Numerical Methods and Programming
Assignment-2

- Problem 1: The differentiation of $f(x)$ can be calculated using the following formulas

  Forward difference:

  $$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{1}$$

  Central difference $O(h^2)$:

  $$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \tag{2}$$

  Central difference $O(h^4)$:

  $$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} \tag{3}$$

  show that truncation errors for these formulas are order of $O(h)$, $O(h^2)$ and $O(h^4)$ respectively.

- Problem 2: Write a C program to evaluate derivative of $f(x) = \cos(x)$ using Eq. 1, Eq. 2 and Eq. 3. Compute the absolute error $|f'(x) + \sin(x)|$ and plot the errors as a function of x (choosing $x \in [0, 2\pi]$) when step sizes are

  a) h=0.1
  b) h=0.01
  c) h=0.001

- Problem 3: Consider integration of the function $f(x) = \sin(x)$ over a fixed interval $[0, \pi/2]$,

  $$I = \int_0^{\pi/2} \sin(x)\ dx \tag{4}$$

  Write a C program to integrate the function using
  a) Trapezoidal rule
  b) Simpson's rule
  taking the number of points 5, 11 and 21. Compare your result with the exact value of the integral .

- Problem 4: Consider the integral

  $$I = \int_5^{12} \frac{dx}{x} \tag{5}$$

  Convert the integral to Gauss Quadrature form and integrate using five-point Gauss Quadrature method. The zeros and weights are given below

Zeros	Weights
-0.90617975	0.23692689
-0.53846931	0.47862867
0.0	0.56888889
0.53846931	0.47862867
0.90617975	0.23692689

- Problem 5: Consider the equation of motion of **Simple Harmonic Oscillator**

$$\frac{d^2x}{dt^2} + w^2 x = 0.0 \tag{6}$$

Solve the above differential equation using **second order** Runge Kutta method when $\omega = 1$ and choosing initial condition $x|_{t=0} = 1.0$ and $\frac{dx}{dt}|_{t=0} = 0.1$. Plot
a) x as a function of t
b) $\frac{dx}{dt}$ as a function of t
c) $\frac{dx}{dt}$ as a function of x (this is known as phase space plot).

- Problem 6: Solve the equation of motion of **Simple Pendulum**

$$\frac{d^2\theta}{dt^2} + \sin(\theta) = 0.0 \tag{7}$$

using **fourth order** Runge Kutta method choosing initial condition $\theta|_{t=0} = 0.5$ and $\frac{d\theta}{dt}|_{t=0} = 0.3$. Plot
a) θ as a function of t
b) $\frac{d\theta}{dt}$ as a function of t
c) $\frac{d\theta}{dt}$ as a function of θ (phase space plot).
d) Change the initial condition $\frac{d\theta}{dt}|_{t=0}$ and observe the difference.

- Problem 7: Find the root of

$$x \sin x - 1.0 = 0.0$$

using Newton-Raphson method and false-position method. Use $x_0 = 0.5$ as the initial guess for Newton-Raphson-method. For false-position method, start with the interval [0, 2].

Compare the number of iterations needed in both the methods to find the root x_R such that $x_R \sin x_R - 1.0 \leq 10^{-10}$.

- Problem 8: Compare the rate of convergence (number of iterations) needed to find the positive root of the equation

$$x^{10} - 1.0 = 0.0$$

using Bisection method (starting with the interval [0, 2]) and Newton-Raphson method (with the initial guess $x_0 = 0.5$). Use the stopping criteria $x_n^{10} - 1.0 \leq 10^{-10}$ to stop after n iterations in both the cases. Which method takes more number of iterations?

- Problem 9: The heat equation in one dimension is given by

$$\frac{\partial u}{\partial t} = \frac{k}{c\rho}\left(\frac{\partial^2 u}{\partial x^2}\right)$$

where $u(x,t)$ is the temperature at x at time t. k, c and ρ are the thermal conductivity, the heat capacity and the density of the material respectively. Solve for the temperatures at t = 2.06 sec for a steel plate that is 2 cm thick. For steel, $k = 0.13$ cal/(sec cm °C), $c = 0.11$ cal/(g °C), and $\rho = 7.8$ g/cm^3. Neglect lateral flow of heat and consider only the flow perpendicular to the faces of the plate. The initial temperatures are given by

$$u(x,t) = 100.0 \sin\left(\frac{\pi x}{2}\right)$$

The boundary conditions are, $u(x = 0, t) = 0$ and $u(x = 2, t) = 0$. Use explicit method with $\Delta x = 0.25cm$. Compare the solution with that obtained using implicit method.

- Problem 10: Find the Fourier transform of the data given below using FFT.

x	0.0	0.897	1.794	2.691
f(x)	0.0	1.816	3.236	0.027