NPTEL MOOC Estimation: Assignment #2

1. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le$ N, i.e. number of observations N and IID Gaussian noise samples of variance σ^2. What is the expression for the MMSE estimate $\hat{h}$ of the unknown parameter h
   a. $\hat{h} = \dfrac{\frac{(\mathbf{1}^T\mathbf{y})/N}{\sigma^2} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/\mathrm{N}} + \frac{1}{\sigma_h^2}}$
   b. $\hat{h} = \dfrac{\frac{(\mathbf{1}^T\mathbf{y})/N}{\sigma^2/\mathrm{N}} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/\mathrm{N}} + \frac{1}{\sigma_h^2}}$
   c. $\hat{h} = \dfrac{\frac{(\mathbf{1}^T\mathbf{y})/N}{\sigma^2/\mathrm{N}} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_h^2}}$
   d. $\hat{h} = \dfrac{\frac{(\mathbf{1}^T\mathbf{y})/N}{\sigma^2} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_h^2}}$

   Ans b

2. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le$ N, i.e. number of observations N and IID Gaussian noise samples of dB variance σ^2. As the number of observations N $\rightarrow \infty$, the MMSE estimate $\hat{h}$ of the unknown parameter h tends to,
   a. 0
   b. 1
   c. ML Estimate
   d. Prior Mean

   Ans c

3. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \le k \le 5$, i.e. number of observations $N = 5$ and IID Gaussian noise samples of dB variance σ^2= - 3 dB i.e. $10\log_{10} \sigma^2$= -3. Let the observations be $y(1) = 1$, $y(2) = 1$, $y(3) = 2$, $y(4) = 3/2$, $y(5) = 5/2$. Let $\mu_h = 1/2$ and $\sigma_h^2 = 1/4$. What is the maximum likelihood estimate $\hat{h}$ of the unknown parameter h
   a. 1.6
   b. 1
   c. 1.5
   d. 2

Ans a

4. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \leq k \leq 5$, i.e. number of observations $N = 5$ and IID Gaussian noise samples of dB variance σ^2= - 3 dB i.e. $10\log_{10} \sigma^2$= -3 dB. Let the observations be $y(1) = 1$, $y(2) = 1$, $y(3) = 2$, $y(4) = 3/2$, $y(5) = 5/2$. Let $\mu_h = 1/2$ and $\sigma_h^2 = 1/4$. What is the MMSE estimate $\hat{h}$ of the unknown parameter h
   a. 10/7
   b. 8/7
   c. 5/7
   d. 9/7
   Ans d

5. Consider MMSE estimation for the wireless sensor network (WSN) scenario as described in lectures with each observation $y(k) = h + v(k)$, for $1 \leq k \leq 5$, i.e. number of observations $N = 5$ and IID Gaussian noise samples of dB variance σ^2= - 3 dB i.e. $10\log_{10} \sigma^2$ = -3 dB. Let the observations be $y(1) = 1$, $y(2) = 1$, $y(3) = 2$, $y(4) = 3/2$, $y(5) = 5/2$. Let $\mu_h = 1/2$ and $\sigma_h^2 = 1/4$. What is the posterior probability density function of the unknown parameter h
   a. Gaussian
   b. Exponential
   c. Rayleigh
   d. Uniform
   Ans a

6. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2),\dots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2),\dots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let μ_h, σ_h^2 denote the prior mean, variance of the parameter h. The maximum likelihood estimate of the channel coefficient h is,
   a. $\hat{h} = \frac{\mathbf{x}^T\mathbf{y}}{||\mathbf{x}||}$
   b. $\hat{h} = \frac{\mathbf{y}^T\mathbf{y}}{||\mathbf{x}||^2}$
   c. $\hat{h} = \frac{\mathbf{x}^T\mathbf{y}}{||\mathbf{x}||^2}$
   d. $\hat{h} = \frac{\mathbf{y}^T\mathbf{y}}{||\mathbf{x}||}$

Ans c

7. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2),\ldots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2),\ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let μ_h, σ_h^2 denote the prior mean, variance of the parameter h. The covariance matrix R_{yy} of the output vector y is,
   a. $R_{yy} = \sigma^2\mathbf{x}\mathbf{x}^T + \sigma_h^2 I$
   b. $R_{yy} = \sigma_h^2\mathbf{x}\mathbf{x}^T + \sigma^2 I$
   c. $R_{yy} = \mathbf{x}\mathbf{x}^T + \sigma^2 I$
   d. $R_{yy} = \sigma_h^2\mathbf{x}\mathbf{x}^T + I$

   Ans b

8. Given the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = [x(1), x(2),\ldots, x(N)]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [y(1), y(2),\ldots, y(N)]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let μ_h, σ_h^2 denote the prior mean, variance of the parameter h. The MMSE estimate of the channel coefficient h is,
   a. $\hat{h} = \dfrac{\frac{(\mathbf{x}^T\mathbf{y})/\|\mathbf{x}\|^2}{\sigma^2/\|\mathbf{x}\|^2} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2} + \frac{1}{\sigma_h^2}}$
   b. $\hat{h} = \dfrac{\frac{(\mathbf{x}^T\mathbf{y})/\|\mathbf{x}\|^2}{\sigma^2} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/\|\mathbf{x}\|^2} + \frac{1}{\sigma_h^2}}$
   c. $\hat{h} = \dfrac{\frac{(\mathbf{x}^T\mathbf{y})/\|\mathbf{x}\|^2}{\sigma^2/\|\mathbf{x}\|} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/\|\mathbf{x}\|} + \frac{1}{\sigma_h^2}}$
   d. $\hat{h} = \dfrac{\frac{(\mathbf{x}^T\mathbf{y})/\|\mathbf{x}\|^2}{\sigma^2/\|\mathbf{x}\|^2} + \frac{\mu_h}{\sigma_h^2}}{\frac{1}{\sigma^2/\|\mathbf{x}\|^2} + \frac{1}{\sigma_h^2}}$

   Ans d

9. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = \begin{bmatrix} \frac{1}{2} & 2 & 1 & \frac{3}{2} \end{bmatrix}^T$ denote the pilot vector of transmitted

pilot symbols and $\mathbf{y} = [-2 \quad 2 \quad -1 \quad 1]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and dB variance σ^2= 3 dB. Let μ_h= 1, σ_h^2= 1/2 denote the prior mean, variance of the parameter h. The MMSE estimate of the channel coefficient h is,

a. 15/23
b. 20/23
c. 21/23
d. 17/23

Ans a

10. Consider the fading channel estimation problem where the output symbol $y(k)$ is $y(k) = hx(k) + v(k)$, with h, $x(k)$, $v(k)$ denoting the *real* channel coefficient, pilot symbol and noise sample respectively. Let $\mathbf{x} = \left[\frac{1}{2} \quad 2 \quad 1 \quad \frac{3}{2}\right]^T$ denote the pilot vector of transmitted pilot symbols and $\mathbf{y} = [-2 \quad 2 \quad -1 \quad 1]^T$ denote the corresponding received symbol vector. Let $v(k)$ be IID Gaussian noise with zero-mean and variance σ^2. Let μ_h= 1, σ_h^2= 1/2 denote the prior mean, variance of the parameter h. As the noise variance $\sigma^2 \rightarrow \infty$, the MMSE estimate of the channel coefficient h becomes,

a. 15/23
b. 7/4
c. 1
d. 0

Ans c