NPTEL MOOC Estimation: Assignment #6

1. Consider the LMMSE (Linear Minimum Mean Squared Error) MIMO channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1 \le k \le N$ and N received symbol vectors $\mathbf{y}(k) = [y_1(k), y_2(k),\ldots,y_R(k)]^T$, $1 \le k \le N$. Let the channel coefficients be IID Gaussian with variance \sigma_h^2 and noise variance is \sigma^2. The expression for the LMMSE estimate of the MIMO channel matrix H is,
   a. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2}+\frac{\mathbf{I}}{\mathrm{R}{\sigma_h}^2}\right)^{-1}\frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}\sigma^2}$
   b. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}{\sigma_h}^2}+\frac{\mathbf{I}}{\mathrm{R}\sigma^2}\right)^{-1}\frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}\sigma^2}$
   c. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2}+\frac{\mathbf{I}}{\mathrm{R}{\sigma_h}^2}\right)^{-1}\frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}{\sigma_h}^2}$
   d. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}{\sigma_h}^2}+\frac{\mathbf{I}}{\mathrm{R}\sigma^2}\right)^{-1}\frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}{\sigma_h}^2}$

   Ans a

2. Consider the MIMO channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1 \le k \le N$ and N received symbol vectors $\mathbf{y}(k) = [y_1(k), y_2(k),\ldots,y_R(k)]^T$, $1 \le k \le N$. Let the channel coefficients be IID Gaussian with variance \sigma_h^2 and noise variance is \sigma^2. The expression for the ML estimate of the MIMO channel matrix H is, [Hint: This can be obtained by assuming \sigma_h^2 X^TX >> \sigma^2 I],
   a. $(\mathbf{X}\mathbf{X}^{\mathrm{T}})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}$
   b. $(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}$
   c. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2}+\frac{\mathbf{I}}{\mathrm{R}{\sigma_h}^2}\right)^{-1}\frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}\sigma^2}$
   d. $\frac{{\sigma_h}^2}{\sigma^2}\mathbf{X}^{\mathrm{T}}\mathbf{Y}$

   Ans b

3. Consider the LMMSE (Linear Minimum Mean Squared Error) MIMO channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1 \le k \le N$ and N received symbol vectors $\mathbf{y}(k) = [y_1(k), y_2(k),\ldots,y_R(k)]^T$, $1 \le k \le N$. Let the channel coefficients be IID Gaussian with variance \sigma_h^2 and noise variance is \sigma^2. The expression for the error covariance of the LMMSE estimate of the MIMO channel matrix H is,
   a. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\sigma^2}+\frac{\mathbf{I}}{{\sigma_h}^2}\right)^{-1}$
   b. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}{\sigma_h}^2}+\frac{\mathbf{I}}{\mathrm{R}\sigma^2}\right)^{-1}$

c. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\sigma_h{}^2} + \frac{\mathbf{I}}{\sigma^2}\right)^{-1}$
d. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2} + \frac{\mathbf{I}}{\mathrm{R}\sigma_h{}^2}\right)^{-1}$
Ans d

4. Consider the MIMO channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1\leq k \leq N$ and N received symbol vectors $\mathbf{y}(k) = [y_1(k), y_2(k),\ldots,y_R(k)]^T$, $1\leq k \leq N$. Let the channel coefficients be IID Gaussian with variance \sigma_h^2 and noise variance is \sigma^2. The expression for the covariance of the ML estimate of the MIMO channel matrix H is, [Hint: This can be obtained by assuming \sigma_h^2 X^TX >> \sigma^2 I]
   a. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\sigma^2}\right)^{-1}$
   b. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma_h{}^2}\right)^{-1}$
   c. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2}\right)^{-1}$
   d. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\sigma_h{}^2}\right)^{-1}$
   Ans c

5. Consider the MIMO channel estimation problem with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. The received output vectors y are $\mathbf{y}(1) = [1, -1, 1]^T$, $\mathbf{y}(2) = [2, -2, 1]^T$, $\mathbf{y}(3) = [-2, 1, -2]^T$, $\mathbf{y}(4) = [1, 2, -1]^T$. Let the channel coefficients be IID Gaussian with dB variance \sigma_h^2 = 0 dB and dB noise variance \sigma^2 = -3 dB. The LMMSE estimate of the MIMO channel matrix H is,
   a. $\begin{bmatrix} 0 & 2/15 & -2/5 \\ -4/5 & 4/15 & -4/15 \end{bmatrix}$
   b. $\begin{bmatrix} 0 & 1/7 & -3/7 \\ -6/7 & 2/7 & -2/7 \end{bmatrix}$
   c. $\begin{bmatrix} 0 & 2 & -6 \\ -12 & 4 & -4 \end{bmatrix}$
   d. $\begin{bmatrix} 0 & 1/2 & -3/2 \\ -3 & 1 & -1 \end{bmatrix}$
   Ans a

6. Consider the MIMO channel estimation problem with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. The received output vectors y are $\mathbf{y}(1) = [1, -1, 1]^T$, $\mathbf{y}(2) = [2, -2, 1]^T$, $\mathbf{y}(3) = [-2, 1, -2]^T$, $\mathbf{y}(4) = [1, 2, -1]^T$. Let the channel coefficients be IID Gaussian with dB variance \sigma_h^2 = 0 dB and dB noise variance \sigma^2 = -3 dB. The ML estimate of the MIMO channel matrix H is,

a. $\begin{bmatrix} 0 & 2/15 & -2/5 \\ -4/5 & 4/15 & -4/15 \end{bmatrix}$
b. $\begin{bmatrix} 0 & 1/7 & -3/7 \\ -6/7 & 2/7 & -2/7 \end{bmatrix}$
c. $\begin{bmatrix} 0 & 2 & -6 \\ -12 & 4 & -4 \end{bmatrix}$
d. $\begin{bmatrix} 0 & 1/2 & -3/2 \\ -3 & 1 & -1 \end{bmatrix}$

Ans b

7. Consider the MIMO channel estimation problem with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. The received output vectors y are $\mathbf{y}(1) = [1, -1, 1]^T$, $\mathbf{y}(2) = [2, -2, 1]^T$, $\mathbf{y}(3) = [-2, 1, -2]^T$, $\mathbf{y}(4) = [1, 2, -1]^T$. Let the channel coefficients be IID Gaussian with dB variance \sigma_h^2 = 0 dB and dB noise variance \sigma^2 = -3 dB. The error covariance of the LMMSE estimate of the MIMO channel matrix H is,
   a. $\begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}$
   b. $\begin{bmatrix} 3/14 & 0 \\ 0 & 3/14 \end{bmatrix}$
   c. $\begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix}$
   d. $\begin{bmatrix} 1/14 & 0 \\ 0 & 1/14 \end{bmatrix}$

   Ans c

8. Channel equalization refers to
   a. Making all the channel gains equal
   b. Making all the transmit powers equal
   c. Making the channels of different users equal
   d. Removing the effect of ISI

   Ans d

9. Consider an Inter Symbol Interference channel $y(k) = \frac{3}{2}x(k) - \frac{1}{2}x(k-1) + v(k)$. Let an r = 3 tap channel equalizer be designed for this scenario based on symbols $y(k+2)$, $y(k+1)$, $y(k)$ to detect $x(k)$. What is the effective channel matrix **H** for this scenario
   a. $\mathbf{H} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$

b. $\mathbf{H} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

c. $\mathbf{H} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

d. $\mathbf{H} = \begin{bmatrix} 0 & 0 & \frac{3}{2} & -\frac{1}{2} \\ 0 & \frac{3}{2} & -\frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{1}{2} & 0 & 0 \end{bmatrix}$

Ans c

10. Consider the LMMSE (Linear Minimum Mean Squared Error) multi-antenna channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_M(k)]^T$, $1 \leq k \leq N$ and N received symbols $y(1), y(2), \ldots, y(N)$. Let the channel vector be $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$. Let the channel covariance E{hh^T} = R_h and noise covariance E{vv^T} = R_v. The expressions for the LMMSE estimate, error covariance matrix of **h** respectively are,

a. $\left(\mathbf{R}_h^{-1} + \mathbf{X}^{\mathrm{T}}\mathbf{R}_v^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{R}_v^{-1}\boldsymbol{y}, \left(\mathbf{R}_h^{-1} + \mathbf{X}^{\mathrm{T}}\mathbf{R}_v^{-1}\mathbf{X}\right)^{-1}$

b. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2} + \frac{\mathbf{I}}{\mathrm{R}\sigma_h{}^2}\right)^{-1} \frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}\sigma^2}, \left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma^2} + \frac{\mathbf{I}}{\mathrm{R}\sigma_h{}^2}\right)^{-1}$

c. $\left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma_h{}^2} + \frac{\mathbf{I}}{\mathrm{R}\sigma^2}\right)^{-1} \frac{\mathbf{X}^{\mathrm{T}}\mathbf{Y}}{\mathrm{R}\sigma_h{}^2}, \left(\frac{\mathbf{X}^{\mathrm{T}}\mathbf{X}}{\mathrm{R}\sigma_h{}^2} + \frac{\mathbf{I}}{\mathrm{R}\sigma^2}\right)^{-1}$

d. $\left(\mathbf{R}_v^{-1} + \mathbf{X}^{\mathrm{T}}\mathbf{R}_h^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{R}_v^{-1}\boldsymbol{y}, \left(\mathbf{R}_v^{-1} + \mathbf{X}^{\mathrm{T}}\mathbf{R}_h^{-1}\mathbf{X}\right)^{-1}$

Ans a