

## Chapter 4

### Estimation of wing loading and thrust loading - 4

#### Lecture 12

#### Topics

##### 4.8 Other considerations for selection of wing loading

- 4.8.1 Selection of wing loading and thrust loading based on take-off balanced field length

##### Example 4.8

- 4.8.2 Selection of wing loading based on wing weight
- 4.8.3 Introductory remarks on selection of wing loading based on specific excess power and turn rate
- 4.8.4 Selection of wing loading based on specified acceleration or sustained turn rate
- 4.8.5 Selection of wing loading based on sensitivity to turbulence

##### Example 4.9

##### 4.8 Other considerations for selection of wing loading

Reference 1.6 gives the aforesaid five criteria for choice of wing loading viz. landing distance( $s_{land}$ ), prescribed flight speed ( $V_p$ ), absolute ceiling ( $H_{max}$ ), maximum rate of climb  $(R/C)_{max}$  and range ( $R$ ). Presently other aspects like take-off balanced field length, wing weight, rate of turn, excess specific power and sensitivity to turbulence are also considered (Ref.1.12, Vol I, chapter 3; Ref.1.15, chapter 5; Ref.1.18 chapter 5; Ref.1.19, chapter 11, and Ref.1.20, chapter 6). These aspects are briefly discussed in subsections 4.8.1 to 4.8.5.

##### 4.8.1 Selection of wing loading and thrust loading based on take-off balanced field length

Figure 3.4 shows the phases of take-off flight. Methods to estimate the take-off distance are available in the literature (see for example section 10.4 in Ref.3.3) These methods permit understanding the effects of wing loading, thrust loading,  $C_{LMax}$  and altitude on take-off distance. However, take-off is a critical phase of

flight operation and various eventualities are also taken into account. In the case of multi-engined airplanes, the possibility of the failure of one of the engines during take-off needs to be considered. If the engine failure takes place during initial stages of ground run, then the pilot can apply brakes and bring the airplane to a halt. If the engine failure takes place after the airplane has gained sufficient speed, then there are two possibilities viz. (a) apply brakes and stop the airplane, but this may need much longer runway length than in the case of take-off without engine failure, (b) instead of applying brakes, continue to fly with one engine off and take-off; but the take-off distance would be longer than when there is no engine failure. These two alternatives indicate the possibility of a speed, called “decision speed”. If the engine failure occurs at the decision speed then the distance required to stop the airplane is the same as that required to take-off with one engine inoperative. The take-off distance required when the engine failure takes place at the decision speed is called ‘Balanced field length (BFL)’. It is estimated as follows.

FAR 25 (see Ref.4.5) are used as a set of regulations for jet airplanes. They prescribed a procedure to obtain the balanced field length (BFL). Reference 3.2 has estimated BFL for many jet airplanes and found that BFL is a function of take-off parameter (TOP). The parameter is defined as:

$$TOP = \frac{(W/S)_{TO}}{\sigma C_{LTO} \left( \frac{T}{W} \right)_{TO}} \quad (4.60)$$

where, suffix ‘To’ refers to the take-off condition and T = sea level static thrust.

Based on this data the BFL in feet, when W/S in lbs/ft<sup>2</sup> is given as (Ref.1.12, Pt.I, chapter 3):

$$BFL \text{ (in ft)} = 37.5 TOP \text{ (in lbs/ft}^2\text{)} \quad (4.61)$$

When SI units are used the expression is:

$$BFL \text{ (in m)} = 0.2387 \frac{\left( \frac{W}{S} \right)_{TO}}{\sigma C_{LTO} \left( \frac{T}{W} \right)_{TO}} \quad (4.62)$$

where, W/S is in N/m<sup>2</sup>.

**Remarks:**

(i) Effect of number of engines on BFL

The data in Ref. 3.2 on which Eq.(4.62) is based, shows some scatter (Fig.3.7 of Ref.3.2). However, the data for airplanes with two, three and four engines show some definite trend; the BFL is more as the number of engines decrease. This is expected as for an airplane with two engines, when one engine is inoperative the thrust available would decrease to half of the full thrust, whereas for an airplane with four engines, with one engine inoperative, the thrust available would be three fourth of the full thrust. Consequently, BFL would be less for a four engined airplane as compared to that for a two engined airplane. Perhaps, based on this argument, Ref.1.18, chapter 5, suggests three different lines for BFL vs TOP curves for airplane with two, three and four engines. In SI units these lines can be expressed as:

$$\text{For two engined airplane, BFL (in m) = 0.2613 TOP (in N/m}^2\text{)} \quad (4.63)$$

$$\text{For three engined airplane, BFL (in m) = 0.2387 TOP (in N/m}^2\text{)} \quad (4.64)$$

$$\text{For four engined airplane, BFL (in m) = 0.2196 TOP (in N/m}^2\text{)} \quad (4.65)$$

(ii)The definition of TOP involves  $(W/S)_{TO}$ ,  $C_{LTO}$  and  $\left(\frac{T}{W}\right)_{TO}$ . Among these,  $C_{LTO}$

is generally taken as  $0.8 C_{Lmax}$ , (Ref.1.18, chapter 5) where  $C_{Lmax}$  equals maximum lift coefficient in landing configuration. The value of  $C_{Lmax}$  is known as the high lift devices to be used on the airplane have been tentatively chosen. Thus, when the balanced field length is prescribed, a value for the quantity  $\{(W/S)_{TO}/(T/W)_{TO}\}$ . can be obtained from Eqs.(4.63) to (4.65). Thus, a higher value of  $(W/S)$  would require a higher  $(T/W)$ . Generally a value of  $(T/W)$  is selected based on the data collection and one can arrive at the wing loading from take-off consideration. This is illustrated with the help of example 4.8.

**Example 4.8**

For the airplane considered in examples 4.1, 4.2 and 4.3, obtain the wing loading from take-off balanced field length (BFL) consideration. It is prescribed that the BFL be 2150 m for take-off at sea level. Further, the value of  $(T/W)_{TO}$  be limited to 0.3 and it be assumed that the airplane has two jet engines.

**Solution:**

From Eq.(4.63), the BFL for a twin engined configuration is given by:

$$\text{BFL (in m)} = 0.2613 \text{ TOP (in N/m}^2\text{)}$$

From example 4.1, the airplane has a  $C_{L_{\max}}$  of 3.0. Hence,

$$C_{L_{\text{TO}}} = 0.8 \times 3 = 2.4$$

Hence,

$$2150 = 0.2613 \text{ TOP}$$

$$\text{Or TOP} = 2150/0.2613 = 8228$$

Consequently,

$$8228 = \frac{(W/S)_{\text{TO}}}{1 \times 2.4 \times (T/W)_{\text{TO}}}$$

$$\text{Or } \frac{(W/S)_{\text{TO}}}{(T/W)_{\text{TO}}} = 8228 \times 2.4 = 19,747 \text{ N/m}^2$$

$$\text{If } (T/W)_{\text{TO}} = 0.3$$

$$\text{Then, } (W/S)_{\text{TO}} = 19747 \times 0.3 = 5924 \text{ N/m}^2$$

If BFL can be permitted to vary by  $\pm 10\%$  or between 1935 to 2365 m, then

$(W/S)_{\text{TO}}$  can be between 5332 to 6516  $\text{N/m}^2$  (Fig.4.9).

Note that BFL is directly proportional to  $(W/S)_{\text{TO}}$  or BFL vs  $(W/S)_{\text{TO}}$  is a straight line.

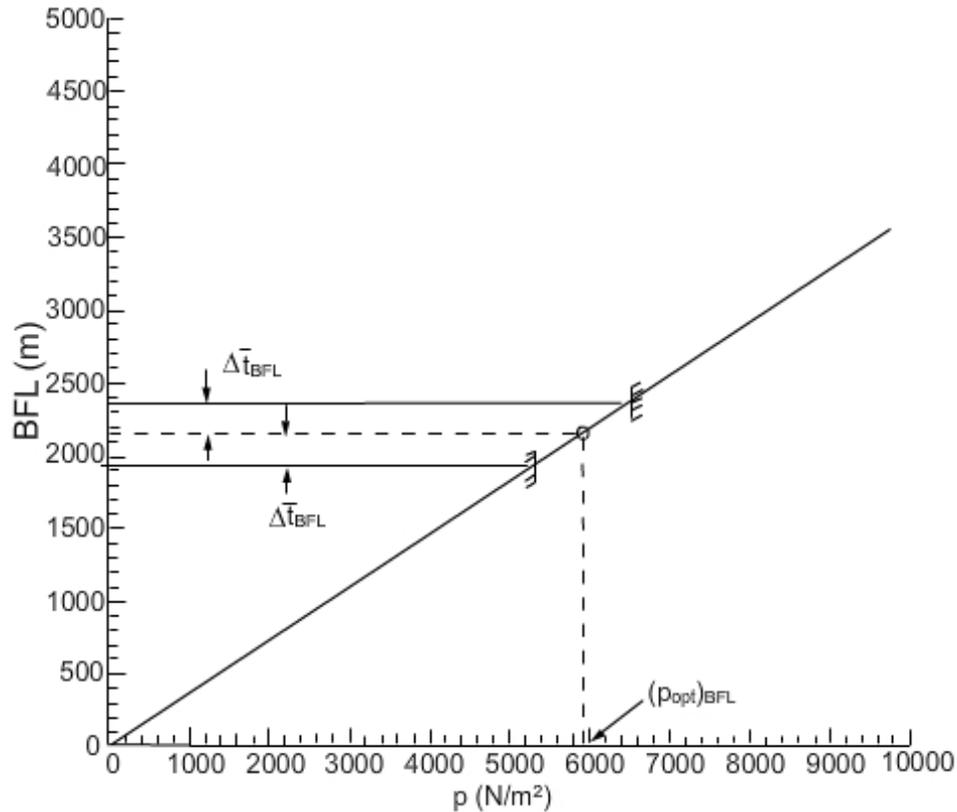


Fig.4.9 Variation of BFL with wing loading

#### 4.8.2 Wing loading based on consideration of wing weight

The aim of airplane design is to arrive at a configuration, which satisfies the design requirements, with minimum gross weight. This is achieved by minimizing the weights of components like wing, fuselage, etc. Reference 1.15, chapter 6; Ref. 1.18, chapter 15; Ref. 1.19, chapter 8 and Ref. 1.20, chapter 20 give expressions for wing weight ( $W_w$ ). It ( $W_w$ ) depends on wing parameters like aspect ratio, taper ratio, sweep, airfoil thickness etc. But it is also proportional to  $S^n$  where 'n' lies between 0.62 to 0.76 depending on the type of airplane. Thus, higher the wing area, larger is the wing weight and in turn the airplane weight. Thus, a smaller wing area or higher ( $W/S$ ) is suggested by this consideration.

#### 4.8.3 Introductory remarks on selection of wing loading based on specific excess power and turn rate

Though the attention in this course is focussed on subsonic airplanes, the criteria for selecting the wing loading for military airplanes are also briefly described here.

In addition to the requirements considered earlier, the military airplanes have the following requirements.

- (a) Rapid acceleration through a specified Mach number range for interceptor role  
(b) Rapid rate of turn in air-to-air combat role.

Before describing the choice of wing loading to satisfy these requirements, three concepts viz. specific excess energy, sustained rate of turn and instantaneous rate of turn are explained below.

**Specific excess energy:**

Consider an airplane in an accelerated climb. The forces acting on the airplane are shown in Fig.3.2. The equations of motion are:

$$T - D - W \sin \gamma = \frac{W}{g} a \quad (4.66)$$

where, a = acceleration along the flight path

$$L - W \cos \gamma = 0 \quad (4.67)$$

Further,

$$R/C = V_c = V \sin \gamma \quad (4.68)$$

Multiplying Eq.(4.66) by 'V' and writing 'a' as dV/dt gives :

$$TV - DV - WV \sin \gamma = \frac{W}{g} V \frac{dV}{dt}$$

$$\text{Or } TV = DV + W \frac{dh}{dt} + \frac{W}{g} \frac{d}{dt} \left( \frac{V^2}{2} \right) \quad (4.69)$$

In Eq.(4.69) the term 'TV' represents the available energy provided by the propulsion system. The term 'DV' represents the energy dissipated in overcoming the drag. The term 'W  $\frac{dh}{dt}$ ', represents the rate of change of potential

energy of the airplane and the term '(W/g){d(V<sup>2</sup>/2)/dt}' represents the rate of change of kinetic energy of the airplane. Thus, the total available energy is utilized in three ways viz. (a) overcoming the drag, (b) change of potential energy and (c) change of kinetic energy. If the flight takes place at V<sub>max</sub> is level flight, then entire energy is utilized in overcoming the drag and no energy is available

for acceleration or climb. Only at speeds in between  $V_{\min}$  and  $V_{\max}$  can an airplane climb and / or accelerate and the excess power  $(T-D)V$  has to be shared for increase of potential energy or kinetic energy or both. If climb takes place at  $V_{(R/C)\max}$  then no acceleration is possible. This leads us to the concepts of energy height and specific excess power.

### Energy height and specific excess power

Equation (4.69) can be rewritten as:

$$\frac{(T-D)V}{W} = \frac{d}{dt} \left( h + \frac{V^2}{2g} \right) \quad (4.70)$$

The term  $\left( h + \frac{V^2}{2g} \right)$  is denoted by  $h_e$  and called 'Energy height' as it has the dimension of height. It is the sum of potential energy and kinetic energy divided by the airplane weight ( $W$ ) or

$$h_e = h + \frac{V^2}{2g} \quad (4.71)$$

The term  $\frac{(T-D)V}{W}$  is called 'Specific excess power' and denoted by  $P_s$  i.e.

$$P_s = \frac{(T-D)V}{W} = \frac{dh_e}{dt} \quad (4.72)$$

Following observations are made, at this juncture.

(i) An airplane can accelerate only when  $P_s > 0$ . But in accelerated level flight, the lift equals weight and the drag is equal to that in level flight ( $D_L$ ) i.e.  $T_r > D_L$ ;

$T_r$  = thrust required.

(ii) From the discussion on turning flight in sub section 3.2.5, it is noted that in a turn the lift is more than the weight of the airplane and hence drag is more than that in level flight. i.e.  $T_r > D_L$ .

### Sustained rate of turn

From section 3.2.5, in a steady level, co-ordinated-turn,

$$T-D = 0 \quad (4.73)$$

$$L = W/\cos \phi \text{ or } n = \frac{L}{W} = \frac{1}{\cos \phi} \quad (4.74)$$

$$L \sin \phi = \frac{W}{g} \frac{V^2}{r} \quad (4.75)$$

$$\text{radius of turn} = r = V^2 / (g \tan \phi) \quad (4.76)$$

$$\text{rate of turn} = \dot{\psi} = (g \tan \phi) / V = \frac{g \sqrt{n^2 - 1}}{V} \quad (4.77)$$

$$\text{Expressing } D = qS(C_{D_0} + KC_L^2); q = \frac{1}{2} \rho V^2$$

$$\text{and noting that in a turn, } C_L = \frac{nW}{qS}, \quad (4.78)$$

$$\text{The thrust required is } T_r = D = qS \left\{ C_{D_0} + K \frac{n^2 W^2}{q^2 S^2} \right\} \quad (4.79)$$

Thus, for a given value of thrust available ( $T_a$ ) :

$$n = \frac{q}{W/S} \sqrt{\frac{1}{K} \left( \frac{T_a}{qS} - C_{D_0} \right)} \quad (4.80)$$

A sustained turn is a steady, level, co-ordinated - turn. Depending on the available maximum thrust and choice of  $q$  and  $W/S$ , the value of maximum sustained load factor ( $n_{ms}$ ) can be obtained from Eq.(4.80) substituting this in Eq.(4.80) gives the maximum sustained turn rate ( $\dot{\psi}_{ms}$ ). Taking into account, the variations of (a) thrust available at different flight speeds (or Mach number) and (b)  $C_{D_0}$  and  $K$  with Mach number, a plot of  $\dot{\psi}_{ms}$  vs  $V$  can be obtained. Similar plots can be obtained at different altitudes.

### Instantaneous turn rate

In a combat, it is advantageous to have a higher rate of turn. The turn rate can be higher than the sustained turn rate if the restrictions of (a) steady flight ( $T = D$ ) and (b) level flight in turn ( $L = W/\cos \phi$ ) are relaxed. Actually, the additional energy in a tighter turn is obtained by losing kinetic energy (slowing the airplane) and / or losing potential energy (losing height). The rate of turn in such a turn is called 'Instantaneous turn rate ( $\dot{\psi}_i$ )'. The load factor ( $n_i$ ) in such a turn would be higher than the maximum sustained load factor ( $n_{ms}$ ). However, the maximum

instantaneous turn rate ( $\dot{\psi}_i$ ) is limited by the following factors. (A) Maximum lift is  $L = qSC_{Lmax}$ . Hence,  $\psi_i$  is limited by  $C_{Lmax}$  at the Mach number during the flight.

(B) Allowable load factor ( $n_{max}$ ) for the airplane which could be as high as 8 or 9.

**Remarks:**

(i) In some fighter airplanes the direction of thrust vector can be altered in flight and this helps in higher rates of turn (see Ref.1.18, chapter 17 for further details).

(ii) From the above discussion it is evident that the difference between the available power (TV) and the power required to overcome the drag (DV) is crucial for acceleration and for maneuvers. The specific excess power ( $P_s$ ) is a quantity which includes both these factors. At the same time the load factor ( $n$ ) is unity in an accelerated level flight or climb but 'n' is high in manoeuvres. Hence, plots of  $P_s$  vs  $V$  at various altitudes with (a)  $n = 1$  and (b)  $n =$  say 5 are obtained. These plots are indicative of the performance of an airplane.

$$\text{when, } n = 1 : P_s = V \left[ \frac{T}{W} - \frac{qC_{DO}}{W/S} - \frac{K}{g} \frac{W}{S} \right] \frac{K}{q} (W/S)^2 \quad (4.81)$$

$$\text{when, } n > 1: P_s = V \left[ \frac{T}{W} - \frac{qC_{DO}}{W/S} - \frac{K}{g} n^2 \frac{W}{S} \right] \frac{K}{q} n^2 (W/S)^2 \quad (4.82)$$

Reference 1.20 presents plots for  $n = 5$  in chapter 3 and  $n = 1$  in chapter 4 for fighter airplanes.

#### 4.8.4 Selection of wing loading based on specified acceleration or sustained turn rate

The optimisation of the wing loading from these considerations is carried out in terms of  $P_s$ .

In an accelerated flight,  $n = 1$  and

$$(P_s)_{acc} = V \left( \frac{T}{W} - \frac{D}{W} \right)$$

$$\text{Let, } \bar{t} = \frac{T}{W}$$

$$D = qSC_D = qS \{ F_1 + F_2 p + F_3 p^2 \}$$

Then,

$$(P_s)_{acc} = V \left\{ \bar{t} - \frac{qS}{W} (F_1 + F_2 p + F_3 p^2) \right\}$$

$$\text{Or } (\bar{t})_{acc} = \frac{(P_s)_{acc}}{V} + q \left( \frac{F_1}{p} + F + F_3 p \right)$$

The desired value of  $P_s$  is prescribed at certain Mach number and altitude. Thus,  $P_s$ ,  $V$  and  $q$  are known. The optimum wing loading to minimize  $\bar{t}$  is obtained by:

$$\frac{d\bar{t}_{acc}}{dp} = 0, \text{ which gives,}$$

$$(p_{opt})_{acc} = \sqrt{\frac{F_1}{F_3}} = q \sqrt{\frac{F_1}{K}} \quad (4.83)$$

In a sustained turn  $n = n_{ms}$

Hence,

$$(P_s)_{st} = V \left\{ \bar{t} - \frac{qS}{W} (F_1 + F_2 p + n_{ms}^2 F_3 p^2) \right\}$$

Proceeding in manner similar to that for  $(P_{opt})_{acc}$ , gives

$$(p_{opt})_{st} = \frac{q}{n_{ms}} \sqrt{\frac{F_1}{K}} \quad (4.84)$$

**Remark:**

Though the values of  $F_1$  and  $K$  may be different during acceleration and turn, Eqs.(4.83) and (4.84) do show that the wing loadings for the two specifications are considerably different. The military airplanes would generally have both these requirements, and the designer has to strike appropriate compromise. In this context the reader can refer to section 2.3.3 of Ref.1.18. An exercise to design a light weight supersonic fighter is carried out there.

The specification, among other items, include (a) accelerate from  $M = 0.5$  to  $1.4$  in 30 sec. at 35000 ft (10700 m) (b)  $P_s = 0$  at  $n = 5$  at 30,000 ft (9144 m) at  $M = 0.9$  and  $1.4$  and (c)  $\dot{\psi} \geq 20^\circ / s$  (0.349 radian/s) at 350 knots (651 kmph) at 20000'(6096 m). The wing loading and thrust loading are optimized for take-off, landing, and sustained turn rate. It is found that  $T/W$  for desired acceleration in 30 s is too high. Hence, the target was revised to acceleration in 50 s.

The reader may also see descriptions of F-22 and F-35 airplanes on the internet.(www.google.com)

#### 4.8.5 Selection of wing loading based on sensitivity to turbulence

During its flight an airplane encounters turbulence due to various reasons. The response of the airplane to turbulence depends on (a) flight speed, (b) altitude, and (c) wing loading, slope of lift curve ( $C_{L\alpha}$ ) and structural properties of the wing.

For the purpose of preliminary design, Ref.1.15, chapter 5 gives the following guidelines for the choice of wing loading.

$$W/S \geq \frac{2.7 V_{MD} A}{\left(0.32 + \frac{0.16A}{\cos \Lambda_{\frac{1}{4}}}\right) \left\{1 - \left(M_{MD} \cos \Lambda_{\frac{1}{4}}\right)\right\}^{\frac{1}{2}}} \quad (4.85)$$

where,

$V_{MD}$  = maximum design speed

A = aspect ratio of wing

$\Lambda_{\frac{1}{4}}$  = quarter chord sweep of wing

$M_{MD}$  = Mach number corresponding to  $V_{MD}$

$V_{MD}$  is  $1.25 V_{cr}$  for low and medium speed airplanes. For subsonic jet airplanes,  $V_{MD}$  corresponds to  $M_{MD}$  at cruising altitude and  $M_{MD} = M_{cr} + 0.05$ . The procedure is illustrated through example 4.9.

#### Example 4.9

Consider a jet airplane having  $M_{cr} = 0.8$ ,  $h_{cr} = 11$  km,  $A = 9$  and  $\Lambda_{\frac{1}{4}} = 30^\circ$ . Obtain

the wing loading so that the airplane has adequate ride comfort.

#### Solution:

$h_{cr} = 11$  Km, hence speed of sound = 295.1 m/s

$M_{cr} = 0.8$ , hence  $M_{MD} = 0.85$

Consequently,  $V_{MD} = 0.85 \times 295.1 = 250.84$  m/s

Equation (4.85) gives :

$$\frac{W}{S} \geq \frac{2.7 \times 250.84 \times 9}{\left(0.32 + 0.16 \times \frac{9}{\cos 30^\circ}\right) \left\{1 - (0.85 \times \cos 30^\circ)^2\right\}^{1/2}}$$

Or  $W/S \geq 4650 \text{ N/m}^2$

**Answer: The wing loading should be greater than 4650 N/m<sup>2</sup>**

**Remark:**

Reference 1.14, chapter 11, defines a quantity called gust response parameter (GRP) as:

$$\text{GRP} = \frac{C_{L\alpha} A}{W/S} \quad (4.86)$$

GRP should lie below a value which depends on cruise Mach number. In other words  $W/S$  should be higher than certain value.