1. For an Inter Symbol Interference channel, received symbol at kth instant is given by,

$$y(k) = \frac{3}{2}\,x(k) - \frac{1}{2}\,x(k-1) + v(k).$$

Comparing the above equation with L=2 tap wireless channel given below,

$$y(k) = h(0)x(k) + h(1)x(k-1) + v(k),$$

we obtain, $h(0) = \frac{3}{2}$ and $h(1) = -\frac{1}{2}$. Considering a $r = 2$ tap equalizer based on $y(k)$ and $y(k+1)$, model can be formulated as,

$$\begin{bmatrix} y(k+1) \\ y(k) \end{bmatrix} = \begin{bmatrix} h(0) & h(1) & 0 \\ 0 & h(0) & h(1) \end{bmatrix} \begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \end{bmatrix} + \begin{bmatrix} v(k+1) \\ v(k) \end{bmatrix}$$

$$\mathbf{y}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{v}(k).$$

So, the effective channel matrix $\mathbf{H}$ for this scenario can be written as,

$$\mathbf{H} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ 0 & 3/2 & -1/2 \end{bmatrix}.$$

Ans (b)

2. Covariance matrix of input

$$E\{\mathbf{x}\mathbf{x}^T\} = E\{\begin{bmatrix} x(k+1) \\ x(k) \\ x(k-1) \end{bmatrix} \begin{bmatrix} x(k+1) & x(k) & x(k-1) \end{bmatrix}\}$$

It is given that $x(k)$'s are IID zero mean with dB power $P_d = 10dB = 10$. Hence,

$$E\{\mathbf{x}\mathbf{x}^T\} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

Noise Covariance matrix,

$$E\{\mathbf{v}\mathbf{v}^T\} = E\{\begin{bmatrix} v(k+1) \\ v(k) \end{bmatrix} \begin{bmatrix} v(k+1) & v(k) \end{bmatrix}\}$$

It is given that $v(k)$'s are IID zero mean with variance $\sigma^2 = 3dB = 2$. Hence,

$$E\{\mathbf{v}\mathbf{v}^T\} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Ans(a)

3. Given data: dB power $P_d = 10$ dB $= 10$ and dB noise variance $\sigma^2 = 3$ dB $= 2$. The effective channel matrix $\mathbf{H}$ is given by,

$$\mathbf{H} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ 0 & 3/2 & -1/2 \end{bmatrix},$$

$$\mathbf{H}\mathbf{H}^T = \begin{bmatrix} 3/2 & -1/2 & 0 \\ 0 & 3/2 & -1/2 \end{bmatrix} \begin{bmatrix} 3/2 & 0 \\ -1/2 & 3/2 \\ 0 & -1/2 \end{bmatrix} = \begin{bmatrix} 5/2 & -3/4 \\ -3/4 & 5/2 \end{bmatrix},$$

$$(P_d\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I})^{-1} = \frac{4}{2691}\begin{bmatrix} 27 & 15/2 \\ 15/2 & 27 \end{bmatrix},$$

$$\mathbf{1}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

The LMMSE equalizer vector $\mathbf{c}$ for the estimation of $x(k)$ is given by,

$$\begin{aligned} \mathbf{c} &= (P_d\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I})^{-1} P_d \mathbf{H}\mathbf{1}_1 \\ &= \begin{bmatrix} -90/2691 \\ 1470/2691 \end{bmatrix} \\ &= \begin{bmatrix} -0.03344 \\ 0.5463 \end{bmatrix}. \end{aligned}$$

Ans (c)

4. The resulting LMMSE equalizer is,

$$\hat{x}(k) = \mathbf{c}^T\mathbf{y}$$

where $\mathbf{c} = \begin{bmatrix} -90/2691 \\ 1470/2691 \end{bmatrix}$ (calculated in Q3), and $\mathbf{y} = \begin{bmatrix} y(k+1) \\ y(k) \end{bmatrix}$

Thus,

$$\hat{x}(k) = \frac{-90}{2691} y(k+1) + \frac{1470}{2691} y(k)$$

Ans (d)

5. The MSE for LMMSE equalization is given by,

$$\begin{aligned} \mathrm{E}\{(\hat{x}(k) - x(k))^2\} &= P_d - P_d \mathbf{1}_1^T \mathbf{H}^T (P_d\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I})^{-1} P_d \mathbf{H}\mathbf{1}_1 \\ &= P_d - P_d \mathbf{1}_1^T \mathbf{H}^T \mathbf{c} \\ &= 10 - 10 \begin{bmatrix} -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} -90/2691 \\ 1470/2691 \end{bmatrix} \\ &= 1.6388. \end{aligned}$$

Ans (a)

6. Considering an Inter Symbol Interference channel $y(k) = \frac{3}{2}x(k) - \frac{1}{2}x(k-1) + v(k)$ with r = 2 tap . Symbols $y(k+1), y(k)$ are used to detect $x(k+1)$ instead of $x(k)$. Given that, symbols

$x(k)$ be IID zero-mean with Power $P_d = 10dB = 10$ and noise variance $\sigma^2 = 3dB = 2$. The effective channel matrix $\mathbf{H}$ is given by,

$$\mathbf{H} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ 0 & 3/2 & -1/2 \end{bmatrix},$$

$$\mathbf{H}\mathbf{H}^T = \begin{bmatrix} 3/2 & -1/2 & 0 \\ 0 & 3/2 & -1/2 \end{bmatrix} \begin{bmatrix} 3/2 & 0 \\ -1/2 & 3/2 \\ 0 & -1/2 \end{bmatrix} = \begin{bmatrix} 5/2 & -3/4 \\ -3/4 & 5/2 \end{bmatrix},$$

$$(P_d\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I})^{-1} = \frac{4}{2691}\begin{bmatrix} 27 & 15/2 \\ 15/2 & 27 \end{bmatrix},$$

$$\mathbf{1}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The LMMSE equalizer vector $\mathbf{c}$ for the estimation of $x(k)$ is given by,

$$\begin{aligned} \mathbf{c} &= (P_d\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I})^{-1} P_d \mathbf{H}\mathbf{1}_0 \\ &= \begin{bmatrix} 0.602 \\ 0.1672 \end{bmatrix} \end{aligned}$$

Ans (a)

7. The MSE for LMMSE equalization is given by,

$$\begin{aligned} \mathrm{E}\{(\hat{x}(k) - x(k))^2\} &= P_d - P_d \mathbf{1}_0^T \mathbf{H}^T (P_d\mathbf{H}\mathbf{H}^T + \sigma^2\mathbf{I})^{-1} P_d \mathbf{H}\mathbf{1}_0 \\ &= P_d - P_d \mathbf{1}_0^T \mathbf{H}^T \mathbf{c} \\ &= 10 - 10 \begin{bmatrix} 3/2 & 0 \end{bmatrix} \begin{bmatrix} 0.602 \\ 0.1672 \end{bmatrix} \\ &= 0.9699. \end{aligned}$$

Ans (c)

8. OFDM is a technology which is used in 4G LTE.

   Ans (a)

9. The acronym OFDM stands for Orthogonal Frequency Division Multiplexing. **Ans (d)**

10. For an L-tap channel, what is the minimum length of cyclic prefix needed to lead to a circular convolution of the channel and input at the receiver is $L - 1$ **Ans (b)**