NPTEL MOOC Estimation: Assignment #8

1. Consider an N = 4 subcarrier OFDM system with pilot symbols $X(0)$ = 3+3j, $X(1)$ = -2-2j, $X(2)$ = -1+j, $X(3)$ = 2-j. Let the corresponding received samples in the time domain be $y(0)$ = -1-j, $y(1)$ = 2+2j, $y(2)$ = 3-2j, $y(3)$ = 3-2j. Let the noise samples $v(k)$, $0 \le k \le 3$ be zero-mean IID Gaussian with variance σ^2 = -3 dB. There are L = 2 IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The transmitted block of samples in the time domain with cyclic prefix is,
    a. $\frac{1}{2}+\frac{1}{4}j, \frac{5}{4}-\frac{1}{2}j, \frac{1}{2}+\frac{7}{4}j, \frac{3}{4}+\frac{3}{2}j$
    b. $\frac{1}{2}+\frac{1}{4}j, \frac{1}{2}+\frac{1}{4}j, \frac{5}{4}-\frac{1}{2}j, \frac{1}{2}+\frac{7}{4}j, \frac{3}{4}+\frac{3}{2}j$
    c. $\frac{3}{4}+\frac{3}{2}j, \frac{1}{2}+\frac{1}{4}j, \frac{5}{4}-\frac{1}{2}j, \frac{1}{2}+\frac{7}{4}j, \frac{3}{4}+\frac{3}{2}j$
    d. $\frac{1}{2}-\frac{1}{4}j, \frac{5}{4}+\frac{1}{2}j, \frac{1}{2}+\frac{7}{4}j, \frac{3}{4}+\frac{3}{2}j$
    Ans c

2. Consider an N = 4 subcarrier OFDM system with pilot symbols $X(0)$ = 3+3j, $X(1)$ = -2-2j, $X(2)$ = -1+j, $X(3)$ = 2-j. Let the corresponding received samples in the time domain be $y(0)$ = -1-j, $y(1)$ = 2+2j, $y(2)$ = 3-2j, $y(3)$ = 3-2j. Let the noise samples $v(k)$, $0 \le k \le 3$ be zero-mean IID Gaussian with variance σ^2 = -3 dB. There are L = 2 IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The received output symbols across the subcarriers in the frequency domain are,
    a. 7+3j, 2j, -3-3j, -8
    b. 7-3j, 2j, -3-3j, -8
    c. -7+3j, -2j, 3+3j, 8
    d. 7-3j, 2j, -3+3j, -8
    Ans b

3. Consider an N = 4 subcarrier OFDM system with pilot symbols $X(0)$ = 3+3j, $X(1)$ = -2-2j, $X(2)$ = -1+j, $X(3)$ = 2-j. Let the corresponding received samples in the time domain be $y(0)$ = -1-j, $y(1)$ = 2+2j, $y(2)$ = 3-2j, $y(3)$ = 3-2j. Let the noise samples $v(k)$, $0 \le k \le 3$ be zero-mean IID Gaussian with variance σ^2 = -3 dB. There are L = 2 IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The LMMSE estimates of the channel coefficients across subcarriers 0, 1 respectively are,
    a. $j, -\frac{8}{3}-\frac{4}{3}j$
    b. $-\frac{1}{3}-\frac{1}{3}j, -\frac{8}{3}-\frac{4}{3}j$

c. $j, \frac{6}{11} - \frac{15}{11}j$
d. $\frac{6}{11} - \frac{15}{11}j, -\frac{1}{3} - \frac{1}{3}j$
Ans d

4. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The LMMSE estimates of the channel coefficients across subcarriers 2, 3 respectively are,
   a. $j, -\frac{16}{9} - \frac{8}{9}j$
   b. $-\frac{1}{3} - \frac{1}{3}j, -\frac{8}{3} - \frac{4}{3}j$
   c. $j, \frac{6}{11} - \frac{15}{11}j$
   d. $\frac{6}{11} - \frac{15}{11}j, -\frac{1}{3} - \frac{1}{3}j$
   Ans a

5. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The ML estimates of the channel coefficients across subcarriers 2, 3 are,
   a. $3j, \frac{16}{5} + \frac{8}{5}j$
   b. $3j, -\frac{16}{5} - \frac{8}{5}j$
   c. $\frac{2}{3} + \frac{5}{3}j, -\frac{1}{2} + \frac{1}{2}j$
   d. $\frac{2}{3} - \frac{5}{3}j, -\frac{1}{2} - \frac{1}{2}j$
   Ans b
6. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The MSEs of LMMSE estimation of the channel coefficients across subcarriers 0, 1, 2, 3 respectively are,

a. $\frac{1}{11}, \frac{1}{6}, \frac{1}{3}, \frac{2}{9}$
b. $\frac{18}{11}, \frac{4}{3}, \frac{2}{3}, \frac{10}{9}$
c. $\frac{2}{11}, \frac{1}{6}, \frac{2}{3}, \frac{2}{9}$
d. $\frac{1}{11}, \frac{4}{6}, \frac{1}{3}, \frac{2}{9}$

Ans a

7. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The noise samples $V(l)$ on the l^{th} subcarrier are,
   a. Zero-mean, Gaussian, variance ½
   b. Zero-mean, Non-Gaussian, variance ½
   c. Zero-mean, Gaussian, variance $\frac{1}{8}$
   d. Zero-mean, Gaussian, variance 2

   Ans d
8. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The channel coefficients $H(l)$ on the l^{th} subcarrier are,
   a. Zero-mean, Gaussian, variance ½
   b. Zero-mean, variance ½
   c. Zero-mean, Gaussian, variance $\frac{1}{8}$
   d. Zero-mean, variance 2

   Ans b
9. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. The LMMSE estimates of the channel taps h(0), h(1) respectively are,
   a. $j, -\frac{8}{3} - \frac{4}{3}j$

b. $\frac{6}{11} - \frac{15}{11}j, -\frac{1}{3} - \frac{1}{3}j$

c. $-\frac{155}{396} - \frac{157}{396}j, -\frac{1}{396} - \frac{91}{396}j$

d. $\frac{27}{44} - \frac{67}{132}j, \frac{15}{132} + \frac{1}{132}j$

Ans c

10. Consider an $N = 4$ subcarrier OFDM system with pilot symbols $X(0) = 3+3j$, $X(1) = -2-2j$, $X(2) = -1+j$, $X(3) = 2-j$. Let the corresponding received samples in the time domain be $y(0) = -1-j$, $y(1) = 2+2j$, $y(2) = 3-2j$, $y(3) = 3-2j$. Let the noise samples $v(k)$, $0 \leq k \leq 3$ be zero-mean IID Gaussian with variance $\sigma^2 = -3$ dB. There are $L = 2$ IID channel taps with zero-mean, **Gaussian (only for this part)** and dB variance \sigma_h^2 = -6 dB. The length of cyclic prefix is the minimum required. If the block of pilot symbols above is repeatedly transmitted, what is the number of OFDM pilot blocks M required such that the real and imaginary parts of the estimate $\hat{H}(1)$ lie within a radius $\frac{1}{8}$ of the real and imaginary parts of the true parameter $H(1)$ with probability greater than 99.99%

a. $\left(\sqrt{2}Q^{-1}\left(5\times10^{-6}\right)\right)^2$

b. $\frac{16}{3}\left(Q^{-1}\left(5\times10^{-6}\right)\right)^2$

c. $\left(Q^{-1}\left(5\times10^{-5}\right)\right)^2$

d. $\frac{16}{3}\left(Q^{-1}\left(5\times10^{-5}\right)\right)^2$

Ans d