Module 3: Physics of Combustion

# Lecture 16: Boundary layer solutions

## The Lecture Contains:

## Boundary layer solution

### Approximate solution for steady 2D incompressible flow over a flat plate

Mass conservation

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

Momentum conservation

$$\frac{\partial(\rho V_x V_x)}{\partial x} + \frac{\partial(\rho V_x V_y)}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial V_x}{\partial x}\right) + \frac{\partial}{\partial x}\left(\mu \frac{\partial V_x}{\partial y}\right)$$

$$\frac{\partial(\rho V_x V_y)}{\partial x} + \frac{\partial(\rho V_y V_y)}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial V_y}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial V_y}{\partial y}\right)$$

By boundary layer approximations,

$$V_x > V_y; \frac{\partial V_x}{\partial x} \gg \frac{\partial V_x}{\partial y}; \frac{\partial V_y}{\partial y} \gg \frac{\partial V_y}{\partial x}$$

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## Boundary layer solution

By carrying out order of magnitude analysis,

Mass conservation

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

Momentum conservation

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + V\frac{\partial^2 V_x}{\partial y^2}$$

$$\frac{\partial P}{\partial y} = 0$$

From the above equation, pressure remains constant along 'y' direction.
Analytical method of Blasius gives exact solution of the above equations.

Relation between B. L. thickness and Re is $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$

Drag coefficient for laminar flow over flat plate: $C_D = \frac{b\int_0^L \tau_w dx}{0.5\rho V_x^2 bL} = \frac{1.328}{\sqrt{Re_L}}$

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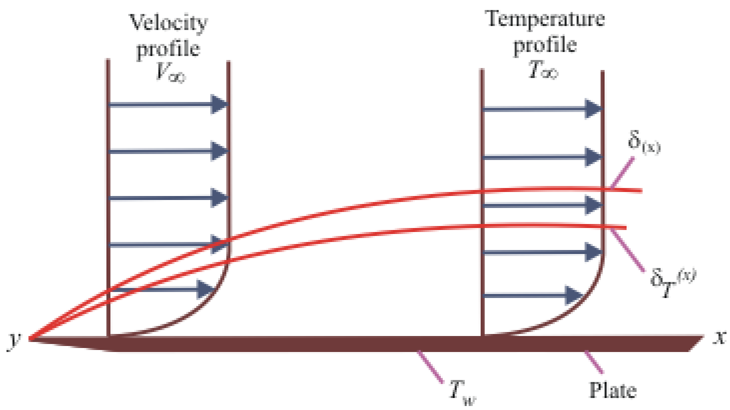
## Thermal Boundary layer

- Free stream temperature $T_\infty$
- Flat plate temperature $T_w$ $(T > T_w)$
- Heat transferred from fluid to plate




(Figure 16.1)

Thermal boundary layer thickness , $\delta_T \rightarrow$ Value of y for which

$$(T - T_w)/(T_\infty - T_w) = 0.99$$

Thermal boundary layer grows with increase in distance from the leading edge

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Local heat flux due to convection,

$$\dot{q}''_w = -h(T_\infty - T_w)$$ (Newton's law of cooling)

Local heat flux at the wall,

$$\dot{q}''_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0}$$ (Fourier's law of conduction)

Combining there two equations, the convective heat transfer coefficient ***(h)*** is given by,

$$h = \frac{-k\left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_\infty - T_w)}$$

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Using Pohlhausen method, Nusselt number (Nu) can be expressed as

$$Nu_x = 0.322 Re_x^{1/2} Pr^{1/3}$$

Valid for $Pr > 0.5$ can be used for most of the gases

For laminar fully developed pipe flow, $Nu_D = 3.36$ Valid for constant temperature

For laminar fully developed pipe flow, $Nu_D = 4.36$ Valid for constant heat flux

Average Nusselt number for developing pipe flow,

$$\overline{Nu}_D = 1.86 \left(\frac{Re_D Pr}{L/D}\right)\left(\frac{\mu}{\mu_w}\right)^{0.14} \quad \begin{cases} Pr > 0.5 \\ 0.0044 < \dfrac{\mu}{\mu_w} < 9.95 \end{cases}$$

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