NPTEL MOOC Estimation: Assignment #5

1. Consider the LMMSE (Linear Minimum Mean Squared Error) multi-antenna channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1\leq k \leq N$ and N received symbols $y(1), y(2), ..., y(N)$. Let the channel vector be $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$. Let the channel coefficients be IID Gaussian with variance σ_h^2 and noise variance is σ^2. The optimization problem for MMSE estimation of $\mathbf{h}$ is,
   a. $\min\left|\mathbf{y}-\mathbf{Xh}\right|$
   b. $\min\left\|\mathbf{y}-\mathbf{Xh}\right\|^2$
   c. $\min E\left\{\left\|\hat{\mathbf{h}}-\mathbf{h}\right\|^2\right\}$, where $\hat{\mathbf{h}}=\mathbf{c}^T\mathbf{y}$
   d. $\min E\left\{\left|\mathbf{y}-\mathbf{Xh}\right|^2\right\}$

   Ans c
2. Consider the LMMSE (Linear Minimum Mean Squared Error) multi-antenna channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1\leq k \leq N$ and N received symbols $y(1), y(2), ..., y(N)$. Let the channel vector be $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$. Let the channel coefficients be IID Gaussian with variance σ_h^2 and noise variance is σ^2. The LMMSE estimate of $\mathbf{h}$ is,
   a. $\sigma_h^2(\sigma_h^2\boldsymbol{X}^T\boldsymbol{X}+\sigma^2\boldsymbol{I}_{MXM})^{-1}\boldsymbol{X}^T\boldsymbol{y}$
   b. $\sigma^2(\sigma_h^2\boldsymbol{X}^T\boldsymbol{X}+\sigma^2\boldsymbol{I}_{MXM})^{-1}\boldsymbol{X}^T\boldsymbol{y}$
   c. $\sigma_h^2(\sigma_h^2\boldsymbol{X}\boldsymbol{X}^T+\sigma^2\boldsymbol{I}_{MXM})^{-1}\boldsymbol{X}^T\boldsymbol{y}$
   d. $\sigma_h^2(\sigma_h^2\boldsymbol{X}\boldsymbol{X}^T+\sigma^2\boldsymbol{I}_{NXN})^{-1}\boldsymbol{X}^T\boldsymbol{y}$

   Ans a

3. Consider the LMMSE (Linear Minimum Mean Squared Error) multi-antenna channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1\leq k \leq N$ and N received symbols $y(1), y(2), ..., y(N)$. Let the channel vector be $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$. Let the channel coefficients be IID Gaussian with variance σ_h^2 and noise variance is σ^2. The LMMSE estimates of $\mathbf{h}$ at high and low SNRs are approximately,
   a. $(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}, \sigma_h^2\boldsymbol{X}^T\boldsymbol{y}$
   b. $(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}, \frac{\sigma_h^2}{\sigma^2}\boldsymbol{X}^T\boldsymbol{y}$
   c. $(\boldsymbol{X}\boldsymbol{X}^T)^{-1}\boldsymbol{X}^T\boldsymbol{y}, \frac{\sigma_h^2}{\sigma^2}\boldsymbol{X}^T\boldsymbol{y}$
   d. $(\boldsymbol{X}\boldsymbol{X}^T)^{-1}\boldsymbol{X}^T\boldsymbol{y}, \sigma_h^2\boldsymbol{X}^T\boldsymbol{y}$

   Ans b

4. Consider the LMMSE (Linear Minimum Mean Squared Error) multi-antenna channel estimation problem with N transmitted pilot vectors $\mathbf{x}(k) = [x_1(k), x_2(k),\ldots,x_M(k)]^T$, $1\leq k \leq$

N and N received symbols $y(1), y(2), ..., y(N)$. Let the channel vector be $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$. Let the channel coefficients be IID Gaussian with variance σ_h^2 and noise variance is σ^2. The error covariance for the LMMSE estimate of $\mathbf{h}$ is,

a. $\left(\frac{\boldsymbol{X}\boldsymbol{X}^T}{\sigma^2} + \frac{\boldsymbol{I}}{\sigma_h^2}\right)^{-1}$

b. $\left(\frac{\boldsymbol{X}^T\boldsymbol{X}}{\sigma_h^2} + \frac{\boldsymbol{I}}{\sigma^2}\right)^{-1}$

c. $\left(\frac{\boldsymbol{X}^T\boldsymbol{X}}{\sigma^2} + \frac{\boldsymbol{I}}{\sigma_h^2}\right)^{-1}$

d. $\left(\frac{\boldsymbol{X}\boldsymbol{X}^T}{\sigma_h^2} + \frac{\boldsymbol{I}}{\sigma^2}\right)^{-1}$

Ans c

5. Consider a multi-antenna channel estimation scenario with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. Let the corresponding received vector be $\mathbf{y} = [2, 1, 1, 3]^T$. Let the channel coefficients be IID Gaussian with dB variance σ_h^2 = 0 dB and dB noise variance σ^2= -3 dB. The LMMSE estimate of $\mathbf{h}$ is,

a. $\begin{bmatrix} 8/15 \\ -2/15 \end{bmatrix}$

b. $\begin{bmatrix} -16/15 \\ 4/15 \end{bmatrix}$

c. $\begin{bmatrix} 8/15 \\ 2/15 \end{bmatrix}$

d. $\begin{bmatrix} 16/15 \\ -4/15 \end{bmatrix}$

Ans d

6. Consider a multi-antenna channel estimation scenario with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. Let the corresponding received vector be $\mathbf{y} = [2, 1, 1, 3]^T$. Let the channel coefficients be IID Gaussian with dB variance σ_h^2 = 0 dB and dB noise variance σ^2= -3 dB. The ML estimate of $\mathbf{h}$ is,

a. $\begin{bmatrix} 8/15 \\ -4/15 \end{bmatrix}$

b. $\begin{bmatrix} 8/7 \\ -2/7 \end{bmatrix}$

c. $\begin{bmatrix} 8/7 \\ 2/7 \end{bmatrix}$

d. $\begin{bmatrix} 16/15 \\ -4/15 \end{bmatrix}$

Ans b

7. Consider a multi-antenna channel estimation scenario with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. Let the corresponding received vector

be **y** = [2, 1, 1, 3]^T. Let the channel coefficients be IID Gaussian with dB variance σ_h^2 = 0 dB and dB noise variance σ^2= -3 dB. The error covariance of the LMMSE estimate of **h** is,

a. $\begin{bmatrix} \frac{1}{15} & 0 \\ 0 & \frac{1}{15} \end{bmatrix}$

b. $\begin{bmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{16} \end{bmatrix}$

c. $\begin{bmatrix} \frac{1}{7} & 0 \\ 0 & \frac{1}{7} \end{bmatrix}$

d. $\begin{bmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{8} \end{bmatrix}$

Ans a

8. Consider a multi-antenna channel estimation scenario with N = 4 pilot vectors **x**(1) = [1, 1]^T, **x**(2) = [1, -2]^T, **x**(3) = [2, 1]^T, **x**[4] = [1, -1]^T. Let the corresponding received vector be **y** = [2, 1, 1, 3]^T. Let the channel coefficients be IID Gaussian and dB noise variance σ^2 = -3 dB. Let $\sigma_h^2 \boldsymbol{X}^T\boldsymbol{X}$>> σ^2I. The error covariance of the LMMSE estimate of **h** reduces to,

a. $\begin{bmatrix} \frac{1}{14} & 0 \\ 0 & \frac{1}{14} \end{bmatrix}$

b. $\begin{bmatrix} \frac{1}{15} & 0 \\ 0 & \frac{1}{15} \end{bmatrix}$

c. $\begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}$

d. $\begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

Ans a

9. Consider the MIMO channel estimation problem with N = 4 pilot vectors **x**(1) = [1, 1]^T, **x**(2) = [1, -2]^T, **x**(3) = [2, 1]^T, **x**[4] = [1, -1]^T. The received output vectors y are **y**(1) = [1, -1, 1]^T, **y**(2) = [2, -2, 1]^T, **y**(3) = [-2, 1, -2]^T, **y**(4) = [1, 2, -1]^T . Let the channel coefficients be IID Gaussian with dB variance σ_h^2 = 0 dB and dB noise variance σ^2 = -3 dB. The size of the MIMO system is,

a. 4x3

b. 4x2

c. 2x3
d. None of these
Ans c

10. Consider the MIMO channel estimation problem with N = 4 pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. The received output vectors y are $\mathbf{y}(1) = [1, -1, 1]^T$, $\mathbf{y}(2) = [2, -2, 1]^T$, $\mathbf{y}(3) = [-2, 1, -2]^T$, $\mathbf{y}(4) = [1, 2, -1]^T$. Let the channel coefficients be IID Gaussian with dB variance σ_h^2 = 0 dB and dB noise variance σ^2= -3 dB. The covariance matrices $\boldsymbol{R}_{HH}$ of the channel matrix and $\boldsymbol{R}_{VV}$ of the noise matrix respectively are,
a. $3\boldsymbol{I}_{2X2}$, $\frac{3}{2}\boldsymbol{I}_{4X4}$
b. $\frac{3}{2}\boldsymbol{I}_{4X4}$, $3\boldsymbol{I}_{2X2}$
c. $\frac{3}{2}\boldsymbol{I}_{2X2}$, $3\boldsymbol{I}_{4X4}$
d. $2\boldsymbol{I}_{2X2}, 4\boldsymbol{I}_{2X2}$
Ans a