The Lecture Contains:

Previous Next

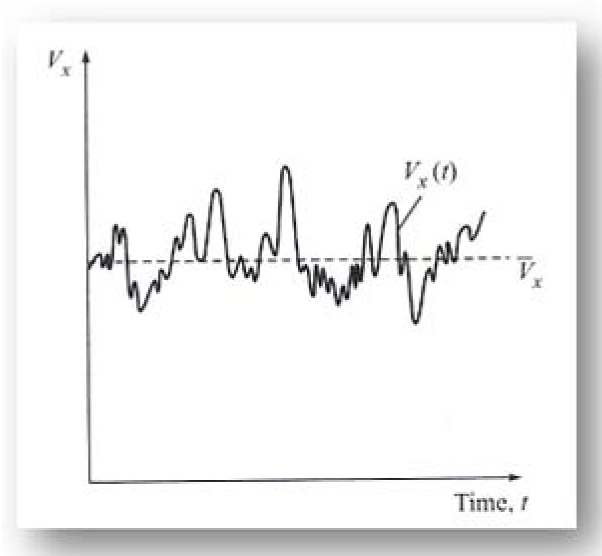
Transport in Turbulent Flow

Turbulent Flow:

- At high Reynolds and Grashof's number, the properties, velocity and temperature exhibits random variation.
- Eddies move randomly back and forth across the adjacent fluid layers.
- Turbulence reduces the B.L. thickness.
- Enhanced mass, momentum, and energy transfer rates.

$$V_x = \bar{V}_x + V_x'$$

$$V_y = V_y + V_y'$$



(Figure 17.1)

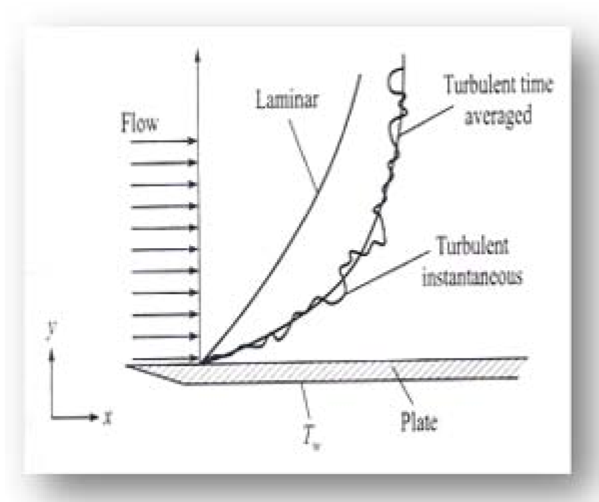
Where, $\bar{V}$ – Time averaged value of velocity

V' – Fluctuating component of velocity

Turbulent diffusivity is given by,

$$\tau_T = \rho V_T \frac{d\bar{V}_x}{dy}; \quad \dot{q}_T'' = -\rho C_p \alpha_T \frac{d\bar{T}}{dy};$$

$$\dot{m}_{AT}'' = -\rho D_T \frac{d\bar{Y}_A}{dy}$$



(Figure 17.2)

Previous Next

Characterization of Turbulent Flow

Length scale of Turbulence:

- The distance covered by an eddy before it disappears or loses its identity.

Intensity of Turbulence:

- Measure of violence of eddies.

Turbulence Intensity:

$$I = \frac{\sqrt{(V_x'^2 + V_y'^2 + V_z'^2)/3}}{\bar{V}}$$

Length Scales used in Turbulent Flow:

1. Macroscopic scale, L (Characteristic width of flow)
2. Integral Scale, l_0
3. Taylor micro scale, l_λ
4. Kolmogorov length Scale, l_K

Previous Next

Taylor microscale, $l_\lambda = \dfrac{V'_{x,rms}}{\left[\overline{\left(\dfrac{\partial V_x}{\partial x}\right)^2}\right]^{0.5}}$

where, $\overline{\left(\dfrac{\partial V_x}{\partial x}\right)^2}$ is the mean strain rate

Kolmogorov length scale, $l_K = \left[\dfrac{2v^3 l_0}{3V'^3_{rms}}\right]^{1/4}$

Note: Kolmogorov length scale (l_K) is related to integral length scale (l_0)
(l_K) - Thickness of the smallest vortex present in turbulent flow

Turbulent Reynolds number based on the length scales

$Re_L = \dfrac{V'_{rms} L}{v}$ $\quad Re_{l_0} = \dfrac{V'_{rms} l_0}{v}$ $\quad Re_{l_\lambda} = \dfrac{V'_{rms} l_\lambda}{v}$ $\quad Re_{l_K} = \dfrac{V'_{rms} l_K}{v}$

Note: V'_{rms} is the characteristic velocity

Previous Next

Turbulent Boundary layer

Consider 2D steady incompressible turbulent flow over a flat plate,

Momentum equation in x direction is given by,

$$\rho\left(\bar{V}_x\frac{\partial \bar{V}_x}{\partial x}+\bar{V}_y\frac{\partial \bar{V}_x}{\partial y}\right)=-\frac{\partial \bar{P}}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{V}_x}{\partial y}-\rho\overline{V_x'V_y'}\right)$$

The term $\rho\overline{V_x'V_y'}$ is known as *Reynolds stress*

Energy equation for turbulent boundary layer is given by,

$$\rho C_p\left(\bar{V}_x\frac{\partial \bar{T}}{\partial x}+\bar{V}_y\frac{\partial \bar{T}}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial \bar{T}}{\partial y}-\rho C_p\,\overline{V_x'T}\right)$$

A simple model for *Reynolds stress* suggested by Bossinesq,

$$-\rho\overline{V_x'V_y'}=\rho v_T\frac{\partial \bar{V}_x}{\partial y};$$ where, v_T is the turbulent diffusivity

Similarly,

$$-\overline{V_x'T}=\propto_T\frac{\partial \bar{T}}{\partial y};$$ where, $\propto_T$ is the eddy diffusivity

Previous Next

In analogy to kinetic theory of gases, Prandtl suggested an expression for turbulent diffusivity

$$v_T = l_m T$$

Where, l_m is the mixing length, and ***I*** is the turbulence intensity

$$I \propto l_m \frac{\partial \bar{V}_x}{\partial y}$$

Combining these two equations,

$$v_T = C l_m^2 \frac{d\bar{V}_x}{dy}$$

C, is the constant, obtained from the experimental data

Previous Next