NPTEL MOOC Estimation: Assignment #4

1. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *complex* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [2+j \quad -1-j \quad 1-2j \quad -1+j]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [2+2j \quad -j \quad -2+j \quad 1-j]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance σ^2 = 3 dB. Let mu_h = 1 + j, \sigma_h^2 = 1/2 denote the prior mean, variance of the complex symmetric Gaussian parameter h. The MSE of the MMSE estimate of $\hat{h}$ is,
   a. 1/9
   b. 1/8
   c. 1/10
   d. 1/7
   Ans a

2. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *complex* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [2+j \quad -1-j \quad 1-2j \quad -1+j]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [2+2j \quad -j \quad -2+j \quad 1-j]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance σ^2 = 3 dB. Let mu_h = 1 + j, \sigma_h^2 = 1/2 denote the prior mean, variance of the complex symmetric Gaussian parameter h. The MSEs of the real, imaginary parts of $\hat{h}$ are,
   a. 1/2
   b. 1/9
   c. 1/18
   d. 1/4
   Ans c

3. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [2+j \quad -1-j \quad 1-2j \quad -1+j]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [2+2j \quad -j \quad -2+j \quad 1-j]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance σ^2 = 3 dB. Let mu_h = 1 + j, \sigma_h^2 = 1/2 denote the prior mean, variance of the parameter h. What is the probability that both the real and imaginary parts

of the MMSE estimate $\hat{h}$ lie within a radius of ½ from the real and imaginary parts of the unknown complex symmetric Gaussian parameter h respectively

a. $\left(1-2Q\left(\frac{9}{2}\right)\right)^2$

b. $\left(1-2Q\left(\sqrt{\frac{9}{2}}\right)\right)^2$

c. $\left(1-2Q\left(\sqrt{\frac{9}{2}}\right)\right)$

d. $\left(1-2Q\left(\frac{9}{2}\right)\right)$

Ans b

4. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le N$, i.e. number of observations N and independent **non-identical** Gaussian noise samples $v(k)$ of variance σ_k^2. What is the expression for the MMSE estimate $\hat{h}$ of the unknown Gaussian parameter h

a. $\hat{h} = \frac{\frac{(\mathbf{1}^T\mathbf{y})/N}{\sigma^2}+\frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/N}+\frac{1}{\sigma_h^2}}$

b. $\hat{h} = \frac{\sum_{k=1}^{N}\frac{y(k)}{2\sigma_k^2}+\frac{\mu_h}{\sigma_h^2}}{\sum_{k=1}^{N}\frac{1}{\sigma_k^2}+\frac{1}{\sigma_h^2}}$

c. $\hat{h} = \frac{\sum_{k=1}^{N}\frac{y(k)}{2\sigma_k^2}+\frac{\mu_h}{\sigma_h^2}}{\sum_{k=1}^{N}\frac{1}{2\sigma_k^2}+\frac{1}{\sigma_h^2}}$

d. $\hat{h} = \frac{\sum_{k=1}^{N}\frac{y(k)}{\sigma_k^2}+\frac{\mu_h}{\sigma_h^2}}{\sum_{k=1}^{N}\frac{1}{\sigma_k^2}+\frac{1}{\sigma_h^2}}$

Ans d

5. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le N$, i.e. number of observations N and independent **non-identical** Gaussian noise samples $v(k)$ of variance σ_k^2. What is the expression for the MSE of the MMSE estimate $\hat{h}$ of the unknown Gaussian parameter h

a. $\left(\sum_{k=1}^{N}\frac{1}{\sigma_k^2}+\frac{1}{\sigma_h^2}\right)^{-1}$

b. $\left(\sum_{k=1}^{N}\frac{1}{\sigma_k^2}+\frac{2}{\sigma_h^2}\right)^{-1}$
c. $\sum_{k=1}^{N}\frac{2}{\sigma_k^2}+\frac{1}{\sigma_h^2}$
d. $\sum_{k=1}^{N}\frac{1}{\sigma_k^2}+\frac{1}{\sigma_h^2}$
Ans a

6. Which of the following statements is true about the LMMSE estimate
   a. LMMSE estimate is always the MMSE estimate
   b. MMSE estimate is always the LMMSE estimate
   c. The LMMSE estimate is identical to the MMSE estimate for a Gaussian parameter
   d. The MMSE and LMMSE estimates are always different
   Ans c

7. Consider LMMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le N$, i.e. number of observations N and IID Gaussian noise samples of variance σ^2. What is the expression for the LMMSE estimate $\hat{h}$ of the unknown parameter h, which is not necessarily Gaussian
   a. $\hat{h} = \dfrac{\frac{(\mathbf{x}^T\mathbf{y})/\|\mathbf{x}\|^2}{\sigma^2/\|\mathbf{x}\|^2}+\frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2}+\frac{1}{\sigma_h^2}}$
   b. $\hat{h} = \dfrac{\frac{(\mathbf{1}^T\mathbf{y})/N}{2\sigma^2}+\frac{\mu_h}{\sigma_h^2}}{\frac{1}{2\sigma^2/\mathrm{N}}+\frac{1}{\sigma_h^2}}$
   c. $\hat{h} = \dfrac{\frac{(\mathbf{1}^T\mathbf{y})/N}{\sigma^2}+\frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/\mathrm{N}}+\frac{1}{\sigma_h^2}}$
   d. $\hat{h} = \boldsymbol{r}_{hy}\boldsymbol{R}_{yy}^{-1}\mathbf{y}$
   Ans d

8. Consider a multi-antenna channel estimation scenario with $N = 4$ pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. Let the corresponding received vector be $\mathbf{y} = [2, 1, 1, 3]^T$. Let the channel coefficients be IID Gaussian with dB variance \sigma_h^2 = -3 dB and dB noise variance \sigma^2 = 3 dB. The number of antennas M in this scenario is,
   a. 1
   b. 2
   c. 3
   d. 4

Ans b

9. Consider a multi-antenna channel estimation scenario with N = 4 pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. Let the corresponding received vector be $\mathbf{y} = [2, 1, 1, 3]^T$. Let the channel coefficients be IID Gaussian with dB variance \sigma_h^2 = -3 dB and dB noise variance \sigma^2 = 3 dB. The corresponding pilot matrix **X** is
   a. $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -1 \end{bmatrix}$
   b. $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 1 \\ -2 \\ 1 \\ -1 \end{bmatrix}$
   c. $\begin{bmatrix} 1 & 1 & 2 & 1 & 1 & -2 & 1 & -1 \end{bmatrix}$
   d. $\begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$

   Ans d

10. Consider a multi-antenna channel estimation scenario with N = 4 pilot vectors $\mathbf{x}(1) = [1, 1]^T$, $\mathbf{x}(2) = [1, -2]^T$, $\mathbf{x}(3) = [2, 1]^T$, $\mathbf{x}[4] = [1, -1]^T$. Let the corresponding received vector be $\mathbf{y} = [2, 1, 1, 3]^T$. Let the channel coefficients be IID Gaussian with dB variance \sigma_h^2 = -3 dB and dB noise variance \sigma^2 = 3 dB. The pilot matrix **X** for this scenario satisfies the property that
    a. It is invertible
    b. It has identical columns
    c. It has orthogonal columns
    d. None of the above

    Ans c