Department of Physics, IITM

Scattering, Atomic Collisions and Spectroscopy

PCD_STiAP_P05

1.

a) Show that the Hamiltonian for the Central Field Potential is given by

$$\hat{H} = \left(\frac{1}{2m}\right)\left(p_r^2 + \frac{\hbar^2 l^2}{r^2}\right) + U(r)$$

Where $p_r \psi = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\psi$

b) If the solution to Schrodinger equation with above Hamiltonian for the special case of zero potential is written as

$$\psi_{klm} = R_{kl}(r) Y_{lm}(\theta, \phi)$$

Then show that the stationary state solutions satisfy the following normalization condition: $\int_0^\infty r^2 R_{k'l'} R_{kl} dr = 2\pi\delta(k' - k)$

c) If the normalization is done on the energy scale instead of $\left(\frac{k}{2\pi}\right)$ scale then show that

$$R_{El} = \left(\frac{1}{\hbar}\right)\sqrt[2]{\left(\frac{m}{2\pi k}\right)} R_{kl}$$

d) Prove that the energy normalization eigen function for a free particle in one dimension (along x axis) is given by

$$\left(\frac{m}{8\pi^2\hbar^2 E}\right)^{\frac{1}{4}} e^{\pm ikx}$$

2.

a) Show that for a free particle $R_{k0} = \frac{2\sin kr}{r}$

b) $R_{kl} = (-1)^l \frac{2r^l}{k^l}\left(\frac{1}{r}\frac{d}{dr}\right)^l \frac{\sin kr}{r}$

c) $R_{kl} = 2kj_l(kr)$

where $j_l(kr)$ are spherical Bessel functions.

3. Subject the solution to the Schrodinger equation for scattering of an electron by a central potential, given by (outgoing wave boundary conditions):

$$\psi^{+}_{Tot}\left(\vec{r},t\right)]_{r\to\infty} \to e^{+i(kz-\omega t)} + \frac{e^{+i(kr-\omega t)}}{r}\left\{\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left[e^{2i\delta_l(k)}-1\right]P_l\left(\cos\theta\right)\right\}$$

To time-reversal symmetry and show clearly that the solution for photoelectron ejection is given by (ingoing wave boundary conditions):

$$\psi^{-}_{Tot}\left(\vec{r},t\right)]_{r\to\infty} \to e^{+i(kz+\omega t)} + \frac{e^{+i(kr+\omega t)}}{r}\left\{\frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left[e^{2i\delta_l(k)}-1\right]P_l\left(\cos\theta\right)\right\}$$

4. Obtain the phase shifts δ_l produced by a repulsive potential $V(r)=\frac{A}{r^2}$ with $A>0$. What kind of angular distribution do you get? Is the scattering cross section finite?

5. If the scattering potential is attractive $V(r)=\frac{A}{r^2}$ with $A<0$, would the radial equation $\left[\frac{d^2}{dr^2}+k^2-\frac{l(l+1)}{r^2}-U(r)\right]u_l(k,r)=0$ have solutions for all negative values or would there be any further restriction? Here, $U(r)=\frac{2mV(r)}{\hbar^2}$ and $u_l(k,r)=rR_l(k,r)$.

6. If the differential scattering cross section is written in the form

$$\frac{d\sigma}{d\Omega}=A+BP_1(\cos\theta)+CP_2(\cos\theta)$$

express the coefficients $A, B \& C$ in terms of the phase shifts δ_l.

7.

a) Show that the general solution to the Schrodinger equation,

$$\nabla^2\psi+\frac{2m}{\hbar^2}\left[E-V(r)\right]\psi=0$$

Can be written as $\psi=\sum c_{lm}\frac{F_l(\rho)}{\rho}Y_l^m(\hat{r})$ where $F_l(\rho)$ is a solution of

$$\frac{d^2F_l}{d\rho^2}+\left\{1-\frac{V(r)}{E}-\frac{l(l+1)}{\rho^2}\right\}F_l=0$$

b) Prove that $F_l(\rho\to\infty)=\sin\left(\rho-\frac{l\pi}{2}+\delta_l\right)$

8.

1. Prove that the scattering amplitude $f(\theta)$ given in the asymptotic form

$$\psi(r \to \infty) = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$$

Is related to the differential cross section $\frac{d\sigma}{d\Omega}$ by the following relation:

$$\frac{d\sigma}{d\Omega} = \left| f(k,\theta,\phi) \right|^2$$

2. Prove that the total scattering cross-section is given by $\sigma_{tot} = \left(\frac{4\pi}{k}\right) \text{Im } f(\theta = 0)$.

<u>References:</u>

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4. Sakurai J.J Modern Quantum Mechanics; Adisson-Wesley Publishing Company Inc., 1994
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