NPTEL MOOC Estimation: Assignment #3

1. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le N$, i.e. number of observations N and IID Gaussian noise samples of variance σ^2. What is the expression for the MSE (Mean Squared Error) of the ML estimate $\hat{h}$ of the unknown Gaussian parameter h
   a) $\frac{\sigma^2}{N}$
   b) $\frac{\sigma_h^2}{N}$
   c) $\frac{1}{\frac{1}{\frac{\sigma^2}{N}}+\frac{1}{\sigma_h^2}}$
   d) $\frac{1}{\sigma_h^2}$

Ans (a)

2. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le N$, i.e. number of observations N and IID Gaussian noise samples of variance σ^2. What is the expression for the MMSE i.e. Minimum Mean Squared Error in the estimate $\hat{h}$ of the unknown Gaussian parameter h
   a) $\frac{1}{\sigma_h^2}$
   b) $\frac{1}{\frac{1}{\frac{\sigma^2}{N}}+\frac{1}{\sigma_h^2}}$
   c) $\frac{\sigma^2}{N}$
   d) None of these
   e)

Ans (b)

3. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le 5$, i.e. number of observations $N = 5$ and IID Gaussian noise samples of dB variance $\sigma^2 = -3$ dB i.e. $10\log_{10}\sigma^2 = -3$. Let the observations be $y(1) = 1$, $y(2) = 1$, $y(3) = 2$, $y(4) = 3/2$, $y(5) = 5/2$. Let mu_h = ½ and \sigma_h^2 = ¼, h be Gaussian. The MSEs of the ML and MMSE estimates are given as,
   a) 1/14,1/10
   b) 1/10,1/14
   c) 1/10,1/4
   d) 1/10,1/8

Ans (b)

4. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \leq k \leq N$, i.e. number of observations is N and IID Gaussian noise samples of dB variance σ^2 = - 3 dB i.e. $10\log_{10}\sigma^2 = -3$. Let \sigma_h^2 = 1/4, h be Gaussian. What is the minimum number of observations N required such that the true parameter h is within \sigma/4 of the estimate \hat{h} with probability greater than 99.9%?
   a) 172
   b) 170
   c) 171
   d) 174

Ans (a)

5. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1),x(2),\ldots,x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2),\ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let mu_h, \sigma_h^2 denote the prior mean, variance of the Gaussian parameter h. What is the expression for the MSE (Mean Squared Error) of the ML estimate $\hat{h}$ of the unknown parameter h
   a) $\frac{1}{\frac{1}{\frac{\sigma^2}{N}}+\frac{1}{\sigma_h^2}}$
   b) $\frac{1}{\frac{1}{\frac{\sigma^2}{\|x\|^2}}+\frac{1}{\sigma_h^2}}$
   c) $\frac{\sigma^2}{N}$
   d) $\frac{\sigma^2}{\|\boldsymbol{x}\|^2}$

Ans (d)

6. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1),x(2),\ldots,x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2),\ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let mu_h, \sigma_h^2 denote the prior mean, variance of the Gaussian parameter h. What is the expression for the MMSE i.e. Minimum Mean Squared Error in the estimate $\hat{h}$ of the unknown parameter h

a) $\frac{1}{\frac{1}{\frac{\sigma^2}{N}}+\frac{1}{\sigma_h^2}}$
b) $\frac{1}{\frac{1}{\frac{\sigma^2}{\|x\|^2}}+\frac{1}{\sigma_h^2}}$
c) $\frac{\sigma^2}{N}$
d) $\frac{\sigma^2}{\|x\|^2}$

Ans (b)

7. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2), \ldots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2), \ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let mu_h, \sigma_h^2 denote the prior mean, variance of the Gaussian parameter h. As the number of samples N becomes very large, what is the expression for the MMSE i.e. Minimum Mean Squared Error in the estimate $\hat{h}$ of the unknown parameter h
   a) σ_h^2
   b) $\frac{\sigma^2}{\|x\|^2}$
   c) $\frac{\sigma^2}{N}$
   d) None of these

Ans (b)

8. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = \begin{bmatrix} \frac{1}{2} & 2 & 1 & \frac{3}{2} \end{bmatrix}^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = \begin{bmatrix} -2 & 2 & -1 & 1 \end{bmatrix}^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance σ^2 = 3 dB. Let mu_h = 1, \sigma_h^2 = 1/2 denote the prior mean, variance of the Gaussian parameter h. The MSEs of the ML and MMSE estimates are given as,
   a) 4/15,4/23
   b) 4/11,4/19
   c) 4/19,4/11
   d) 4/23,4/15

Ans (a)

9. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2), \ldots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2), \ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let mu_h, \sigma_h^2 denote the prior mean, variance of the Gaussian parameter h. What is the minimum number of observations N required such that the true parameter h is within \sigma/8 of the estimate \hat{h} with probability greater than 99.99%? Assume the pilot symbols are constant modulus with |x(k)|^2 = 4.

   a) $\frac{1}{4}\left((Q^{-1}(5\times10^{-4}))^2 - \frac{\sigma^2}{\sigma_h^2}\right)$
   b) $\frac{1}{4}\left((16Q^{-1}(5\times10^{-5})) - \frac{\sigma^2}{\sigma_h^2}\right)$
   c) $\frac{1}{2}\left((64Q^{-1}(5\times10^{-5}))^2 - \frac{\sigma^2}{4\sigma_h^2}\right)$
   d) $\frac{1}{4}\left((8Q^{-1}(5\times10^{-5}))^2 - \frac{\sigma^2}{\sigma_h^2}\right)$

Ans (d)

10. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *complex* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [2+j \quad -1-j \quad 1-2j \quad -1+j]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [2+2j \quad -j \quad -2+j \quad 1-j]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance σ^2 = 3 dB. Let mu_h = 1 + j, \sigma_h^2 = 1/2 denote the prior mean, variance of the complex symmetric Gaussian parameter h. The MMSE estimate of $\hat{h}$ is,

   a) 1/12
   b) 1/7
   c) 1/9
   d) None of these

Ans (c)