, a value of 110 kgf per passenger (Ref.1.18, chapter 9) can be taken for long range airplanes (82 kgf for passengers plus the cabin baggage and 28 kgf for the check-in baggage). The value of 16 kgf for check-in baggage can be taken for short and medium range airplanes.

(v)For long range airplanes the weight of flight and cabin crew can be taken as 110 kgf. For short range airplanes it could be 85 kgf (Ref.1.15 chapter 6).

(vi)The weight of the trainee and the instructor in trainer airplanes can be taken of as 80 kgf. In combat airplanes the weight of the pilot could be 100 kgf due to the additional weight of protection gear.

(vii)In the approach of Ref.1.18 the empty weight is the gross weight of the airplane minus the weight of crew, payload and fuel. In some other approaches, in passenger airplanes (Refs. 1.15 and 1.16), the weights of operational items like food, water etc., are not included in the empty weight of the airplane.

Thus, W_{crew} & $W_{payload}$ are known from the design specifications.

W_f & W_e depend on gross weight (W_0).

Hence, Eq.(3.23) is rewritten as:

$$W_0 = W_{crew} + W_{pay} + \left(\frac{W_f}{W_0}\right)W_0 + \left(\frac{W_e}{W_0}\right)W_0$$

$$\text{Or } W_0 = \frac{W_{crew} + W_{pay}}{1 - \left(\frac{W_f}{W_0}\right) - \left(\frac{W_e}{W_0}\right)} \qquad (3.24)$$

The next two sections deal with the determination of W_e/W_0 and W_f/W_0.

3.4 Estimation of empty-weight fraction (W_e/W_0)

Reference 1.18, chapter 3 has analyzed the data on empty weights of different types of airplanes. When the data are plotted as (W_e / W_0) vs $\log_{10}(W_0)$ the resulting curves are roughly straight lines. This suggests that these curves can be approximated by an equation of the type:

$$\frac{W_e}{W_0} = A\, W_0^C \qquad (3.25)$$

where, W_0 = Take- off gross weight in kgf. The quantities A and c depend on the type of the airplane.

The values of A and c are presented in Table 3.1. The last column refers to the range of W_0 over which Eq.(3.25) can be used.

Type of airplane	A (W_0 in kgf)	c	Range of validity (W_0 in kgf)
Sailplane-unpowered	0.83	-0.05	150-700
Sailplane-powered	0.88	-0.05	200-1100
Homebuilt-metal/wood	1.11	-0.09	250-1800
Homebuilt-composite	1.07	-0.09	200-900
General aviation-single engine	2.05	-0.18	750-2300
General aviation-Twin engine	1.40	-0.10	1800-4000
Agricultural aircraft	0.72	-0.03	1300-7000
Twin turboprop	0.92	-0.05	3000-26000
Flying boat	1.05	-0.05	1200-9500
Jet Trainer	1.47	-0.10	2400-7400
Jet fighter*	2.11	-0.13	8200-58000
Military cargo/bomber*	0.88	-0.07	10000-400,000
Jet Transport	0.97	-0.06	10000-450000

* See remark (i) below.

Table 3.1 Values of A and c in Eq.(3.25)

(Adapted from Ref.1.18, chapter 3)

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Remarks:

i) Fighter and bomber airplanes may have wing with variable sweep. Such airplanes are heavier than those with constant sweep wings. Hence, multiply the value of W_e / W_0 by 1.04 in this case.

ii) Components of airplane made out of composites are lighter than those made out of aluminum. A reduction in weight by 20% can be expected when composite material is used in place of aluminum in a particular component. However, there are other components with metallic materials. Hence, an overall reduction of 5% is reasonable and the values of W_e / W_0 given in the above table be multiplied by

0.952. For homebuilt airplane using composites, a larger reduction in structural weight is expected and W_e / W_0 given by the above expression is multiplied by 0.85.