Chapter 3

Weight estimation - 3

Lecture 8

Topics

3.5.9 Drag polar of a typical turboprop airplane.

Typical values of the parameters, which influence the drag polar of such airplanes, could be (a) A =12, (b) λ = 0.4, (c) $\Lambda_{1/4} = 0$, (d) t/c = 18% (conventional airfoil), (e) M = 0.5, and (f) C_{lf} = 0.

From Ref.1.15, chapter 6 the values of R_w , T_f and A_f for this type of airplanes are:

R_w = 5, T_f = 1.4 , A_f = 0.75,

The other quantities in Eq. (3.42) are evaluated below.

(I) $\left(1-\frac{2C_{lt}}{R_w}\right) = 1.0$

(II) $\tau = \left[\left(\frac{R_w\text{-}2}{R_w}\right) + \frac{1.9}{R_w}\left\{1+0.526\left(\frac{t/c}{0.25}\right)^3\right\}\right]$

$$= \left(\frac{5\text{-}2}{5}\right) + \frac{1.9}{5}\left\{1+0.526\left(\frac{0.18}{0.25}\right)^3\right\} = 1.0546$$

(III) $1\text{-}0.2M + 0.12\left[\frac{M\times\cos^{1/2}\Lambda_{\frac{1}{4}}}{A_f\text{ - }(t/c)}\right]^{20}$

$$= 1\text{-}0.2\times0.5 + 0.12\left\{\frac{0.5\times1}{0.75-0.18}\right\}^{20} = 0.9087$$

Consequently, for this category of airplanes,

$C_{D0} = 0.005\times1.0546\times5\times1.4\times1\times0.9087\,S^{-0.1}$

$= 0.03354\ S^{-0.1}$ (3.50)

Remark:

Reference 1.12, volume VI, chapter 5 gives drag polar of Fokker F-27. This airplane has S = 70 m^2 and A = 12. From the figures in the aforesaid reference, it is observed that for this airplane C_{D0} = 0.022 and $(L/D)_{max}$ of 17.6. These two values indicate a value of 0.0359 for K.

From Eq.(3.50) a value of S = 70 m^2 would give C_{D0} of 0.02192, which is almost same as the actual value of 0.022.

Further in this case,

$$f(\lambda) = 0.005\left\{1+1.5(0.4-0.6)^2\right\} = 0.0053$$

Hence, $f(\lambda)\times A\times(10t/c)^{0.33} = 0.0053\times12\times(10\times0.18)^{0.33} = 0.07721$

Consequently,

$$1+\frac{0.142+f(\lambda)A(10t/c)^{0.33}}{\cos^2\Lambda_{1/4}} = 1+\frac{0.142+0.07721}{1} = 1.2192$$

Further, $\frac{0.1(3N_e+1)}{(4+A)^{0.8}} = \frac{0.1(0+1)}{(4+12)^{0.8}} = 0.01088$ and

$1 + 0.12\ M^6 = 1+0.12(0.5)^6 = 1.002$

From Eq.(3.43), the value of K for this category of airplanes, is :

$$K = \frac{1}{\pi \times A}\left[1.002\left\{1.2192 + 0.01088\right\}\right]$$

$$= \frac{1.2325}{\pi \times A}; \qquad (3.51)$$

with A = 12, K = 0.0327.

Remarks :

(i) This value of K appears to be about 10% lower than the value of 0.0359 for Fokker F-27.

(ii) C_{D0}=0.02192 and K = 0.0327 would give $(L/D)_{max}$ of 18.7. Ref.1.18, chapter 3, indicates that for a twin engined turbo-prop airplanes $(L/D)_{max}$ could be only about 17. Hence, it appears that the value of K in Eq.(3.51) should be modified to :

$$K = 1.356/\left(\pi A\right) \qquad (3.52)$$

Thus, for estimation of fuel fraction the drag polar of turboprop airplanes can be taken as :

$$C_D = 0.03354\, S^{-0.1} + \frac{1.356}{\pi A} \qquad (3.53)$$

(iii) An improved estimate of the drag polar would be obtained, after the geometrical parameters of wing, fuselage and tail surfaces are refined.(see section 9.1 of Appendix 10.2).

3.5.10 Drag polar of a typical low subsonic general aviation airplane with fixed landing gear

In this case the typical values of airplane parameters can be taken as:

$M = 0.2$, $A = 6$, $\lambda = 1$, $\Lambda_{1/4} = 0$, t/c = 0.15 (conventional airfoil), $C_{lf} = 0$

From Ref.1.15 chapter 6 the following values are obtained.

$R_W = 4$, $T_f = 2.0$, $A_f = 0.75$

In this case the other values would be:

$$\left(1 - \frac{2C_{lf}}{R_w}\right) = 1.0$$

$$1-0.2M+0.12\left\{\frac{M(\cos\Lambda_{1/4})^{1/2}}{A_f\ -\ (t/c)}\right\}^{20}$$

$$= 1-0.2\times0.2+0.12\left\{\frac{0.2\times1}{0.75-0.15}\right\}^{20} = 0.96$$

$$\tau = \left[\frac{4\text{-}2}{4}+\frac{1.9}{4}\left\{1+0.526\left(\frac{0.15}{0.25}\right)^3\right\}\right] = 1.029$$

Consequently,

$$C_{D0} = 0.005\times1\times1.029\times0.96\times4\times2S^{-0.1} = 0.03951S^{-0.1} \qquad (3.54)$$

For a typical value of $S = 15\ m^2$,

$C_{D0} = 0.03951 \times 15^{-0.1} = 0.0301$

Estimation of K

In this case the following values are obtained.

$1 + 0.12\ M^6 = 1$

$$f(\lambda) = 0.005\left[1+1.5(1-0.6)^2\right] = 0.0062,$$

$$1+\frac{0.142+f(\lambda)A\left(10\frac{t}{c}\right)^{0.33}}{(\cos\Lambda_{1/4})^2}$$

$$= 1+\frac{0.142+0.0062\times6\times(10\times0.15)^{0.33}}{1} = 1.1845, \text{ and}$$

$$\frac{0.1(3N_e+1)}{(4+A)^{0.8}} = \frac{0.1\times1}{(4+6)^{0.8}} = 0.01585$$

Hence, $K = \frac{1}{\pi A}\left[1\times\{1.1845+0.01585\}\right] = \frac{1.2003}{\pi A}$ or e = 0.833

Section 2.8.1 in Appendix A of Ref.3.3 has presented C_{Do}, A and $(L/D)_{max}$ for many low speed airplanes. Estimation of 'e' for general aviation airplanes like Piper Cherokee and Cessna sky hawk indicate that a value of 0.75 for 'e' is more appropriate. Taking e = 0.75 gives:

$$K = \frac{1.333}{\pi A} \qquad (3.55)$$

Thus, typical drag polar of this category of airplanes can be written as:

$$C_D = 0.03951S^{-0.1} + \frac{1.333}{\pi A} C_L^2 \qquad (3.56)$$

For A = 6 and S = 15 m^2, Eq.(3.56) gives :

From this $C_D = 0.0301 + 0.0708\, C_L^2$

These values of C_{D0} and K would give:

$$(L/D)_{max} = \frac{1}{2\sqrt{0.0301 \times 0.0708}} = 10.83$$

which is typical of $(L/D)_{max}$ for such airplanes (Ref.1.18, chapter 3).

Remark:

The value of C_{DO} could be lower by about 10% for airplanes made of FRP with very smooth surface.

General remarks on drag polar :

At this stage of preliminary design, from data collection, the first estimates of the wing area (S), the aspect ratio (A) and wing quarter chord sweep ($\Lambda_{1/4}$)are known. Based on this information C_{DO} and K can be obtained from the following formulae.

(A) High subsonic speed jet airplanes:

$$C_D = 0.02686\, S^{-0.1} + \frac{1}{\pi A}\left\{1.0447 + \frac{0.2078}{\cos^2 \Lambda_{1/4}}\right\} C_L^2 \qquad (3.57)$$

Note:

The value of 0.02686 in Eq.(3.56) is for $\Lambda_{1/4} = 30^\circ$. For $25^\circ \le \Lambda_{1/4} \le 35^\circ$ decrease this value by 0.4% for each 1° increase in $\Lambda_{1/4}$ or increase it by 0.4% for each 1° decrease in $\Lambda_{1/4}$.

(B) For airplanes with turboprop engine:

$$C_D = 0.03354S^{-0.1} + \frac{1.356}{\pi A} C_L^2 \qquad (3.58)$$

This expression in Eq.(3.58) is for fuselage shape typical of passenger airplanes. For cargo airplanes with rectangular fuselage, increase C_{DO} by about 20% and K by about 5%. The basis for increasing C_{DO} is that the contribution of fuselage to

C_{DO} is about 40% in passenger airplanes and this contribution would go up by about 50% for a rectangular fuselage. The change in the value of 'K' due to the change in fuselage cross section is small.

C) Airplanes with piston engine:

$$C_D = 0.03951S^{-0.1} + \frac{1.333}{\pi A} C_L^2 \tag{3.59}$$

Note:

Using the above expressions for C_{DO} & K the drag polar is obtained. Then the value of $(L/D)_{max}$, which is needed for obtaining the fuel fraction is given by Eq.(3.46) as :

$$(L/D)_{max} = \frac{1}{2\sqrt{C_{DO} K}}$$

3.5.11 Introduction to estimation of BSFC, η_p and TSFC

As mentioned earlier, here the attention is confined to subsonic airplanes. The piston engines are used in low subsonic airplanes ($M \leq 0.3$), the turboprop engines are used in the range of flight Mach numbers from 0.4 to 0.7 and the turbofan engines are used in the flight Mach number range of 0.7 to about 0.9. The details regarding the propellers and the engines are given in chapter 4. Here, the values of propeller efficiency and BSFC/TSFC are presented. These can be used as guidelines at this stage of design process.

3.5.12 BSFC and η_p of typical piston engined airplanes

The propeller efficiency (η_p) depends on the pitch setting (β), engine r.p.m, power output and the advance ratio (J = V/nd , where V is the flight velocity in m/s, n is the revolutions per second of the propeller and d is the propeller diameter). These airplanes may have (i) a fixed pitch propeller whose pitch setting could generally be chosen to give best efficiency in cruise or (ii) a propeller with two or three pitch setting and would give good efficiency both during take-off and cruise.

The BSFC of a piston engine depend on the r.p.m and power output. Generally the BSFC is higher at lower power settings. The power setting and flight velocity are lower in loiter than in cruise. At this stage of design, the following values of η_p and BSFC are suggested as ballpark values for calculations of fuel fraction. These values are based on (i) recommendations in chapter 3, Ref.1.18 (ii) calculation of performance of Piper Cherokee in Appendix A of Ref.3.3 and (iii) calculations of Rotax engine with three pitch propeller for a general aviation airplane.

(a)Fixed pitch propeller :

$$\text{Loiter}: \eta_p \approx 0.6, \text{BSFC} \approx 3.0 \text{ N/kW - hr} \quad (3.60)$$

$$\text{Cruise}: \eta_p \approx 0.8, \text{BSFC} \approx 2.7 \text{ N/kW - hr} \quad (3.61)$$

(b)Variable pitch propeller :

$$\text{Loiter}: \eta_p \approx 0.7, \text{BSFC} \approx 3.0 \text{ N/kW - hr} \quad (3.62)$$

$$\text{Cruise}: \eta_p \approx 0.8, \text{ BSFC} \approx 2.7 \text{ N/kW - hr} \quad (3.63)$$

3.5.13 BSFC and η_p of a typical turboprop powered airplanes

These airplanes are used as medium range transport airplanes. They have modern variable pitch propellers. The BSFC decreases slightly with increase in engine rating and with flight Mach number. Older turboprop engines had BSFC around 3.2 to 3.5 N/kW-hr (See for example, engine data in ch.6 of Ref.3.4). However, the current engines have BSFC around 2.9 N/kW-hr. (See for example, expression in ch.3 of Ref.1.15)

Keeping these aspects in mind the following values of η_p and BSFC are suggested as ballpark values for calculation of fuel fraction at this stage of design.

$$\text{Loiter}: (M \approx 0.3 \text{ at s.l.}): \eta_p \approx 0.75, \text{BSFC} \approx 2.85 \text{ N/kW - hr} \quad (3.64)$$

$$\text{Cruise}: (M \approx 0.5 \text{ at } h \approx 5 \text{ km}): \eta_p \approx 0.85, \text{ BSFC} \approx 2.7 \text{ N/kW - hr} \quad (3.65)$$

3.5.14 TSFC of a typical turbofan engine

The turbofan engines have lower, TSFC at lower altitudes and lower Mach numbers. Besides, the flight altitude and Mach number, the TSFC also depends on the bypass ratio (μ) of the engine. This ratio is the ratio of the air mass that passes through the bypass duct to the mass of air that passes through the gas generator (see also section 4.15.2). Older engines had bypass ratio around 3 but present day engines have this ratio between 5 to 8 and engines with this ratio of 13 are also planned.(Ref.1.21 and ch. 9 of Ref.1.14)

Reference 1.15 chapter 3 gives the following formula for the TSFC of high bypass ratio engines.

$$\text{TSFC} = c\left\{1-0.15\,\mu^{0.65}\right\}\left[1+0.28\left(1+0.063\,\mu^2\right)M\right]\sigma^{0.08} \qquad (3.66)$$

where, c = 0.7 , μ = bypass ratio , σ = density ratio = $\rho/\rho_{sealevel}$.

Note : Equation (3.66) is valid for h < 11 km. At h > 11 km TSFC is same as that at h = 11 km.

Taking typical values of M = 0.8 and h = 11 km, the following variation of TSFC with by-pass ratio is obtained.

Bypass ratio (μ)	5	8	10
TSFC (hr^{-1})	0.574	0.569	0.552

Table 3.2 Typical variation of TSFC with bypass ratio μ ; h = 11 km, M = 0.8.

Consider loiter at sea level and M = 0.3, Then Eq.(3.66) gives the following variation of TSFC with bypass ratio.

Bypass ratio (μ)	5	8	10
TSFC (hr^{-1})	0.488	0.419	0.373

Table 3.3 Typical variation of TSFC with bypass ratio (μ) ; h = sea level, M =0.3.

Thus, to choose the values of TSFC for cruise and loiter obtain the bypass ratio of the probable engine and then use Tables 3.2 and 3.3 or Eq.(3.66).

Remarks:

(i) The value of c = 0.7 given in Eq.(3.66) is based on data for large subsonic turbofan engines. If the TSFC of the engine, likely to be used on the airplane under design, is known under certain flight condition, then the constant 'c' in

Eq.(3.66) can be evaluated. Subsequently, Eq.(3.66) with this value of ‘c’ can be used to evaluate TSFC under desired conditions. For example, Boeing 787, dreamliner, is expected to have a TSFC of 0.54 hr^{-1} in cruise.

(ii) For TSFC of engines used on supersonic airplanes, see Ref.1.15, chapter 3 and Ref.1.18, chapter 3.

3.5.15 Fuel fraction for descent, landing and taxing

Following guidelines can be given based on the data in Ref.1.12 vol.I, chapter 2. The homebuilt, low speed single engined and agricultural airplanes generally fly close to the ground and the value of fuel fraction, for this phase of flight, of 0.99 is suggested. For other types of airplanes, except supersonic cruise airplane, the suggested value is 0.98. For supersonic cruise airplanes, the descent phase would be longer and consequently the suggested value is 0.937.

3.5.16 Fuel fraction for the mission

After calculating the fuel fractions in various phases of the mission, the weight of the airplane at the end of the mission is given by:

$$\frac{W_n}{W_0} = \frac{W_1}{W_0} \times \frac{W_2}{W_1} \times --- \times \frac{W_{n-1}}{W_{n-2}} \times \frac{W_n}{W_{n-1}} \qquad (3.67)$$

Consequently, the mission fuel fraction is :

$$1 - (W_n/W_0) \qquad (3.68)$$

Remark :

Generally an allowance of 6 % is provided for trapped fuel. Thus,

$$\frac{W_f}{W_0} = 1.06\left[1 - \frac{W_n}{W_0}\right] \qquad (3.69)$$

3.6 Iterative procedure for take-off weight calculation

Having obtained (W_f / W_0) and (W_e / W_0) the take-off weight can now be calculated. However, the expression for (W_e / W_0) involves W_0 and an iterative procedure is needed. This is illustrated through example 3.1.

Example 3.1

For the airplane considered in example 2.1, obtain the revised estimate of the gross weight. The specifications are reproduced below.

Type: Regional transport airplane with turboprop engine

No. of passengers: 60

V_{cruise}: Around 500 kmph at around 4.5 km altitude,

Safe range: 1300 km;

Service ceiling: 8000 m

Balanced field length for take-off : Around 1400 m

Solution:

As mentioned in section 3.3 the revised estimate of the gross weight (W_O) is obtained using the following steps.

(i) Obtain weights of payload and crew.

(ii) Estimate fuel function.

(iii) Estimate empty weight fraction.

(iv) Solve Eq.(3.24) iteratively.

I) Estimation of weights of payload and crew

For a sixty seater airplane the cabin crew consists of two members.

The flight crew would consist of two members – pilot and co-pilot.

Taking 100 kgf(82+18) as weight of passenger + carry on + check-in baggage gives:

W_{pay} = 60 x 100 = 6000 kgf

Taking 85 kgf as weight per crew member yields:

W_{crew} = 4 x 85 = 340 kgf

Thus, W_{pay} + W_{crew} = 6000 + 340 = 6340 kgf

II) Estimation of fuel fraction

(A) Warm up and take-off

From section 3.5.3 the fuel fraction for this phase:

$W_1 / W_O = 0.98$

W_O = take-off weight, W_1 = weight at the end of take-off phase

(B) Fuel fraction for climb

From section 3.5.4 the fuel fraction, in the present case, for this phase is:

$W_2 / W_1 = 0.99$. W_2 = weight at the end of climb.

Note: In the present calculation the horizontal distance covered in climb is ignored.

(C) Fuel fraction for cruise

Let W_3 = Weight at the end of cruise. From Eq.(3.34),

$$\frac{W_3}{W_2} = \exp\left\{\frac{-R \times BSFC}{3600 \times \eta_p \times (L/D)_{max}}\right\}$$

Before evaluating the above equation the following information be noted.

(i) The safe range is specified as 1300 km. However, the airplane may encounter head wind and would require extra amount of fuel. A simple way to take care of this is as follows.

It is assumed that the head wind is 15 m/s or 54 km/hr. The time of flight is 1300/500 = 2.6 hrs. Hence, additional distance to account for head wind would be: 54 x 2.6 = 140 km.

Further, in the event of landing being refused at the destination, the airplane may have to go to alternate airport. It is assumed that the distance would be 300 km. Though the flight to the alternate airport may be at an altitude different from cruising altitude it is assumed, at this stage of design, that flight is under cruise conditions.

Thus, taking into account the head wind and the provision for going to alternate airport, the range (R) would be:

R = 1300 + 140 + 300 = 1740 km

(ii) To get η_p and BSFC during cruise, it is assumed that the airplane has variable pitch propellers of modern design and the engine has BSFC corresponding to the present day engines. From section 3.5.13 :

$\eta_p = 0.85$ and BSFC = 2.7N/kW - hr

(iii) $(L/D)_{max}$

From of section 3.5.9 it is noted that the drag polar of a twin-engined turboprop airplane can be given approximately as:

$$C_D = 0.03354S^{-0.1} + \frac{1.356}{\pi A}C_L^2$$

From example 2.1 it is noted that, for this airplane the wing area (S) is estimated to be 61.43 m^2 and the aspect ratio (A) would be around 12. Thus, the drag polar would approximately be as :

$$C_D = 0.03354(61.43)^{-0.1} + \frac{1.356}{\pi \times 12}C_L^2 = 0.0222 + 0.036\,C_L^2 \qquad \text{(E3.1.1)}$$

Consequently,

$$(L/D)_{max} = \frac{1}{2\sqrt{0.0222 \times 0.036}} = 17.7$$

Hence,

$$\frac{W_3}{W_2} = \exp\left\{\frac{-1740 \times 2.7}{3600 \times 0.85 \times 17.7}\right\} = 0.917$$

(D) Fuel fraction for loiter

It is generally assumed that the airplane would have to wait for about 30 min before permission to land is granted. During this phase the airplane goes around in circular path at a speed corresponding to maximum endurance. Let, W_4 be the weight at the end of the loiter. From Eq.(3.40):

$$\frac{W_4}{W_3} = \exp\left\{\frac{-E \times BSFC \times V}{1000 \times \eta_p \times (L/D)}\right\}$$

From the remarks following Eq.(3.40), it is noted that in the above expression, the quantity "V" corresponds to flight velocity at minimum power which occurs at $C_L = C_{Lmp}$. For a parabolic polar :

$$C_{Lmp} = \sqrt{\frac{3\,C_{D0}}{K}}$$

In the present case,

$$C_{Lmp} = \sqrt{\frac{3 \times 0.0222}{0.036}} = 1.36$$

Velocity in a flight at C_{Lmp} would be :

$$V = \sqrt{\frac{2W_3}{\rho S C_{Lmp}}}$$

$$W_3 = \left(\frac{W_1}{W_0}\right)\left(\frac{W_2}{W_1}\right)\times\left(\frac{W_3}{W_2}\right)W_0 = 0.98 \times 0.99 \times 0.917\ W_0 = 0.889\ W_0$$

$$\text{Hence, } V = \sqrt{\frac{2\times 0.889\times 21500\times 9.81}{1.225\times 61.43\times 1.36}} = 60.56\ \text{m/s} = 218.01\ \text{kmph};$$

Note: ρ during the loiter corresponds to sea level conditions.

From section 3.5.13 (Eq.3.64) it is noted that in the loiter phase:

$\eta_p = 0.75$, BSFC = 2.85 N/kW - hr

Further, $(L/D)_{loiter} = 0.866\ (L/D)_{max}$

$= 0.866 \times 17.7 = 15.33$

Consequently,

$$\frac{W_4}{W_3} = \exp\left\{\frac{-0.5\times 2.85\times 60.56}{1000\times 0.75\times 15.33}\right\} = 0.992$$

(D) Fuel fraction for descent, landing and taxing (W_5/W_4)

Based on section 3.5.15 a value of $W_5/W_4 = 0.98$ is adopted.

(E) Using the above fractions yields:

$$\frac{W_5}{W_0} = \frac{W_1}{W_0}\times\frac{W_2}{W_1}\times\frac{W_3}{W_2}\times\frac{W_4}{W_3}\times\frac{W_5}{W_4}$$

$$= 0.98\times 0.99\times 0.917\times 0.992\times 0.98 = 0.865$$

Allowing 6% for trapped fuel, the fuel fraction is (Eq.3.69):

$$\frac{W_f}{W_0} = 1.06\left(1-\frac{W_5}{W_0}\right) = 1.06\times(1-0.865) = 0.1431 \qquad \text{(E3.1.2)}$$

(I) Empty weight fraction (W_e/W_0):

From Table 3.1, for a twin turboprop airplane (W_e/W_o) is given as :

$$\frac{W_e}{W_0} = 0.92\ W_0^{-0.05} \qquad \text{(E3.1.3)}$$

(II) Revised estimate of gross weight (W_0)

From Eq.(3.24):

$$W_0 = \frac{W_{pay} + W_{crew}}{1 - \dfrac{W_f}{W_0} - \dfrac{W_e}{W_0}}$$

In the present case:

$$W_0 = \frac{6000 + 340}{1 - 0.1431 - 0.92\, W_0^{-0.05}} \qquad \text{(E3.1.4)}$$

Since, the right hand side of Eq.(E 3.1.4) involves W_0, an iterative procedure is used to obtain W_0. A value of W_0, is guessed. This value is substituted in the right hand side of Eq.(E 3.1.4) and W_0 is obtained. If this value differs significantly from the guessed value, the iteration is continued.

The procedure is illustrated in Table E 3.1.1

W_0 (guessed) (kgf)	W_e/W_0 from Eq. (E3.1.3)	W_0 from Eq. (E3.1.4) (kgf)
21500	0.5587	21261
21261	0.5590	21282
21282	0.55897	21280

Table E3.1.1 Iterative procedure to obtain W_0

After the third iteration, the values in the first and third columns are almost the same. The iteration is stopped. Hence, the estimated gross weight (W_0) is :

21280 kgf = 208,757 N.

Remark:

Thus, the important ratios are :

W_e / W_0 = 0.559, W_f / W_0 = 0.143, W_{pay} / W_0 = 0.282 and

W_{crew} / W_0 = 0.016.

3.7 Trade-off studies

A designer would always like to examine as to what would be the changes if the specifications were varied slightly from those chosen earlier. The gross weight of the airplane is a very important quantity as it decides many parameters including the cost of the airplane. Hence, at this stage of preliminary design, it is a general practice to examine the effect of change of specifications on the gross weight. To illustrate this aspect, the following cases are considered and the gross weight is obtained in each case.

I) Let the number of passenger, instead of 60, be 50 or 70.

II) Let the safe range, instead of 1300 km, be 1000 km or 1600 km.

Repeating the calculations is left as an exercise to the reader. The final results are given in the Tables E3.1.2 and E3.1.3 .

No. of passenger	W_{pay} kgf	W_0 kgf	$\frac{W_e}{W_0}$	$\frac{W_f}{W_0}$	$\frac{W_{pay}}{W_0}$	$\frac{W_{crew}}{W_0}$
50	5000	18192	0.563	0.143	0.275	0.019
60	6000	21280	0.559	0.143	0.282	0.016
70	7000	24332	0.555	0.143	0.288	0.014

Table E3.1.2 Effect of number of passengers on W_0 ; Safe range = 1300 km, W_{crew} = 340 kgf

Safe range (km)	1000	1300	1600
Flight Time (hr)	2	2.6	3.2
Allowance for Head wind (km)	108	140	173
Range*	1408	1740	2073
W_3/W_2	0.932	0.917	0.902
W_f/W_0	0.1283	0.1431	0.1579
W_0	20354	21280	22291
W_e/W_0	0.5602	0.559	0.5577
W_{pay}/W_0	0.2948	0.282	0.2691
W_{crew}/W_0	0.0167	0.0160	0.0153

* Including allowance for head wind and 300 km of flight to alternate airport

Table E3.1.3 Effect of change in safe range on W_0; W_{pay} = 6000 kgf, W_{crew} = 340 kgf

Remarks:

(i) From Tables 3.2 and 3.3 the following observations can be made.

As the payload, in this case the number of passengers increases, the gross weight (W_0) increases. The cost of the airplane is generally proportional to the weight of the airplane and hence the price will also increase when W_0 increases. To cater to the needs of different customers, generally the airplane company offers different versions around a baseline configuration with varying payload, fuel capacity, engines etc. In such versions, the wing and empennage are generally common. The fuselage length would change depending on the payload. The engine output required would also change with gross weight and different versions may have different engine rating. All these aspects need to be considered in later stages of design.

(ii) The reader is advised to study features of ATR-72-200 and ATR-72-500 from Ref.1.21 or the internet(www.google.com). In these versions the dimensions of the wing, fuselage and empennage are same but the engines are different ;

ATR – 72 – 500 has a more powerful engine (Table 2.1). The weights of the two airplanes are only slightly different.(Table 2.1). The number of seats are varied from 68 to 74 by varying pitch of seats in the passanger cabin.

A close look at the performance, indicates that the take-off balanced field length (BFL) for ATR-72-500 is 1205 m as compared to 1408 m for the ATR – 72 – 200. The former also has a shorter landing run. Thus, reducing BFL seems to have been the criteria for evolving ATR-72-500 from the earlier version.

(iii) The reader may similarly study differences between De Hewilland

Dash 8 – Q 100, Q 200, Q 300 and Q 400.