Assignment 2

The due date for submitting this assignment has passed.

Due on 2021-02-07, 23:59 IST.

As per our records you have not submitted this assignment.

Instructions to Students: In all the problems below, please report your answer accurate to four significant digits. If the solution is `1.23456`, then please report either `1.234` or `1.235`.

1. Multiple Choice Questions

Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathcal{R}^n$ be three linearly independent vectors in n-dimensional space

1) The above three vectors are linearly independent vectors in n-dimensional space. What are the possible values of n — ***0.5 points***

- [ ] $n = 1$
- [ ] $n = 2$
- [ ] $n = 3$
- [ ] $n = 4$
- [ ] $n = 5$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$n = 3$
$n = 4$
$n = 5$

2) Consider the following three vectors — ***0.5 points***

$\mathbf{u} = \mathbf{x}_1 + \mathbf{x}_2$
$\mathbf{v} = \mathbf{x}_1 + \mathbf{x}_3$
$\mathbf{w} = \mathbf{x}_2 + \mathbf{x}_3$

Are the three vectors **u**, **v** and **w** linearly independent?

- ( ) True
- ( ) False

No, the answer is incorrect.
Score: 0
Accepted Answers:
True

Problem-2: Change of Basis – Vectors

Let $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Express the above vector in terms of new bases,

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The vector in the new basis set is given by $\hat{\mathbf{x}}$. Please report the two elements of this vector below.

3) Please report the first element of the vector $\hat{\mathbf{x}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 3

0.5 points

4) Please report the second element of the vector $\hat{\mathbf{x}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) -1

0.5 points

Problem-3: Change of Basis – Matrix

Recall that the problem of blending two streams was a three-input-two-output problem. The inputs were flowrates F_1, F_2, F_{out} and the outputs were h, x_B. The gain matrix is given by:

$$K = \begin{bmatrix} 2 & 4 & 2 \\ 0.3 & 0.6 & 0 \end{bmatrix}$$

How will this matrix change if the domain space is expressed in terms of the following bases:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

and the co-domain space is expressed in terms of

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

In the questions below, please report the resulting matrix:

$$\hat{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

Please report the values individually below.

5) Please report the value of k_{11}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 1.1,1.2

0.25 points

6) Please report the value of k_{13}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 3.25,3.35

0.25 points

7) Please report the value of k_{22}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.9,-0.8

0.25 points

8) Please report the value of k_{23}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 2.65,2.75

0.25 points

Problem-4: Eigenvalues and Characteristic Equation

Consider the matrix

$$B = \begin{bmatrix} 1 & 0 \\ 8 & 7 \end{bmatrix}$$

Please answer the following questions about the characteristic equations and eigenvalues

9) The characteristic equation is: $\lambda^2 + a\lambda + b = 0$. Please report the value of `a` below.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) -8

0.2 points

10) The characteristic equation is: $\lambda^2 + a\lambda + b = 0$. Please report the value of `b` below.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 7

0.2 points

11) Report the larger of the two eigenvalues

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 7

0.2 points

12) Report the smaller of the two eigenvalues

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 1

0.2 points

13) Substitute the matrix **B** in the characteristic equation and verify that the equation is satisfied. This property of a matrix (i.e., matrix satisfies it's characteristic equation) is known as _________ _________ theorem

No, the answer is incorrect.
Score: 0
Accepted Answers:
String containing all of these (AND): Cayley, Hamilton

0.2 points

Problem-5

Consider the matrix:

$$C = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$

Part-A: Jordan Decomposition

In this problem, we will compute eigenvalues and eigenvectors of C and perform Jordan decomposition to obtain $C = V\Lambda V^{-1}$.

In this part, please answer the following questions:

14) The eigenvalues of `C` are repeated. Please report the repeated eigenvalue, λ, below.

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 2

0.2 points

15) The matrix `C` can be written as $C = V\Lambda V^{-1}$. Here,

$$\Lambda = \begin{bmatrix} \lambda & l_1 \\ l_2 & \lambda \end{bmatrix}$$

In the above, please report the value of l_1

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 1

0.4 points

16) $$\Lambda = \begin{bmatrix} \lambda & l_1 \\ l_2 & \lambda \end{bmatrix}$$

In the above, please report the value of l_2

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Numeric) 0

0.4 points

Part-B: Computing matrix exponent

You can use Jordan decomposition used above for computing the matrix exponent. Note that the matrix exponent is given by

$$e^C = Ve^{\Lambda}V^{-1}$$

We have already computed Jordan decomposition in Part-A above. We can use the Jordan decomposition to compute the matrix exponent.

Note that for matrix exponent

$$\exp\left(\begin{bmatrix} \lambda & a \\ 0 & \lambda \end{bmatrix}\right) = \begin{bmatrix} e^{\lambda} & ae^{\lambda} \\ 0 & e^{\lambda} \end{bmatrix}$$

Please use the above to compute e^C and report the results below

17) $$e^C = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

In the above, please report the value of e_{11}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -0.01,0.01

0.25 points

18) $$e^C = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

In the above, please report the value of e_{12}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 7,7.5

0.25 points

19) $$e^C = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

In the above, please report the value of e_{21}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) -7.5,-7

0.25 points

20) $$e^C = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

In the above, please report the value of e_{22}

No, the answer is incorrect.
Score: 0
Accepted Answers:
(Type: Range) 14,15

0.25 points