NPTEL MOOC Estimation: Assignment #1

1. In the context of the estimation framework considered, the probability density function (PDF) of the unknown parameter h is termed as the
    a. Likelihood Density
    b. Parameter Density
    c. Prior Density
    d. Posterior Density
    Ans c
2. In the context of the estimation framework considered, the probability density function (PDF) of the unknown parameter h conditioned on the observation vector **y** is termed as the
    a. Likelihood Density
    b. Parameter Density
    c. Prior Density
    d. Posterior Density
    Ans d
3. The estimation framework described in the lectures is,
    a. Maximum Cost Estimation
    b. Minimum Mean Squared Error Estimation
    c. Minimum Error Estimation
    d. Maximum Likelihood Estimation
    Ans b
4. Given the parameter h and observation vector **y**, the expression for the MMSE estimate is,
    a. $\hat{h} = E\{h|\boldsymbol{y}\}$
    b. $\hat{h} = E\{h\}$
    c. $\hat{h} = E\{h, \boldsymbol{y}\}$
    d. $\hat{h} = E\{\boldsymbol{y}\}$
    Ans a
5. Consider jointly Gaussian random variables h, y with mean 0 each and variances σ_h^2, σ_y^2. Let ρ denote the correlation coefficient between h,y. The marginal probability density functions of h, y are,
    a. $f_H(h) = \frac{1}{\sqrt{2\pi\sigma_h^2}} e^{\frac{-h^2}{\sigma_h^2}}, f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{\frac{-y^2}{\sigma_y^2}}$
    b. $f_H(h) = \frac{1}{\sqrt{2\pi\sigma_h^2}} e^{\frac{-h^2}{2\sigma_h^2}}, f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{\frac{-y^2}{\sigma_y^2}}$

c. $f_H(h) = \frac{1}{\sqrt{2\pi\sigma_h^2}} e^{\frac{-h^2}{\sigma_h^2}}, f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{\frac{-y^2}{2\sigma_y^2}}$

d. $f_H(h) = \frac{1}{\sqrt{2\pi\sigma_h^2}} e^{\frac{-h^2}{2\sigma_h^2}}, f_Y(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} e^{\frac{-y^2}{2\sigma_y^2}}$

Ans d

6. Consider jointly Gaussian random variables h, y with mean 0 each and variances σ_h^2, σ_y^2. Let ρ denote the correlation coefficient between h, y. The joint probability density function of h, y is,

a. $f_{H,Y}(h,y) = \frac{1}{\sqrt{(2\pi)^2\sigma_y^2\sigma_h^2}} e^{\frac{-(\sigma_y^2h^2+\sigma_h^2y^2)}{2\sigma_y^2\sigma_h^2}}$

b. $f_{H,Y}(h,y) = \frac{1}{\sqrt{(2\pi)^2(1-\rho^2)\sigma_y^2\sigma_h^2}} e^{\frac{-(\sigma_y^2h^2-2\rho\sigma_h\sigma_y hy+\sigma_h^2y^2)}{2(1-\rho^2)\sigma_y^2\sigma_h^2}}$

c. $f_{H,Y}(h,y) = \frac{1}{\sqrt{(2\pi)^2(1-\rho^2)\sigma_y^2\sigma_h^2}} e^{\frac{-(\sigma_y^2h^2+\sigma_h^2y^2)}{2(1-\rho^2)\sigma_y^2\sigma_h^2}}$

d. $f_{H,Y}(h,y) = \frac{1}{\sqrt{(2\pi)^2(1-\rho^2)\sigma_y^2\sigma_h^2}} e^{\frac{-(\sigma_y^2h^2-2\rho\sigma_h\sigma_y hy+\rho^2\sigma_h^2y^2)}{2(1-\rho^2)\sigma_y^2\sigma_h^2}}$

Ans b

7. Consider jointly Gaussian random variables h, y with mean 0 each and variances σ_h^2, σ_y^2. Let ρ denote the correlation coefficient between h, y. The conditional probability density function of h given y is,

a. $f_{H|Y}(h|y) = \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma_h^2}} e^{-\frac{\left(h-\rho\frac{\sigma_h}{\sigma_y}y\right)^2}{2(1-\rho^2)\sigma_h^2}}$

b. $f_{H|Y}(h|y) = \frac{1}{\sqrt{2\pi\sigma_h^2}} e^{-\frac{\left(h-\rho\frac{\sigma_h}{\sigma_y}y\right)^2}{2\sigma_h^2}}$

c. $f_{H|Y}(h|y) = \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma_h^2}} e^{-\frac{(h-\rho y)^2}{2(1-\rho^2)\sigma_h^2}}$

d. $f_{H|Y}(h|y) = \frac{1}{\sqrt{2\pi(1-\rho^2)\sigma_h^2}} e^{-\frac{\left(h-\frac{\sigma_h}{\sigma_y}y\right)^2}{2(1-\rho^2)\sigma_h^2}}$

ans a

8. Consider a scenario with jointly Gaussian random variables h,y, where $\mu_h = 1$, $\mu_y = 3$, $\sigma_h^2 = 4$, $\sigma_y^2 = 9$ and correlation coefficient $\rho = 0.85$. Find the cross-correlation r_{hy}.
   a. 5.0
   b. 2.55
   c. 0.85
   d. 5.1

   ans d
9. Consider a scenario with jointly Gaussian random variables h,y, where $\mu_h = 1$, $\mu_y = 3$, $\sigma_h^2 = 4$, $\sigma_y^2 = 9$ and correlation coefficient $\rho = 0.85$. Find the MMSE estimate $\hat{h}$
   a. 1.275y-2.825
   b. 0.566y+2.43
   c. 0.566y-0.7
   d. 1.275y+1.725

   Ans: 0.566y-0.7
10. Consider a scenario with jointly Gaussian random variables h,y, where $\mu_h = 1$, $\mu_y = 3$, $\sigma_h^2 = 4$, $\sigma_y^2 = 9$ and correlation coefficient $\rho = 0.85$. Find the MMSE r_{ee}
   a. 1.11
   b. 0.99
   c. 1.01
   d. 1

   Ans:1.11