Module 2: Governing Equations and Hypersonic Relations

Lecture -2: Mass Conservation Equation

2.1 The Differential Equation for mass conservation:

Let consider an infinitely small elemental control volume having dimensions dx,dy,dz in X ,Y ,Z directions respectively.

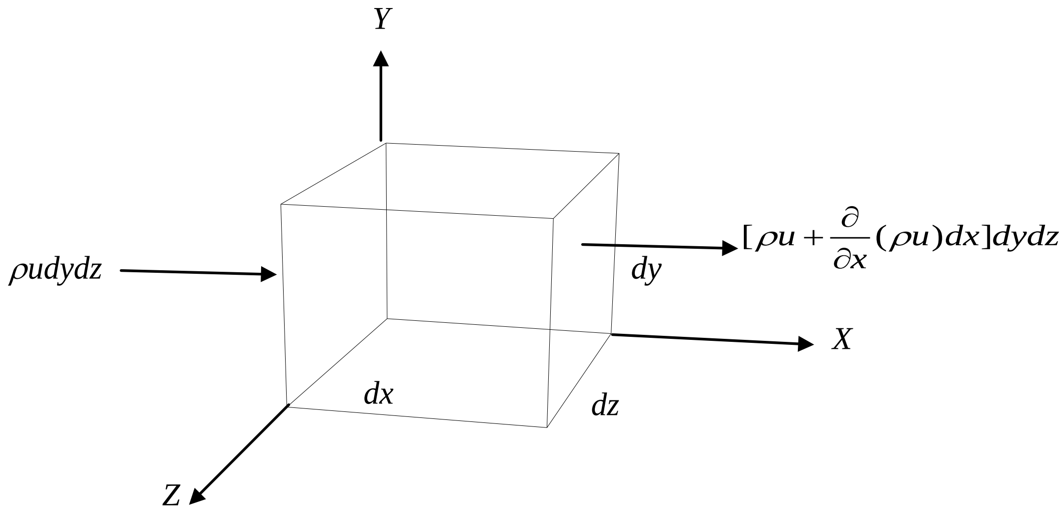

Fig 2.1 Schematic of the elemental control volume with mass flux

The flow through each side of the element is approximately one dimensional hence the approximate mass conservation relation is used here

$$\int_{cv} \frac{\partial \rho}{\partial t} dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0 \qquad (2.1)$$

The element is so small hence we can assume that density is uniform within the small elemental volume and thus the volume integral can be reduced as,

$$\int_{cv} \frac{\partial \rho}{\partial t} dV = \frac{\partial \rho}{\partial t} dxdydz$$

The mass flow terms occur on all six faces which leads to following flux values.

Face	Mass Flux	Face	Mass flux
Y-Z plane passing through origin	$\rho udydz$	Plane parallel to Y-Z plane and separated by dx	$[\rho u + \frac{\partial}{\partial x}(\rho u)dx]dydz$
X-Z plane passing through origin	$\rho vdxdz$	Plane parallel to X-Z plane and separated by dy	$[\rho v + \frac{\partial}{\partial x}(\rho v)dy]dxdz$
X-Y plane passing through origin	$\rho wdxdy$	Plane parallel to X-Y plane and separated by dz	$[\rho w + \frac{\partial}{\partial x}(\rho w)dz]dxdy$

Introducing these terms in into equation (2.1) we get,

$$\frac{\partial \rho}{\partial t}dxdydz + \frac{\partial}{\partial x}(\rho u)dxdydz + \frac{\partial}{\partial y}(\rho v)dxdydz + \frac{\partial}{\partial z}(\rho w)dxdydz = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (2.2)$$

This is the desired equation of conservation of mass for an infinitesimal control volume. This is also called continuity equation. As we know the vector gradient operator as,

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Hence the continuity equation can be written in a comparable form:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \equiv \vec{\nabla}.(\rho\vec{V})$$

We can write in the compact form as:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\vec{V}) = 0 \qquad (2.3)$$

We can as well derive the integral form of the mass conservation equations as

$$\frac{\partial}{\partial t}\oiiint_v \rho d\upsilon + \oiint_s \rho V.ds = 0 \qquad (2.4)$$

This equation is obtained from the Gauss theorem and is valid for non-moving control volume.

Lecture -3: Linear Momentum Conservation Equation

3. 1 The acceleration field of a fluid

The cartesian vector form of velocity field that varies in space and time is

$$\vec{V}(r,t) = \hat{i}u(x,y,z,t) + \hat{j}v(x,y,z,t) + \hat{k}w(x,y,z,t)$$

To write Newton's second law for an infinitesimal fluid system, we need to calculate the acceleration vector field **(** $\vec{a}$ **)** of the flow which can be given as,

$$\vec{a} = \frac{d\vec{V}}{dt} = \hat{i}\frac{du}{dt} + \hat{j}\frac{dv}{dt} + \hat{k}\frac{dw}{dt}$$

Since each scalar component (u,v,w) is a function of four variables (x,yz,t) ,then we can use the chain rule to obtain each scalar time derivative.

$$\frac{du(x,y,z,t)}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + (\vec{V}.\vec{\nabla})u$$

$$a = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}$$

The term $\frac{\partial \vec{V}}{\partial t}$ is called the local acceleration. The other terms are collectively called the convective acceleration, which arises when the particle moves through regions of spatially varying velocity.

3.1 The Differential equation of linear momentum

Let consider an infinitely small elemental control volume having dimensions dx,dy,dz in X ,Y ,Z directions respectively. Schematic of such as

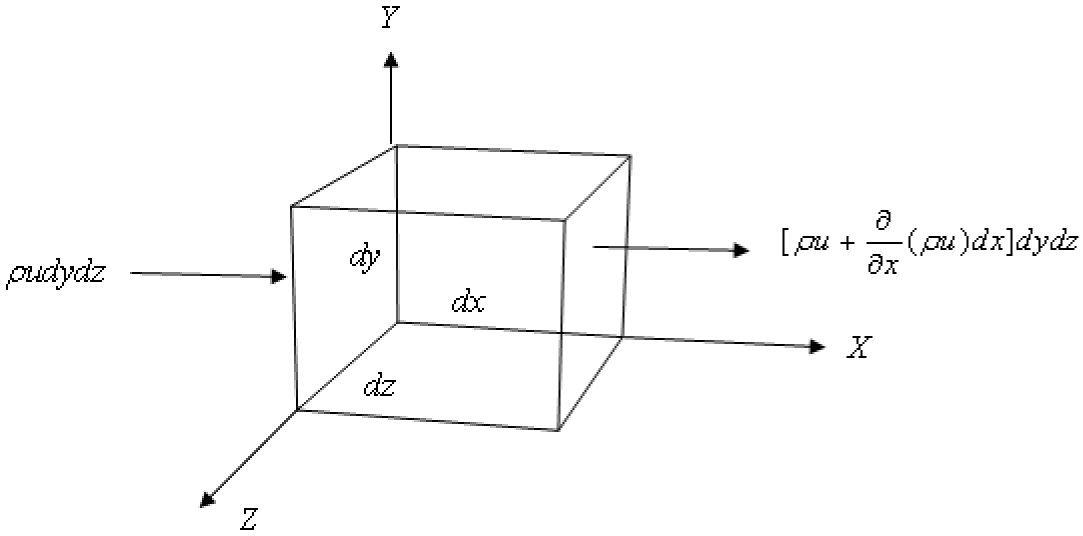

Fig. 3.1. Schematic of the elemental control volume

Considering the control volume as shown in Fig. 3.2 for momentum equation we get

$$\sum \vec{F} = \frac{\partial}{\partial t}\left(\int \vec{V}\rho d\forall\right) + \sum(\dot{m}_i\vec{V}_i)_{out} - \sum(\dot{m}_i\vec{V}_i)_{in} \tag{3.1}$$

Here $\forall$ is the volume of the elemental control volume.

As the control volume is considered to be so small the integral is reduced to a derivative

$$\frac{\partial}{\partial t}\left(\int \vec{V}\rho d\forall\right) = \frac{\partial}{\partial t}(\vec{V}\rho)dxdydz \tag{3.2}$$

$$\sum \vec{F} = \frac{\partial}{\partial t}(\vec{V}\rho)dxdydz + \sum(\dot{m}_i\vec{V}_i)_{out} - \sum(\dot{m}_i\vec{V}_i)_{in}$$

The momentum fluxes at all six faces are

Face	Mass Flux	Face	Mass flux
Y-Z plane passing through origin	$\rho u\vec{V}dydz$	Plane parallel to Y-Z plane and separated by dx	$[\rho u\vec{V}+\frac{\partial}{\partial x}(\rho u\vec{V})dx]dydz$
X-Z plane passing through origin	$\rho v\vec{V}dxdz$	Plane parallel to X-Z plane and separated by dy	$[\rho v\vec{V}+\frac{\partial}{\partial y}(\rho v\vec{V})dy]dxdz$
X-Y plane passing through origin	$\rho w\vec{V}dxdy$	Plane parallel to X-Y plane and separated by dz	$[\rho w\vec{V}+\frac{\partial}{\partial z}(\rho w\vec{V})dz]dxdy$

Using the the inlet momentum flux and outlet momentum flux in the above equation we get

$$\sum\vec{F}=[\frac{\partial}{\partial t}(\vec{V}\rho)+\frac{\partial}{\partial x}(\rho u\vec{V})+\frac{\partial}{\partial y}(\rho v\vec{V})+\frac{\partial}{\partial z}(\rho w\vec{V})]dxdydz \qquad (3.3)$$

Simplification of the right hand side of Eq. 3.3 gives:

$$[\frac{\partial}{\partial t}(\vec{V}\rho)+\frac{\partial}{\partial x}(\rho u\vec{V})+\frac{\partial}{\partial y}(\rho v\vec{V})+\frac{\partial}{\partial z}(\rho w\vec{V})]=\vec{V}[\frac{\partial\rho}{\partial t}+\vec{\nabla}.(\rho\vec{V})]+\rho(\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z})$$

However we know that,

$$\frac{\partial\vec{V}}{\partial t}+u\frac{\partial\vec{V}}{\partial x}+v\frac{\partial\vec{V}}{\partial y}+w\frac{\partial\vec{V}}{\partial z}=\frac{d\vec{V}}{dt}$$

Hence we get,

$$[\frac{\partial}{\partial t}(\vec{V}\rho)+\frac{\partial}{\partial x}(\rho u\vec{V})+\frac{\partial}{\partial y}(\rho v\vec{V})+\frac{\partial}{\partial z}(\rho w\vec{V})]=\vec{V}[\frac{\partial\rho}{\partial t}+\vec{\nabla}.(\rho\vec{V})]+\rho\frac{d\vec{V}}{dt} \qquad (3.4)$$

The first term in above equation is zero due to continuity equation. Hence the momentum equation (Eq. 3.1) reduces to

$$\sum\vec{F}=\rho\frac{d\vec{V}}{dt}dxdydz \qquad (3.5)$$

Equation 3.5 points out that the summation of all the forces acting on the elemental control volume leads to change in momentum of the element. However, these forces are of two types viz. body forces and surface forces. Examples of body forces are gravity and magnetic forces. The only body force we shall be considering in the gravity for demonstration. The gravity force acting on the differential mass $\rho dxdydz$ within the control volume is

$$d\vec{F}_{grav} = \rho\vec{g}dxdydz$$

Body forces acting on the elements are pressure force and shear stress. The total stress tensor is,

$$\sigma_{ij} = \begin{vmatrix} -p+\tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & -p+\tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & -p+\tau_{zz} \end{vmatrix} \tag{3.6}$$

The subscript notation for this stress tensor can be given as in Fig. 3.3,

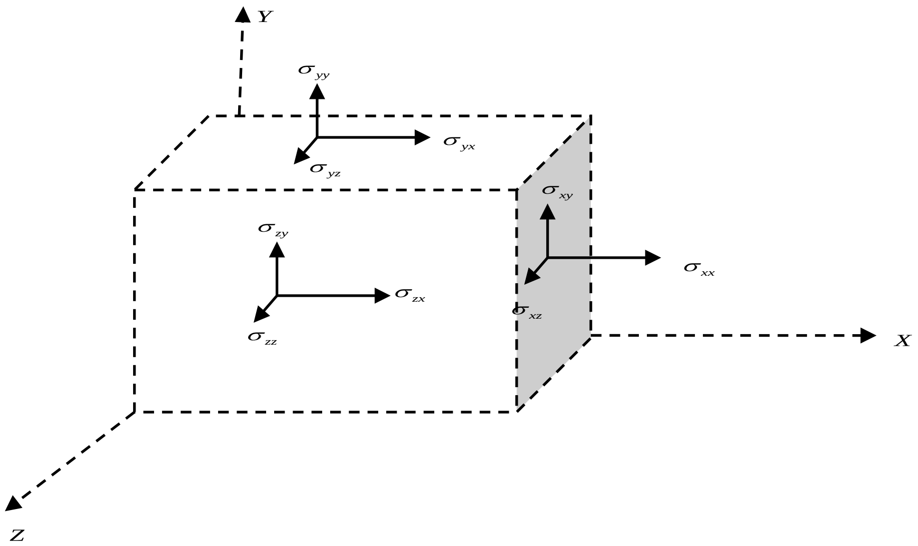

Fig: 3.2 Notation for stresses in relation with the control volume

Lecture -4: Linear Momentum Conservation Equation

4.1 The Differential equation of linear momentum (Continues)

The surface forces acting on the elemental control volume are shown in Fig. 4.1

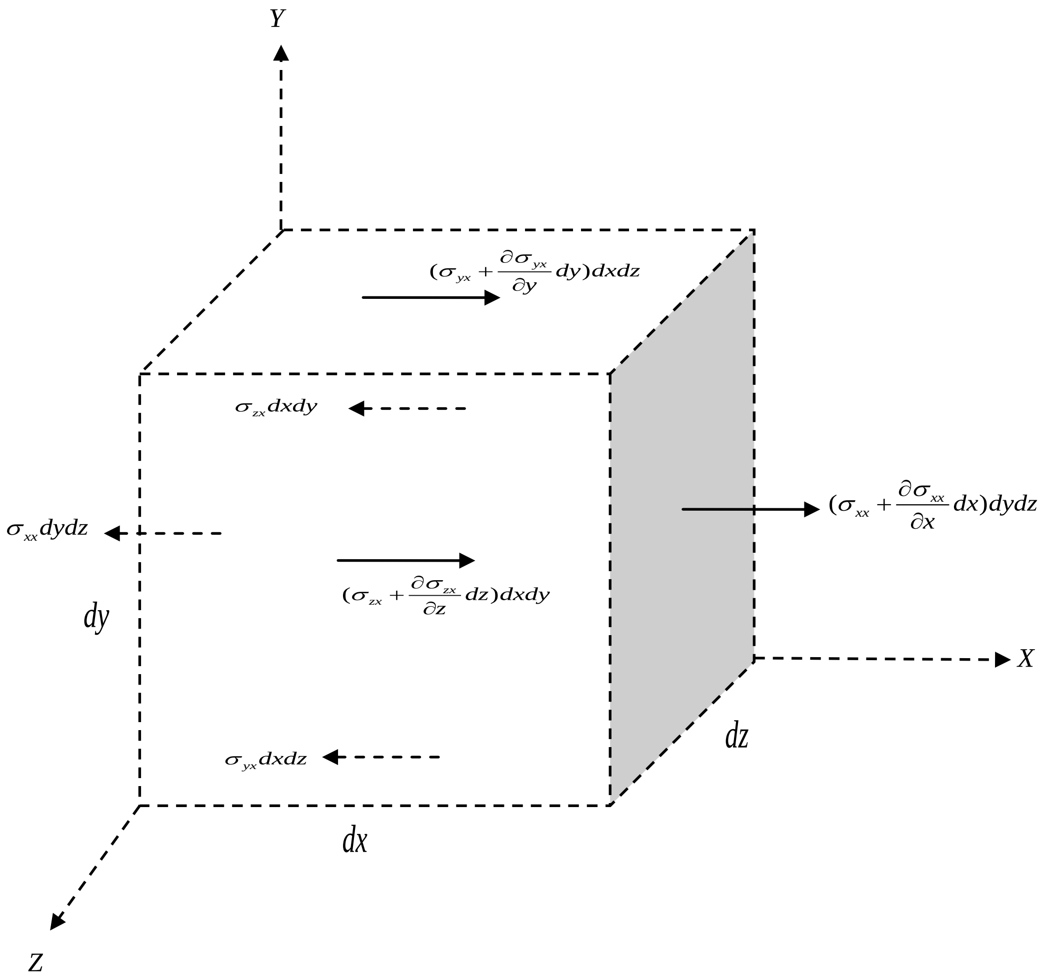

Fig. 4.1 Surface force representation for a control volume.

The net surface force in the X direction is given by:

$$dF_{X,surf} = [\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}]dxdydz \quad (4.1)$$

Which necessarily is,

$$\frac{dF_x}{d\forall} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx} \quad (4.2)$$

Where $d\forall = dxdydz$

In the same way we can derive the Y and Z components of surface forces for the control volume

$$\frac{dF_y}{d\forall} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy} \tag{4.3}$$

$$\frac{dF_z}{d\forall} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz} \tag{4.4}$$

So the net surface force per unit volume is

$$\left(\frac{d\vec{F}}{d\forall}\right)_{surf} = -\nabla p + \left(\frac{d\vec{F}}{d\forall}\right)_{viscous} \tag{4.5}$$

The surface force is thus the sum of pressure gradient vector and the divergence of viscous stress tensor.

Where the viscous force per unit volume is

$$\left(\frac{d\vec{F}}{d\forall}\right)_{viscous} = \hat{i}\left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) + \hat{j}\left(\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right) + \hat{k}\left(\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right)$$
(4.6)

$$\left(\frac{d\vec{F}}{d\forall}\right)_{viscous} = \nabla.\vec{\tau}_{ij} \tag{4.7}$$

Where

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix}$$

The above term is the viscous stress tensor acting on the element

But we know that the Net force =Gravity force +Surface force

$$\sum \vec{F} = d\vec{F}_{grav} + d\vec{F}_{surf} \tag{4.8}$$

The momentum equation 3.5 can be written as,

$$\rho\frac{d\vec{V}}{dt}dxdydz = \rho\vec{g}dxdydz + \left[-\nabla p + \left(\frac{d\vec{F}}{d\forall}\right)_{viscous}\right]dxdydz \tag{4.9}$$

$$\rho\frac{d\vec{V}}{dt} = \rho\vec{g} - \nabla p + \nabla.\vec{\tau}_{ij} \tag{4.10}$$

Where $\frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$

The vector form of the momentum equation (4.10) can be obtained for its scalars as,

$$\rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z})$$

$$\rho g_y - \frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = \rho(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z})$$

$$\rho g_z - \frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} = \rho(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z})$$

4.2 Consideration of momentum equation

Inviscid flow has no consideration of viscosity. Momentum equation for such flow is called as Euler equation. Hence the momentum equation or Euler equation is,

$$\rho\frac{d\vec{V}}{dt} = \rho\vec{g} - \nabla p$$

(4.11)

The general momentum equation represented by Eq. 4.10 is called as Navier-Stokes equation where viscous nature of the flow is explicitly considered. The shear stress terms of that equation can be evaluated for a Newtonian fluid where the viscous stress is assumed to be proportional to the element strain rates and the proportionality coefficient as viscosity. This assumption leads to viscous stresses as,

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} \qquad \tau_{yy} = 2\mu\frac{\partial v}{\partial y} \qquad \tau_{zz} = 2\mu\frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})$$

$$\tau_{xz} = \tau_{zx} = \mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x})$$

$$\tau_{yz} = \tau_{zy} = \mu(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y})$$

By using the above relations in the Eq. (4.10) for its components, we get

$$\rho g_x - \frac{\partial p}{\partial x} + \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) = \rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z})$$

$$\rho g_y - \frac{\partial p}{\partial y} + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}) = \rho(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z})$$

$$\rho g_z - \frac{\partial p}{\partial z} + \mu(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) = \rho(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z})$$

These are the in compressible flow Navier-Stokes equation

For the same control volume, the integral form of the momentum equation can be derived for the x-directional velocity as,

$$\oiiint_v \frac{\partial(\rho u)}{\partial t} d\upsilon + \oiint_s (\rho V.ds)u = -\oiiint_v \frac{\partial P}{\partial x} d\upsilon + \oiiint_v \rho F_{b,x} d\upsilon + F_{viscous,x}$$

Lecture -5: Energy Conservation Equation

5.1 The Differential equation of energy conservation

Consider the schematic of the control volume as shown in Fig. 5.1 for derivation of differential form of the energy equation.

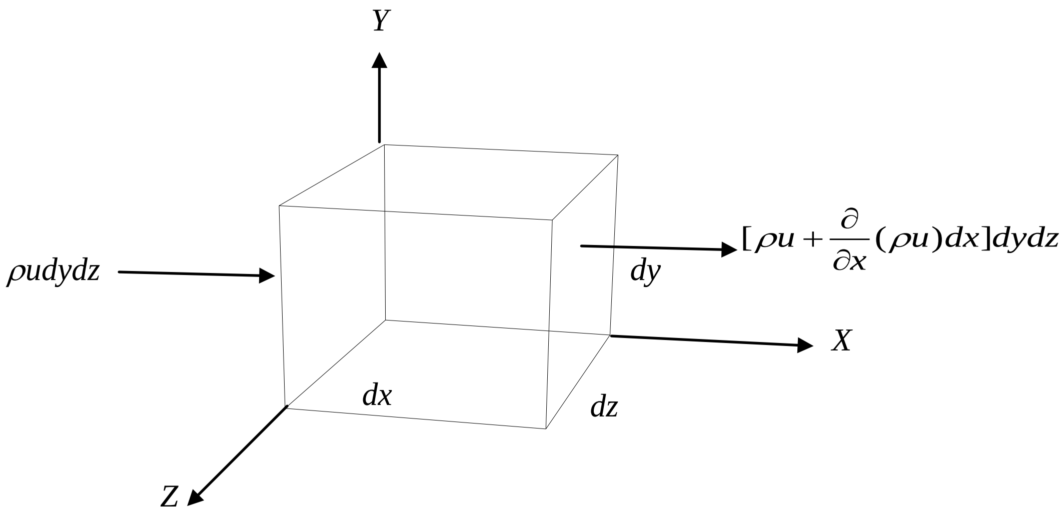

Fig.5.1 Schematic of the elemental control volume

We can write the energy conservation equation for this fixed control volume as

$$\dot{Q}-\dot{W}_S-\dot{W}_V=\frac{\partial}{\partial t}\left(\int_{cv} e\rho d\forall\right)+\int_{cs}\left(e+\frac{p}{\rho}\right)\rho\left(\vec{V}\cdot\hat{n}\right)dA \tag{5.1}$$

Where

$\dot{Q}$ = Rate of head added to the control volume

$\dot{W}_S$ =0, Rate of shaft work done. It is zero in general for the hypersonic applications.

$\dot{W}_V$ =Rate of viscous work done

So the energy equation can be written as :

$$\dot{Q}-\dot{W}_V=\left[\frac{\partial}{\partial t}(\rho e)+\frac{\partial}{\partial x}(\rho u\xi)+\frac{\partial}{\partial y}(\rho v\xi)+\frac{\partial}{\partial z}(\rho w\xi)\right]dxdydz \tag{5.2}$$

Where $\xi = e + \dfrac{p}{\rho}$

We can simplify the above equation using the mass conservation equation which leads to following as the energy equation.

- $\dot{Q} - \dot{W}_v = \left(\rho \dfrac{de}{dt} + \vec{V}.\nabla p + p\nabla.\vec{V} \right) dxdydz$ **(5.3)**

To evaluate $\dot{Q}$ we neglect radiation and consider only heat conduction through the sides of the element. The heat flow by conduction follows Fourier’s law:

- $q = -k\nabla T$

Where k = coefficient of thermal conductivity of the fluid.

The details of heat fluxes from the surface of the element can be derived using the demonstration shown in Fig.5.2

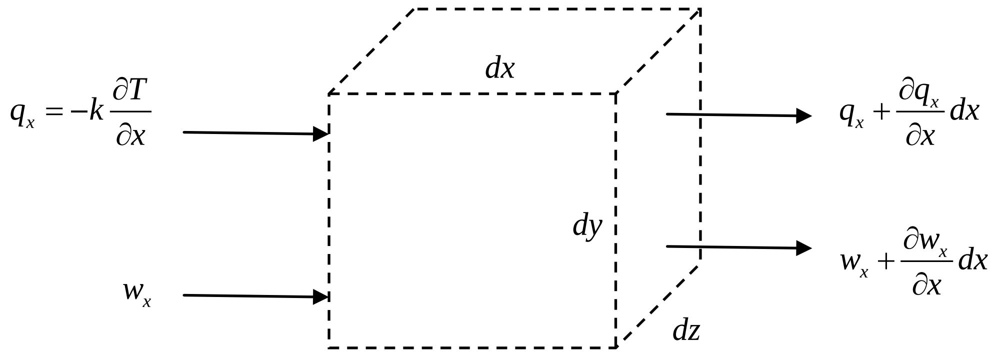

Fig 5.2: Control volume with the x-directional heat flux

Where

$q_x =$ Heat flow per unit area

w_x = viscous work per unit area

$w_x = -\left(u\tau_{xx} + v\tau_{xy} + w\tau_{xz} \right)$

So $\dot{Q} = -\left(\dfrac{\partial q_x}{\partial x} + \dfrac{\partial q_y}{\partial y} + \dfrac{\partial q_z}{\partial z} \right) dxdydz = -\vec{\nabla}.\vec{q}dxdydz$ (5.4)

$$\dot{Q} = -\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) dxdydz = -\vec{\nabla}.\left(k\vec{\nabla}T \right) dxdydz \qquad (5.5)$$

The neat viscous work rate

$$\dot{W}_v = \left[\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z} \right] dxdydz$$

$$\dot{W}_v = -\left[\frac{\partial}{\partial x}\left(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}\right) + \frac{\partial}{\partial y}\left(u\tau_{yx} + v\tau_{yy} + w\tau_{yz}\right) + \frac{\partial}{\partial z}\left(u\tau_{zx} + v\tau_{zy} + w\tau_{zz}\right) \right] dxdydz \tag{5.6}$$

$$\dot{W}_v = -\vec{\nabla}.\left(\vec{V}.\tau_{ij}\right) dxdydz \tag{5.7}$$

By using equation (5.6) and (5.7) we can rewrite equation (5.5) as:

$$\dot{Q} - \dot{W}_v = \left(\rho \frac{de}{dt} + \vec{V}.\nabla p + p\nabla.\vec{V} \right) dxdydz$$

$$\vec{\nabla}.\left(k\vec{\nabla}T\right) dxdydz + \vec{\nabla}.\left(\vec{V}.\tau_{ij}\right) dxdydz = \left(\rho \frac{de}{dt} + \vec{V}.\nabla p + p\nabla.\vec{V} \right) dxdydz$$

$$\vec{\nabla}.\left(k\vec{\nabla}T\right) + \vec{\nabla}.\left(\vec{V}.\tau_{ij}\right) = \left(\rho \frac{de}{dt} + \vec{V}.\nabla p + p\nabla.\vec{V} \right) \tag{5.8}$$

Where

$$e = \hat{u} + \frac{1}{2}V^2 + gz$$

$\hat{u}$ =internal energy

We can as well derive the energy equation in the integral form for the same control volume as

$$\oiiint_v \frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right) \right] d\upsilon + \oiiint_v \nabla.\left[\rho\left(e + \frac{V^2}{2}\right) V \right] d\upsilon - \oiiint_v \dot{q}\,\rho d\upsilon + \oiiint_v \nabla.(PV) d\upsilon - \oiiint_v \rho(F_b.V) d\upsilon - \dot{Q}_{viscous} - \dot{W}_{viscous} = 0 \tag{5.9}$$

Lecture -6: Species Conservation Equation

6.1. Species continuity equation

Apart from the mass, momentum and energy conservation equations, species continuity or conservation equation is an important equation for hypersonic flowfield with dissociated gas or ionized gases. Presence of strong bow shock ahead of the body dissociates or even ionizes the air. This dissociated air or mixture of gases or ions flow over the object of interest. Hence, conservation of mass for each component of the mixture should be accounted. Typical species continuity equation given by Eq. 6.1 is similar to the mass conservation equation. Hence procedure to derive this equation would same as that of the mass conservation equation.

$$\frac{\partial(\rho m_i)}{\partial t}+\frac{\partial(\rho u m_i)}{\partial x}+\frac{\partial(\rho v m_i)}{\partial y}+\frac{\partial(\rho w m_i)}{\partial z}+\frac{\partial(\rho_i u_i)}{\partial x}+\frac{\partial(\rho_i v_i)}{\partial y}+\frac{\partial(\rho_i w_i)}{\partial z}=\dot{w}_i \quad (6.1)$$

$$\frac{\partial(\rho m_i)}{\partial t}+\vec{\nabla}.\rho m_i\vec{V}+\vec{\nabla}.\rho m_i\vec{V}_i=\dot{w}_i$$

Here $m_i V_i$ is the diffusive mass flux of species 'i' due to concentration gradient of the same in the mixture with velocity 'V_i'.

Same equation can be written in integral form as well for the elemental control volume in the flowfield as

$$\frac{\partial}{\partial t}\oiiint_v \rho m_i dv+\oiint_s \rho m_i\vec{V}.d\vec{S}+\oiint_s \rho m_i\vec{V}_i.d\vec{S}=0$$

$$\frac{\partial}{\partial t}\oiiint_v \rho m_i dv+\oiint_s \rho m_i\vec{V}.d\vec{S}+\oiint_s \rho m_i\vec{V}_i.d\vec{S}=\oiiint_v \dot{w}_i dv \qquad (6.2)$$

Here, subscript 'i' is for a particular specie and m_i is the mass fraction of the a specie given by $m_i = \rho_i/\rho$;

This term represents the mass fraction of a particular species in the given control volume. Therefore, first term on left hand side of Eq. 6.2, represents rate of change of mass of a particular species in the control volume. The balance for this mass for the control volume is represented by rest of the terms on left hand side and term on right hand side. Second term on left hand side provides the balance through convection by the virtue of gross fluid motion while third term provides the balance through diffusion of mass due to concentration gradient of that particular species. The term on right hand side provides the reason of either generation or consumption of the species due to chemical reaction in the given control volume. In the absence of chemical reaction or for non reacting mixture of gases, the species continuity equation can be expressed as,

$$\frac{\partial}{\partial t}\oiiint_v \rho m_i dv + \oiint_s \rho m_i \vec{V}.d\vec{S} + \oiint_s \rho m_i \vec{V}_i .d\vec{S} = 0$$

In the present analysis it is assumed that the effect of pressure and temperature on mass diffusion flux velocities (u_i, v_i) is small. Hence these fluxes are neglected. Only the effect of concentration gradient is considered on the mass diffusion fluxes. Diffusion fluxes are calculated by Fick's law, which gives the mechanism of mass diffusion in microscopic point of view. This law correlates mass diffusion flux with concentration gradient as

$$\rho_i u_i = -\rho D_{12}\left[\frac{\partial m_i}{\partial x}\right]$$

$$\rho_i v_i = -\rho D_{12}\left[\frac{\partial m_i}{\partial y}\right]$$

where D_{12} is binary diffusion coefficient of species one and two and u_i and v_i are the diffusion velocities in x and y directions, respectively. The binary diffusion coefficient is function of molecular diameters and temperature.

The above set of governing equations involves two diffusion parameters, which are viscosity and binary diffusion coefficient. In case of single specie, viscosity as the function of temperature can be calculated by Sutherland's formula as,

$$\mu = T^{3/2}\left[\frac{1+S}{T+S}\right]$$

where S is the Sutherland’s constant, which depends upon the fluid. In case of multiple species, viscosity can be calculated by Wilke’s mixture rule, as

$$\mu = \sum \frac{\mu_i X_i}{\sum X_i \phi_{ij}}$$

where ϕ_{ij} is collision integral, X_i are mole fractions and μ_i are species viscosities. The calculation of mixture viscosity needs the viscosity of individual specie. We can find various ways to calculate viscosity of specie using Sutherland’s formula.