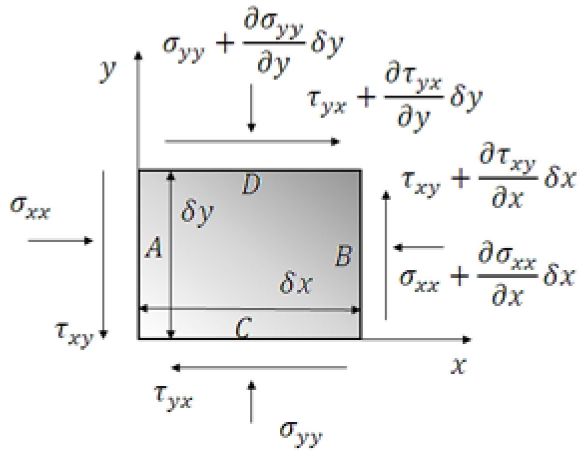
Module 3: Physics of Combustion

Lecture 14: Momentum conservation equation

The Lecture Contains:

Momentum conservation equation



$$\left\{\begin{array}{l}\text{Rate of momentum}\\ \text{Accumulation in fluid element}\end{array}\right\} =$$

$$\left\{\begin{array}{l}\text{Rate of momentum}\\ \text{into fluid element}\end{array}\right\} - \left\{\begin{array}{l}\text{Rate of momentum}\\ \text{out of fluid element}\end{array}\right\} + \left\{\begin{array}{l}\text{Sum of forces acting}\\ \text{on the system}\end{array}\right\}$$

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Momentum conservation equation

Rate of momentum accumulation in x-direction = $\frac{\partial(\rho V_x)}{\partial t}(\delta x \times \delta y \times 1)$

Rate of momentum accumulation in y-direction = $\frac{\partial(\rho V_y)}{\partial t}(\delta x \times \delta y \times 1)$

Momentum in x-direction into fluid element across face A $\Bigg\}$ = $\rho V_x . V_x(\delta y \times 1)$

Momentum in x-direction leaving the fluid element across face B = $\Bigg\}$ $=\rho V_x V_x(\delta y \times 1) + \frac{\partial}{\partial x}[\rho V_x V_x(\delta y \times 1)\delta x]$

Momentum in y-direction entering the fluid element through face C $\Bigg\}$ = $\rho V_x V_y(\delta x \times 1)$

Momentum in y-direction leaving the fluid element across face D $\Bigg\}$ = $\rho V_x V_y(\delta x \times 1) + \frac{\partial}{\partial y}[\rho V_x V_y(\delta x \times 1)\delta y]$

Net forces acting on the fluid element in x-direction =

$$\left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\delta x - \sigma_{xx}\right)(\delta y \times 1) + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\tau_{yx}\right)(\delta x \times 1)$$

$$\Rightarrow \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right)(\delta x \times \delta y)$$

Net body forces acting in fluid element in the x-direction $\Big\} = \rho f_x = (\delta x \times \delta y)$

Momentum equation for fluid element in x-direction $\Big\} = \dfrac{\partial(\rho V_x)}{\partial t} + \dfrac{\partial(\rho V_x V_x)}{\partial x} + \dfrac{\partial(\rho V_x V_y)}{\partial y} = \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \tau_{yx}}{\partial y} + \rho f_x$

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Momentum equation for fluid element in x-direction

$$= \frac{\partial(\rho V_y)}{\partial t} + \frac{\partial(\rho V_x V_y)}{\partial x} + \frac{\partial(\rho V_y V_y)}{\partial y} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \rho f_y$$

where, ρV_x , ρV_y are components of mass velocity vector in x and y direction τ and σ are surface stresses

Applying Stokes viscosity law, the surface stresses are given by

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right)$$

$$\sigma_{xx} = \mu\left(2\frac{\partial V_x}{\partial x} - \frac{2}{3}(\nabla . V)\right) - P \approx -P$$

$$\sigma_{yy} = \mu\left(2\frac{\partial V_y}{\partial y} - \frac{2}{3}(\nabla . V)\right) - P \approx -P$$

Momentum equation for fluid element in x-direction

$$= \frac{\partial(\rho V_x)}{\partial t} + \frac{\partial(\rho V_x V_x)}{\partial x} + \frac{\partial(\rho V_x V_y)}{\partial y}$$

$$= -\frac{\partial P}{\partial_x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial V_x}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial V_x}{\partial y}\right) + \rho f_x$$

Momentum equation for fluid element in y-direction

$$= \frac{\partial(\rho V_y)}{\partial t} + \frac{\partial(\rho V_x V_y)}{\partial x} + \frac{\partial(\rho V_y V_y)}{\partial y}$$

$$= -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left(\mu\frac{\partial V_y}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial V_y}{\partial y}\right) + \rho f_y$$

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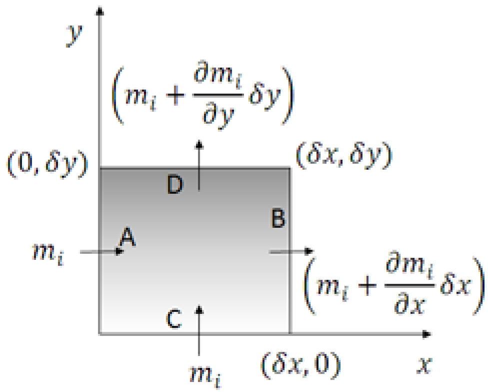
Species transport equation



$$\left\{\begin{array}{l}\text{Rate of accumulation of mass}\\ \text{of species A in fluid element}\end{array}\right\} = \left\{\begin{array}{l}\text{Rate of mass of species A}\\ \text{into fluid element}\end{array}\right\}$$

$$- \left\{\begin{array}{l}\text{Rate of mass of species A}\\ \text{out of fluid element}\end{array}\right\} + \left\{\begin{array}{l}\text{Mass production rate of species A}\\ \text{due to chemical reaction}\end{array}\right\}$$

Rate of accumulation in fluid element= $\dfrac{\partial(\rho . Y_i . \delta x . \delta y . 1)}{\partial t}$

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Rate of accumulation in fluid element= $\dfrac{\partial\left(\rho.Y_i.\delta x.\delta y.1\right)}{\partial t}$

Rate of mass of species A into fluid element across face A = $\dot{m}_i''(\delta y.1)$

By Taylor's series expansion, the rate of mass of species A leaving fluid element across face B $= \left(\dot{m}_i'' + \dfrac{\partial(\dot{m}_i'')}{\partial x}\delta x\right)(\delta y \times 1)$

Net efflux in x direction $=\dfrac{\partial \dot{m}_i''}{\partial x} \times (\delta x \times \delta y)$

Net efflux in y direction = $\dfrac{\partial \dot{m}_i''}{\partial y} \times (\delta x \times \delta y)$

Mass production rate of ith species due to chemical reaction = $\dot{m}_i''' \times (\delta x \times \delta y \times 1)$

According to Fick's law, $\dot{m}_i'' = Y_i \sum \dot{m}_i'' - \rho D \left(\dfrac{\partial Y_i}{\partial x}\right)$

Species transport equation is given by,

$$\frac{\partial(\rho Y_i)}{\partial t} + \frac{\partial(\rho V_x Y_i)}{\partial x} + \frac{\partial\left(\rho V_y Y_i\right)}{\partial y} = \frac{\partial}{\partial x}\left(\rho D \frac{\partial Y_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho D \frac{\partial Y_i}{\partial y}\right) + \dot{m}_i'''$$

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