Week-6 - Assignment-6 Solutions

(1) The displacement from the vertical is given by

$$y' = \left[\frac{1}{3}gt^3 - ut^2\right]\omega\cos\lambda$$

Here, $u = 100\ m/s$
$\lambda = 60°$
$t = 10\ sec.$

$$= \left[\frac{1}{3}\times 9.8\times 10^3 - 100\times 100\right]\times 7.27\times 10^{-5}\cos 60°$$

$$= -0.245\ m = -24.5\ cm$$

$\therefore$ Displacement will be on west and 24.5 cm.

(2) $$C.M. = \frac{\sum_i m_i r_i}{\sum m_i}$$

$$x = \frac{1\times(-1) + 2\times(3) + 3\times(1) + 4\times 3}{1+2+3+4} = \frac{-1+6+3+12}{10} = 2$$

$$y = \frac{1\times(-2) + 2\times 2 + 3\times(-2) + 4\times 1}{1+2+3+4} = \frac{-2+4-6+4}{10} = 0$$

$$z = \frac{1\times 2 + 2\times(-1) + 3\times(4) + 4\times 2}{1+2+3+4} = \frac{2-2+12+8}{10} = 2$$

$\therefore$ C.M. $= (2, 0, 2)$

(3) $$R_{cm} = \frac{2\times r_1 + 1\times r_2 + 3\times r_3}{2+1+3}$$

$$= \frac{1}{6}\Big[\left(10t\hat{i} - 4t^2\hat{j} + (6t-4)\hat{k}\right) + \left((2t-3)\hat{i} + (12-5t^2)\hat{j} + (4+6t-3t^3)\hat{k}\right) + \left((6t-3)\hat{i} + (3t^2+6)\hat{j} - 3t^3\hat{k}\right)\Big]$$

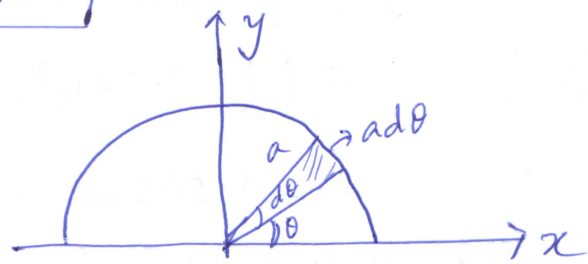
$$= \frac{1}{6}\Big[(10t + 2t - 3 + 6t - 3)\hat{i} + (-4t^2 + 12 - 5t^2 + 3t^2 + 6)\hat{j} + (6t - \cancel{4} + \cancel{4} + 6t - 3t^3 - 3t^3)\hat{k}\Big]$$

$$= \frac{1}{6}\Big[(18t-6)\hat{i} + (18-6t^2)\hat{j} + (12t - 6t^3)\hat{k}\Big]$$

$$R_{cm} = (3t-1)\hat{i} + (3-t^2)\hat{j} + (2t-t^3)\hat{k}$$

Velocity of C.M. $= \dot{R}_{CM} = 3t\,\hat{i} - 2t\,\hat{j} + (2-3t^2)\,\hat{k}$

$$\therefore \boxed{\dot{R}_{CM}\Big|_{t=1\,sec.} = 3\hat{i} - 2\hat{j} - \hat{k}}$$



(4) By symmetry, C.M. of the wire must be on the y-axis.

Let, σ = mass per unit length

$ds = r\,d\theta = a\,d\theta$ = Elementary length

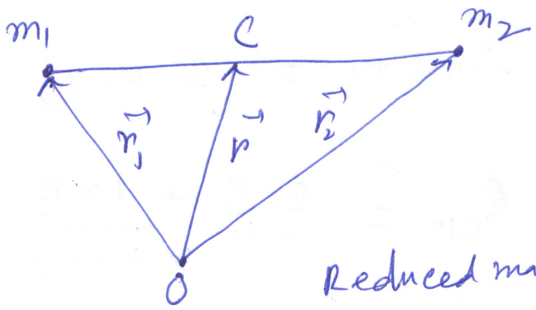
$$\therefore \bar{y} = \frac{\int y\sigma\,ds}{\int \sigma\,ds}\,\hat{y} = \frac{\int_0^{\pi} a\sin\theta \cdot a\,d\theta}{\int_0^{\pi} a\,d\theta}\,\hat{y} = \frac{a^2\left[\cos\theta\right]_{\pi}^{0}}{a\cdot\pi}\,\hat{y}$$

$$\Rightarrow \boxed{\bar{y} = \frac{2a}{\pi}\,\hat{y}}$$

(5) $\vec{r}_1$ = Position vector of mass m_1

$\vec{r}_2$ = " " " " m_2

$\vec{r}$ = " " of C.M. 'C'.



$$\therefore \vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

Reduced mass of the system $\mu = \dfrac{m_1 m_2}{m_1+m_2}$

$$\dot{\vec{r}} = \vec{\bar{v}} = \frac{m_1\dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2}{m_1+m_2} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

$$\Rightarrow m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\,\vec{\bar{v}} \quad \text{—(1)}$$

Relative velocity of m_1 and m_2 $= \vec{v} = \vec{v}_1 - \vec{v}_2$ —(2)

From (1) and (2), we can calculate,

$$\vec{v}_1 = \vec{\bar{v}} + \frac{m_2\vec{v}}{m_1+m_2} \quad \text{and} \quad \vec{v}_2 = \vec{\bar{v}} - \frac{m_1\vec{v}}{m_1+m_2}$$

$$\therefore \text{Total K.E.} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1\left(\vec{\bar{v}} + \frac{m_2\vec{v}}{m_1+m_2}\right)^2 + \frac{1}{2}m_2\left(\vec{\bar{v}} - \frac{m_1\vec{v}}{m_1+m_2}\right)^2$$

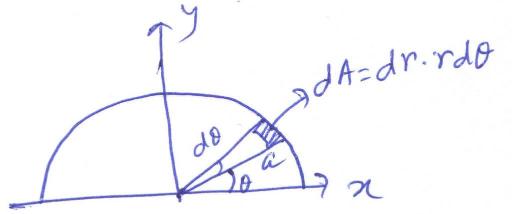
$$= \frac{1}{2}(m_1+m_2)\bar{v}^2 + \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}v^2$$

$$= \frac{1}{2}M\bar{v}^2 + \frac{1}{2}\mu v^2$$

(6) Degrees of freedom of a linear triatomic molecule = 9

(7) D.O.F = 6

(8) By Symmetry, c.m. will be on y-axis.



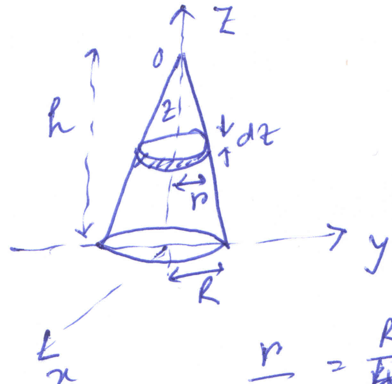
σ = mass per unit area

$dA = dr \cdot r d\theta$

$$\bar{y} = \frac{\iint y \sigma \, dA}{\iint \sigma \, dA} = \frac{\int_{r=0}^{a}\int_{\theta=0}^{\pi} r \sin\theta \cdot dr \cdot r d\theta}{\int_{r=0}^{a}\int_{\theta=0}^{\pi} dr \cdot r d\theta}$$

$$= \frac{\int_0^a r^2 dr \cdot \int_0^\pi \sin\theta \, d\theta}{\int_0^a r dr \int_0^\pi d\theta} = \frac{\frac{a^3}{3} \times 2}{\frac{a^2}{2} \times \pi} = \frac{4a}{3\pi}$$

(9)



$$dm = \frac{M}{\frac{1}{3}\pi R^2 h} \times \pi r^2 dz$$

$$= \frac{3M}{R^2 h} \frac{R^2}{h^2} z^2 = \frac{3M}{h^3} z^2 dz$$

$$\frac{r}{z} = \frac{R}{h}$$

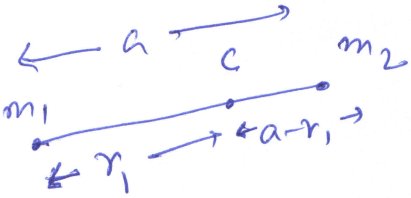
$$\Rightarrow r = \frac{R}{h} z$$

$dI_z = \frac{1}{2} dm \, r^2$ = M.I. of elemental disc.

$$= \frac{1}{2} \cdot \frac{3M}{h^3} z^2 dz \cdot \frac{R^2}{h^2} z^2$$

$$I = \frac{3}{2}\frac{M}{h^5} R^2 \int_0^h z^4 dz = \frac{3}{2}\frac{M}{h^5} R^2 \left.\frac{z^5}{5}\right|_0^h = \frac{3}{10} M R^2$$

(10) Reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$



$\therefore m_1 r_1 = m_2 (a - r_1)$

$\Rightarrow r_1 = \frac{m_2 a}{m_1 + m_2}$ $\quad \therefore a - r_1 = a - \frac{m_2 a}{m_1 + m_2} = \frac{m_1 a}{m_1 + m_2}$

$\therefore$ M.I. about an axis passing through c.m. is $= m_1 r_1^2 + m_2 (a - r_1)^2$

$$= m_1 \left(\frac{m_2 a}{m_1 + m_2}\right)^2 + m_2 \left(\frac{m_1 a}{m_1 + m_2}\right)^2 = \frac{m_1 m_2}{m_1 + m_2} a^2 = \mu a^2$$