Module 6: Diffusion Flame

Lecture 31: Overall mass conservation

The Lecture Contains:

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Overall mass conservation:

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho V) = 0 \qquad \text{-----(1)}$$

$\Rightarrow r^2\dot{m}'' = \text{constant}$ for all r V - bulk velocity; ρ -gas density

Momentum conservation:

$P = \text{constant}$

Species conservation:

$$\rho r^2 V\frac{dY_i}{dr} = \rho D\frac{d}{dr}\left(r^2\frac{dY_i}{dr}\right) + \dot{m}_i'''r^2 \qquad \text{-----(2)}$$

$$\rho r^2 V\frac{(C_pT)}{dr} = \frac{k_g}{C_p}\frac{d}{dr}\left(r^2\frac{C_pT}{dr}\right) + \dot{q}'''r^2 \qquad \text{-----(3)}$$

Single step reaction:

$$fF + Ox \rightarrow (1+f)P \qquad \text{-----(4)}$$

T - Temperature; $\dot{q}'''$ - energy release rate due to chemical reaction

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Liquid Fuel Combustion (Contd.)

Fuel, oxidizer and product can be related to heat release rate as follows

$$-\dot{m}_F''' = \frac{-\dot{m}_F'''}{f} = \frac{\dot{m}_P'''}{1+f} = \frac{\dot{q}'''}{f\Delta\widehat{H}_C} \quad \text{-----(5)}$$

Rearranging,

$$\dot{q}''' + \Delta\widehat{H}_C\dot{m}_F''' = 0 \quad \text{-----(6)}$$

We can rewrite fuel species conservation equation as,

$$\dot{m}''r^2\frac{dY_F}{dr} = \rho D\frac{d}{dr}\left(r^2\frac{dY_F}{dr}\right) + \dot{m}_F'''r^2 \quad \text{-----(7)}$$

Multiply eq.7 by ΔH_c and add by eq.3,

$$\dot{m}''r^2\frac{d(C_pT)}{dr} + \dot{m}''r^2\frac{d(Y_F\Delta\widehat{H}_C)}{dr} = \rho\alpha\frac{d}{dr}\left(r^2\frac{d(C_pT)}{dr}\right) + \rho D\frac{d}{dr}\left(r^2\frac{d(Y_F\Delta\widehat{H}_C)}{dr}\right) + r^2\left(\dot{q}''' + \dot{m}_F'''\Delta\widehat{H}_C\right)$$

-----(8)

Here a is the thermal diffusivity $(k_g/\rho C_p)$; $\alpha = D\,(Le = 1)$

Liquid Fuel Combustion (Contd.)

Using eq.6, eq.8 becomes,

$$\dot{m}''r^2\frac{d\left(C_pT+Y_F\Delta\widehat{H}_C\right)}{dr}=\rho\alpha\frac{d}{dr}\left(r^2\frac{d\left(C_pT+Y_F\Delta\widehat{H}_C\right)}{dr}\right) \qquad \text{-----(9)}$$

Elimination of the non-linear term simplifies the analysis. This simplification is known as Schwab- Zeldovich Transformation.

Dividing eq.9 by $Q_V+\Delta\widehat{H}_C(Y_{Fs}-1)$,

$$\dot{m}''r^2\frac{db_{FT}}{dr}=\rho\alpha\frac{d}{dr}\left(r^2\frac{db_{FT}}{dr}\right) \qquad \text{-----(10)}$$

$$b_{FT}=\frac{C_pT+Y_F\Delta\widehat{H}_C}{Q_V+\Delta H_C(Y_{Fs}-1)}$$

Q_V – Heat input required for vaporization of droplet

Y_{Fs} – Mass fraction of species at the surface of the droplet

$Q_V+\Delta\widehat{H}_V+C_L(Y_{Fs}-1)$

Conserved variable for oxidizer

$$b_{Ox,T}=\frac{C_pT+Y_{OX}\Delta\widehat{H}_C}{Q_V+\Delta H_C fY_{Ox,s}} \qquad b_{F,Ox}=\frac{Y_F+Y_{Ox,}f}{(Y_{Fs}-1)+Y_{Ox,s}f}$$

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Liquid Fuel Combustion (Contd.)

General format of all the equations

$$\dot{m}'' r^2 \frac{db}{dr} = \rho\alpha \frac{d}{dr}\left(r^2 \frac{db}{dr}\right) \qquad \text{-----(11)}$$

Boundary conditions

$$r = r_s \quad \dot{m}_F'' = \rho\alpha \left(\frac{db}{dr}\right)_{r=r_s} \; ; \qquad \mathrm{r} \to \infty; \mathrm{b} = \mathrm{b}_\infty$$

Integrating eq.11 twice and applying the boundary conditions,

$$\frac{\dot{m}_F'' r_s^2}{\rho\alpha} \frac{1}{r} = \ln\left(\frac{b_\infty - b_s + 1}{b - b_s + 1}\right) \qquad \text{-----(12)}$$

At $r = r_s; b = b_\infty$

By applying boundary condition to Eq. 12, We can have,

$$\dot{m}_F'' = \frac{\rho\alpha}{r_s} ln(1 + B)$$

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Liquid Fuel Combustion (Contd.)

The transfer number, B is given by

$$B_{F,T} = \frac{C_p(T_\infty - T_s) + \Delta \hat{H}_C (Y_{F\infty} - Y_{Fs})}{Q_V + \Delta \hat{H}_C (Y_{Fs} - 1)}$$

$$B_{Ox,T} = \frac{C_p(T_\infty - T_s) + \Delta \hat{H}_C (Y_{Ox,\infty} - Y_{Ox,s})}{Q_V + f \Delta \hat{H}_C \left(Y_{Ox,s}\right)}$$

$$B_{F,Ox} = \frac{\left(Y_{F,\infty} - Y_{F,s}\right) + \left(Y_{Ox,s} - Y_{Ox,\infty}\right) f}{\left(Y_{F,s} - 1\right) + f\left(Y_{Ox,s}\right)}$$

Values of transfer number, B for some typical fuel:

Combustion in air	B	Combustion in air	B
ISO-Octane	6.41	Kerosene	3.4
Benzene	5.97	Gas oil	2.5
n-Heptane	5.82	Light fuel oil	2.0
Avation gasoline	5.5	Heavy fuel oil	1.7
Automobile gasoline	5.3		

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