Module 6: Diffusion Flame

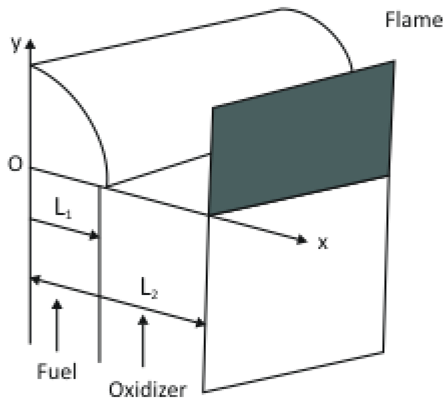
Lecture 29: Theoretical Analysis

The Lecture Contains:

- Theoretical Analysis
- Theoretical Analysis (Contd.)

Theoretical Analysis

Consider a 2D diffusion flame,



(Figure 29.1)

Assumptions:

i. 2D steady laminar inviscid flow.
ii. Velocity above the channel is constant everywhere $\Rightarrow V_x = 0$
iii. Fuel and oxidizer react in stoichiometric proportion at the flame surface with infinite reaction rate (Thin flame approximation).
iv. Binary diffusion between participating species.
v. Mass diffusion is along x-direction only.
vi. Unity Lewis number.
vii. Single step irreversible reaction.
viii. Radiation heat transfer is negligibly small.
ix. Constant thermophysical properties.
x. Mass diffusivity of both fuel and oxidizer are the same.
xi. Buoyancy force is neglected.

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Theoretical Analysis (Contd.)

Conservation equations:

Mass conservation:

$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} = 0$$

Using assumption (ii), we can have,

$$\frac{\partial(\rho V_y)}{\partial y} = 0 \Rightarrow \rho V_y = const.$$

Axial momentum conservation:

$$\frac{\partial(\rho V_x V_y)}{\partial x} + \frac{\partial(\rho V_y V_y)}{\partial y} = \frac{\partial}{\partial x}\left(\mu \frac{dV_y}{dx}\right) + (\rho_\infty - \rho)g$$

The pressure gradient in the y direction is approximated as $-\rho_\infty g$, which is known as Boussinesq approximation.

Species conservation equation:

$$\frac{\partial(\rho V_x Y_F)}{\partial x} + \frac{\partial(\rho V_y Y_F)}{\partial y} = \frac{\partial}{\partial x}\left(\rho D_{12} \frac{\partial Y_F}{\partial x}\right) + \dot{m}_F'''$$

$$\frac{\partial(\rho V_x Y_{0x})}{\partial x} + \frac{\partial(\rho V_y Y_{0x})}{\partial y} = \frac{\partial}{\partial x}\left(\rho D_{12} \frac{\partial Y_{0x}}{\partial x}\right) + \dot{m}_{0x}'''$$

Mass fraction of the product can be found from

$$Y_\rho = 1 - Y_F - Y_{0x}$$

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Theoretical Analysis (Contd.)

By thin flame approximation,

$$V_y \frac{\partial Y_F}{\partial y} = D_{12} \frac{\partial^2 Y_F}{\partial x^2}; \qquad V_y \frac{\partial Y_{0x}}{\partial y} = D_{12} \frac{\partial^2 Y_{0x}}{\partial x^2}$$

Single step irreversible reaction,

$$F + vOx \rightarrow (v+1)P$$

Universal concentration variables,

$$\left.\frac{dY_F}{dx}\right|_{F^-} = -\frac{1}{v}\left.\frac{dY_{Ox}}{dx}\right|_{F^+} \Rightarrow Y_R = Y_F = -\frac{Y_{Ox}}{v}$$

Rate of fuel transport from the centre to the flame surface is equal to stoichiometric rate of oxidizer transport.

Let Y_R be the mass fraction of the reactant,

Instead of solving two equations (For fuel and oxidizer), we can solve a single equation as given below,

$$V_y \frac{\partial Y_R}{\partial y} = D_{12} \frac{\partial^2 Y_R}{\partial x^2}$$

This analysis is known as the Burke-Schumann's analysis

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Theoretical Analysis (Contd.)

Above equation can be converted into a diffusion equation by substituting

$$y = V_y t$$

$$\frac{dY_R}{dt} = D_{12}\frac{d^2 Y_R}{dx^2}$$

Inner wall exists at $x = 0$ and outer wall at $x = L_2$

The initial and boundary conditions are as follows.

$$\text{At } t = 0, Y_R = (Y_R)_0 \;\; \text{At } x = 0, \frac{dY_R}{dx} = 0; \;\; x = L_2, \frac{dY_R}{dx} = 0;$$

Applying boundary conditions, we obtain a closed form series solution

$$\frac{Y_R}{(Y_R)_0} = \frac{(Y_F)_0 L_1}{(Y_R)_0 L_2} - \frac{(Y_{0x})_0}{(Y_R)_0 v}\left(\frac{L_2 - L_1}{L_2}\right) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi L_1}{L_2}\right)\cos\left(\frac{n\pi x}{L_2}\right)\exp\left(\frac{-yn^2\pi^2 D_{12}}{vL_2^2}\right)$$

where, $Y_R/(Y_R)_0$ is the non-dimensional mass fraction of the reactant.

$$\Rightarrow (Y_R)_0 = (Y_F)_0 + (Y_{Ox})_0/v$$

The infinite series must have a constant value at the flame surface as given below

$$E = \frac{(Y_{Ox})_0}{v(Y_R)_0}\left(\frac{L_2 - L_1}{L_2}\right) - \frac{(Y_F)_0}{(Y_R)_0}\left(\frac{L_1}{L_2}\right)$$

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Theoretical Analysis (Contd.)

The series solution depends on L_1/L_2, x/L_2 and ξ

At the burner rim, $\xi = 0$, the series constant (E) becomes a square wave

$$E = \left(\frac{L_1}{L_2}\right) and\ Y_R = (Y_F)_0\ for\ 0 < x \le L_1, \xi = 0$$

$$E = \left(\frac{L_1 - L_2}{L_2}\right) and\ Y_R = \frac{-(Y_{Ox})_0}{v}\ for\ L_1 < x \le L_2, \xi = 0$$

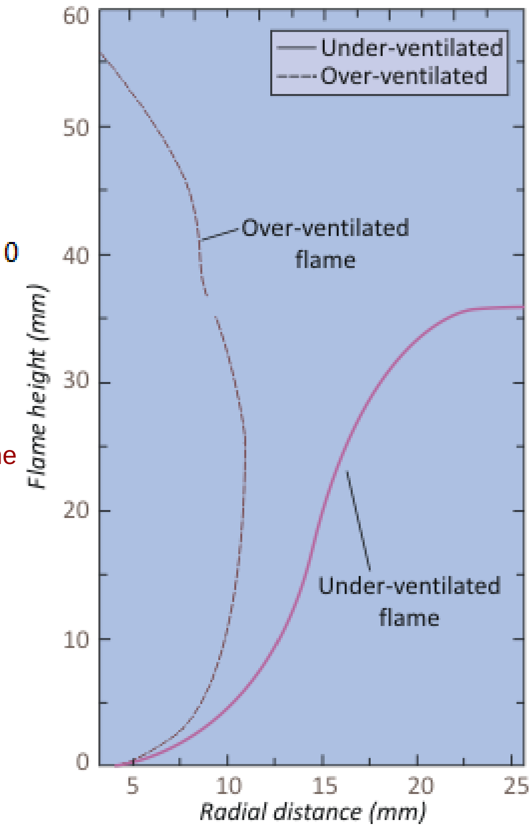
When F/A ratio is stoichiometric E becomes zero.

$$(Y_F)_0 = \frac{(Y_{Ox})_0}{v}\left(\frac{L_2 - L_1}{L_2}\right)$$

Roper extended the Burke-Schumann model by varying the velocity to vary along the axial direction.

The flame height is given by,

$$h_{F,Roper} = \frac{\dot{V}_F (T_\infty/T_F)}{4\pi D_\infty ln\left(1 + \frac{1}{v}\right)}\left(\frac{T_\infty}{T_{ad}}\right)^{0.67}$$



(Figure 29.2)

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