Module-6: Hypersonic Viscous Interaction

Lecture-29: Hypersonic Viscous Interaction

29.1 Introduction

While deriving the special governing equations for the boundary layer, we have assumed the boundary layer thickness to be very small where viscous effects are confined. This assumption in turn specifies that the outer inviscid flow remain unaffected in the presence of the boundary layer. However hypersonic flows regime has trait of large boundary layer thickness where outer inviscid flow gets partly or strongly affected in the presence of thick boundary layer. Changes in outer inviscid flow leads to further changes in boundary layer characteristics and hence provides a feed back to the inviscid flow again. This interaction of outer inviscid flow with the boundary layer is called as viscous interaction. This interaction can be mainly of two types. In one of the interactions, exceptionally thick boundary layer grows over the surface. This interaction is termed as pressure interaction; however this interaction is prominently referred as viscous interaction. In the other interaction, shock impingement takes place on the body due to which boundary layer gets disturbed or even separated depending upon the thickness of the boundary layer and strength of the shock.

Viscous interaction is generally characterized as strong or weak interaction depending upon its effect on the wall properties, outer inviscid flow and growth of boundary layer thickness. Strong interaction region is immediately downstream of the leading edge, where the rate of growth of boundary layer thickness is high as shown in Fig.29.1. This is the main reason of reception of large deflection to the streamlines which in turn increases the strength of the shock wave. Higher value of wall pressure at the leading edge in comparison of the corresponding inviscid wall pressure (Fig. 29.2) is the immediate effect of the same strong interaction between outer inviscid flow and viscous flow in the thick boundary layer [Ref].

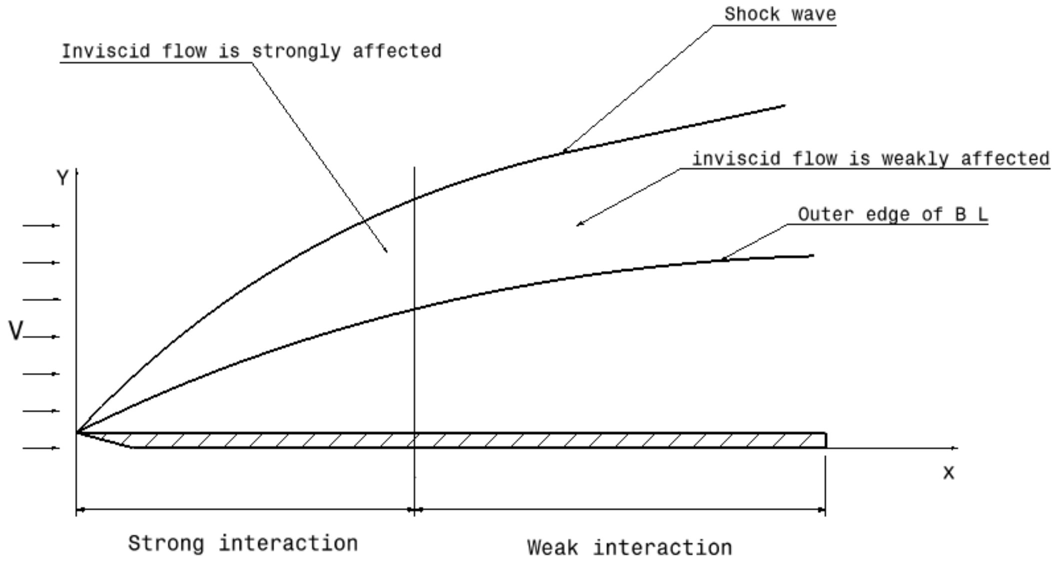

Fig.29.1 Strong and weak interaction regions for hypersonic flow over flat plate.

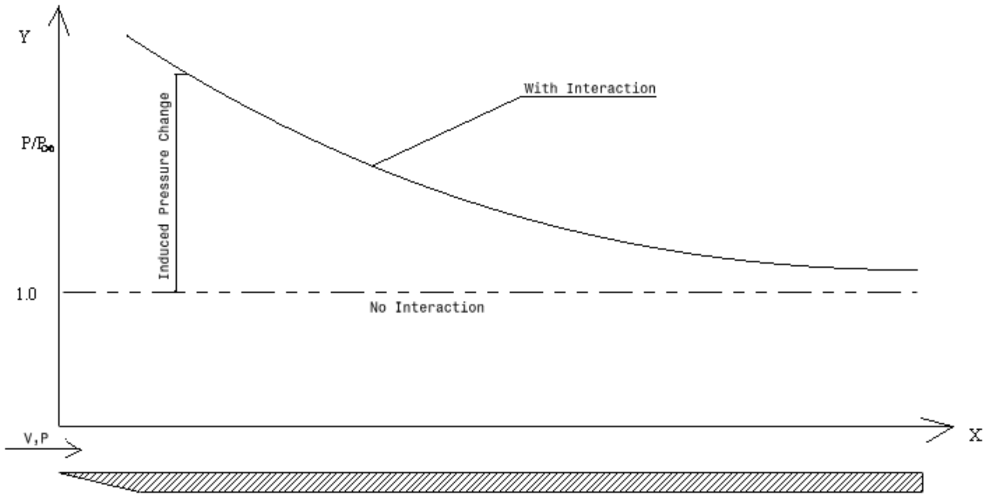

Fig.29.2 Schematic of wall pressure distribution due to viscous interaction [1]

Region of weak interaction lies much downstream of the leading edge of the plate where boundary layer thickness in sufficient large and rate of growth of boundary layer thickness is very low. The strength of the shock generated due to the presence of decelerated fluid in the boundary layer is also low in this region. As a result of this interaction between outer inviscid flow and the boundary layer fluid is much less in this region due to which pressure on the wall approaches the inviscid flow value.

29.2 Hypersonic viscous interaction for flow over flat plate

Consider the laminar hypersonic flow over the flat plate. From the basics of the boundary layer theory we know that, thickness of the boundary layer (δ) is directly proportional with the distance from the leading edge of the plate and inversely proportional with the square root of the Reynolds number.

$$\delta \;\alpha\; \frac{x}{\sqrt{\mathrm{Re}_x}} \tag{29.1}$$

From the definition of Reynold's number we have,

$$\mathrm{Re} = \frac{\rho V L}{\mu}$$

Based on wall thermodynamic properties and freestream velocity, the local Reynold's number is,

$$\mathrm{Re}_x = \frac{\rho_w u_e x}{\mu_w}$$

Hence,

$$\delta \;\alpha\; \frac{x}{\sqrt{\dfrac{\rho_w u_e x}{\mu_w}}}$$

$$\delta \;\alpha\; \frac{x}{\sqrt{u_e x}}\sqrt{\frac{1}{\rho_w}}\sqrt{\frac{\mu_w}{1}}$$

$$\delta \;\alpha\; \frac{x}{\sqrt{\dfrac{\rho_e u_e x}{\mu_e}}}\sqrt{\frac{\rho_e}{\rho_w}}\sqrt{\frac{\mu_w}{\mu_e}}$$

But the Reynolds number based on freestream quantities is,

$$\mathrm{Re}_{\infty x} = \frac{\rho_e u_e x}{\mu_e}$$

Hence,

$$\delta \;\alpha\; \frac{x}{\sqrt{\mathrm{Re}_{\infty x}}}\sqrt{\frac{\rho_e}{\rho_w}}\sqrt{\frac{\mu_w}{\mu_e}} \qquad (29.2)$$

The density ratio can be evaluated using ideal gas equation as,

$$\frac{\rho_e}{\rho_w} = \frac{P_e}{P_w}\frac{T_w}{T_e}$$

However if we assume pressure to be same in normal direction for the boundary layer then wall pressure (P_w) and the pressure at the edge of the boundary layer (P_e) will be same. This leads to,

$$\frac{\rho_e}{\rho_w} = \frac{T_w}{T_e} \qquad (29.3)$$

Let's assume linear dependence of the temperature on viscosity.

$$\frac{\mu_w}{\mu_e} = \frac{T_w}{T_e} \qquad (29.4)$$

Therefore we can express Eq. (29.2) using Eq. (29.3) and (29.4) as,

$$\delta \;\alpha\; \frac{x}{\sqrt{\mathrm{Re}_{\infty x}}}\frac{T_w}{T_e}$$

The boundary layer thickness is directly proportional to the local wall temperature. The wall temperature can be expressed in terms of Mach number under the assumption of adiabatic wall where the wall temperature is closely equal to the freestream total temperature. Hence

$$\frac{T_w}{T_e} = 1 + \frac{\gamma - 1}{2} M_\infty^2$$

Therefore

$$\delta \;\alpha\; \frac{x}{\sqrt{\mathrm{Re}_{\infty x}}} M_\infty^2$$

This results clearly depicts that the local boundary layer thickness is directly proportional to distance from leading of the flat plate, square of freestream Mach number and inversely proportional to the square root of freestream Reynold's number based on distance from leading edge.

Lecture-30: Hypersonic Viscous Interaction

30.1 Understanding of viscous interaction for external flows

Quantification of the interaction is carried out using a parameter defined as “chi bar”,

$$\bar{\chi} = \frac{M_\infty^3 \sqrt{C_\infty}}{\sqrt{\mathrm{Re}_{\infty,x}}} \text{ where, } C_\infty = \left(\frac{\mu_w}{\mu_\infty}\right)\left(\frac{T_e}{T_w}\right) \tag{30.1}$$

It is also called as Chapman-Rusbin parameter for interaction. It turns out as a similarity parameter for comparing the results. Interaction is generally considered as strong for values $\bar{\chi}$ more than 3 while the interaction in treated as weak interaction for values of the same less than 3. This can be noted that the stronger interaction or higher value of $\bar{\chi}$ corresponds to higher wall pressure and larger growth rate of the boundary layer thickness.

We know that the displacement boundary layer thickness at any point on the plate for laminar compressible flow is given as,

$$\delta^* \alpha \frac{x}{\sqrt{\overline{\overline{\mathrm{Re}}}_{\infty x}}} \tag{30.2}$$

Here Reynolds number is calculated from the average boundary layer properties using reference temperature method as,

$$\overline{\mathrm{Re}}_x = \frac{\rho^* V_\infty x}{\mu^*}$$

Here density and viscosity are calculated using reference temperature, where reference temperature is evaluated as,

$$\frac{T^*}{T_e} = 1 + 0.032 M_e^2 + 0.58\left(\frac{T_w}{T_e} - 1\right)$$

Using the reference temperature we can calculate the reference viscosity as,

$$\frac{\mu^*}{\mu_\infty} = C\frac{T^*}{T_\infty} \tag{30.3}$$

Where C is calculated from known temperatures and viscosities as,

$$C = \left(\frac{\mu_w}{\mu_\infty}\right)\left(\frac{T_e}{T_w}\right) \tag{30.4}$$

From the equation of state we can calculate the reference density as,

$$\frac{\rho_\infty}{\rho^*} = \frac{T^*}{T_\infty}\frac{P_\infty}{P^*} = \frac{T^*}{T_\infty}\frac{P_\infty}{P_e} \tag{30.5}$$

Here we have assumed that the pressure at the edge of the boundary layer is equal to the reference pressure. However this pressure will not be equal to the freestream pressure in the presence of viscous interaction.

Thus the displacement thickness can be represented as,

$$\delta^* \alpha x\sqrt{\frac{\mu^*}{\rho^* V_\infty x}} = x\sqrt{\frac{\mu_\infty}{\rho_\infty V_\infty x}}\sqrt{\frac{\rho_\infty}{\rho^*}\frac{\mu^*}{\mu_\infty}} = \frac{x}{\sqrt{\mathrm{Re}_\infty}}\sqrt{\frac{\rho_\infty}{\rho^*}\frac{\mu^*}{\mu_\infty}}$$

The term $\sqrt{\frac{\rho_\infty}{\rho^*}\frac{\mu^*}{\mu_\infty}}$ should be evaluated using Eq. (30.3) and (30.5),

Thus,

$$\delta^* \alpha \frac{x}{\sqrt{\mathrm{Re}}}\sqrt{C\left(\frac{T^*}{T_\infty}\right)^2 \frac{P_\infty}{P_e}}$$

But the temperature ratio follows the proportionality relation,

$$\frac{T^*}{T_e} \alpha M^2{}_\infty$$

Hence,

$$\delta^* \alpha \frac{x}{\sqrt{\mathrm{Re}}} M_\infty{}^2 \sqrt{\frac{C}{P_e / P_\infty}} \tag{30.6}$$

The pressure ratio required to evaluate this expression can be obtained from the oblique shock relations extended for hypersonic Mach numbers, given by Eq. (8.11),

$$\frac{p_2}{p_1} = 1 + \gamma K^2 \sqrt{\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{K^2}} + \frac{K^2\gamma(\gamma+1)}{4} \qquad (30.7)$$

Here k is termed as the similarity parameter and is the product of freestream Mach number and flow deflection angle, $M\theta = k$. However in the present case the flow deflection angle is equivalent with the growth rate of boundary layer thickness or the slope of the tangent to the edge of the boundary layer,

$$K = M_\infty \theta = M_\infty \frac{d\delta^*}{dx}$$

In the presence of strong interaction, K>>1, the growth rate of the boundary layer thickness is very high. Hence the value of k will be large which reduces the Eq. (30.7) as,

$$\frac{P_e}{P_\infty} \approx \frac{\gamma(\gamma+1)}{2} K^2 = \frac{\gamma(\gamma+1)}{2} M_\infty^2 \left(\frac{d\delta^*}{dx}\right)^2 \qquad (30.8)$$

$$K^2 = M_\infty^2 \left(\frac{d\delta^*}{dx}\right)^2 \alpha \frac{M_\infty^3}{\sqrt{\mathrm{Re}}}\sqrt{C} = \overline{\chi}$$

$$\frac{P_e}{P_\infty} = 1 + \frac{\gamma(\gamma+1)}{4}\overline{\chi} + \frac{\gamma(\gamma+1)}{4}\overline{\chi}$$

Or we can written as $\frac{P_e}{P_\infty} = 1 + a_1 \overline{\chi}$

Here, a_1 is a constant.

We can obtain the dependence of boundary layer thickness for the strong interaction case using Eq. (30.6) and (30.8) as,

$$\delta^* \alpha \frac{x}{\sqrt{\mathrm{Re}}} M_\infty^2 \sqrt{C} \frac{1}{M_\infty \left(\frac{d\delta^*}{dx}\right)} \qquad (30.9)$$

From the definition of Reynolds number we have,

$$\mathrm{Re} = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

We can update the Eq. (30.9) as,

$$\delta^* d\delta^* \alpha \sqrt{\frac{C\mu_\infty}{\rho_\infty V_\infty}} M_\infty x^{1/2} dx$$

Integrating this equation leads to,

$$\left(\delta^*\right)^2 \alpha \sqrt{\frac{C\mu_\infty}{\rho_\infty V_\infty}} M_\infty x^{3/2}$$

Or

$$\delta^* \alpha \left(\frac{C\mu_\infty}{\rho_\infty V_\infty}\right)^{1/4} M_\infty^{1/2} x^{3/4} \qquad (30.10)$$

Therefore, boundary layer thickness for strong interaction region is, $\delta^* \alpha x^{3/4}$, where conventional boundary layer thickness is proportional to $x^{1/2}$. Therefore, boundary layer thickness at a particular location in higher in the presence of viscous interaction in comparison with that no viscous interaction.

Differentiating the Eq. (30.10),

$$\frac{d\delta^*}{dx} \alpha \left(\frac{C\mu_\infty}{\rho_\infty V_\infty}\right)^{1/4} M_\infty^{1/2} x^{-1/4} \qquad (30.11)$$

Thus for the region of strong interaction, the rate of growth of boundary layer thickness follows the proportionality as,

$$\frac{d\delta *}{dx} \alpha x^{-1/4}$$

Since $\frac{P_e}{P_\infty}$ is proportional to square of $\frac{d\delta^*}{dx}$, this leads to the proportionality of the pressure ratio as $x^{-1/2}$ which otherwise would have been constant in the absence of viscous interaction.

For strong viscous interaction over a hypersonic flow over a flat plate

1 $\frac{P_e}{P_\infty}$ depends only on the similarity parameter $\overline{\chi}$.

2 $\frac{P_e}{P_\infty}$ vary linearly with $\overline{\chi}$ and $x^{-1/2}$**.**

3. Proportionality of boundary layer thickness is as $\delta^* \ \alpha \ x^{3/4}$

4. For rate of growth of boundary layer thickness, $\frac{d\delta^*}{dx} \ \alpha \ x^{-1/4}$.

For weak interaction where K<1, we can rewrite the pressure equation (30.7) as ,

$$\frac{P_e}{P_\infty} = 1 + \gamma K + \frac{\gamma(\gamma+1)}{4} K^2 \tag{30.12}$$

Which leads to,

$$\frac{P_e}{P_\infty} = 1 + b_1 \overline{\chi} + b \overline{\chi}^2$$

However, $\overline{\chi} << 1$, hence,

$$\frac{P_e}{P_\infty} = 1 + b_1 \overline{\chi}$$

Therefore the proportionality of boundary layer thickness can be obtained from (30.6) and (30.12) for K<1 and K^2<<1 as,

$$\delta^* \alpha \frac{x}{\sqrt{\text{Re}}} M_\infty^2 \sqrt{C} \tag{30.13}$$

Therefore

$\delta^* \ \alpha \ x^{1/2}$, which is very much same as the case of no interaction or conventional boundary layer growth.

Differentiating Eq. (30.13) we get,

$$\frac{d\delta^*}{dx}\alpha\sqrt{\frac{\mu_\infty}{\rho_\infty V_\infty}}M_\infty^2\sqrt{C}x^{-1/2}=\frac{M_\infty^2}{\sqrt{\mathrm{Re}}}\sqrt{C} \tag{30.14}$$

Therefore boundary layer thickness grows as $x^{-1/2}$. Hence the pressure ratio varies inversely to the distance from leading edge. Therefore the effect of viscous interaction gets cancelled quickly away from the leading edge.

Therefore for region of weak interaction over a flat plate,

1. Induced pressure change, $\frac{P_e}{P_\infty} = 1+b_1\overline{\chi}$, linearly varies with $\overline{\chi}$ and distance from the leading edge.

2. Proportionality of boundary layer thickness is, $\delta^* \ \alpha \ x^{1/2}$

3. For rate of growth of boundary layer thickness, $\frac{d\delta^*}{dx} \ \alpha \ x^{-1/2}$

References:

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