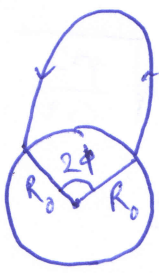


①



$$\text{Distance} = R_0 \times 2\theta = 2R_0\theta \quad [\because s = r\theta]$$

② Angular momentum.

$$\textcircled{3} \quad F \propto \frac{1}{r^3} \Rightarrow F = -\frac{GMm}{r^3}$$

$$\therefore v = -\int \vec{F} \cdot d\vec{r} = GMm \int \frac{dr}{r^3} = GMm \cdot \frac{r^{-3+1}}{-3+1} = -\frac{GMm}{2R^2}$$

$$\therefore \frac{1}{2} m v_e^2 = \frac{GMm}{2R^2} = \frac{GM}{R}$$

$$\Rightarrow \boxed{v_e = \sqrt{gR}}$$

Again,

$$\frac{GMm}{R^3} = mg$$

$$\Rightarrow g = \frac{GM}{R^3}$$

④

$$e = \frac{v_{\max} - v_{\min}}{v_{\max} + v_{\min}} = \frac{v_p - v_a}{v_p + v_a} \Rightarrow \frac{1-e}{1+e} = \frac{2v_a}{2v_p}$$

$$\Rightarrow \frac{0.5}{1.5} = \frac{v_a}{v} \Rightarrow \boxed{v_a = v/3}$$

⑤

$$e = \frac{v_{\max} - v_{\min}}{v_{\max} + v_{\min}} = \frac{30 - 29.2}{30 + 29.2} = \frac{0.8}{59.2} = 0.013$$

⑥

We know,

$$v = \omega r \Rightarrow v = \frac{2\pi}{T} r \Rightarrow T = \frac{2\pi}{v} r$$

$$\Rightarrow 2r = \frac{vT}{\pi} = \frac{220 \times 10^3 \times 14.4 \times 24 \times 3600}{3.14} = 8.7 \times 10^{10} \text{ m}$$

⑦

For elliptic orbit,

$$v = \sqrt{GM \left(\frac{2}{r} - \frac{1}{a} \right)}$$

$$r_{\max} + r_{\min} = 2a$$

$$\therefore v_{\max} = \sqrt{GM \left(\frac{2}{r_{\min}} - \frac{1}{a} \right)}$$

$$= \sqrt{GM \left(\frac{2a - r_{\min}}{a r_{\min}} \right)}$$

$$= \sqrt{\frac{GM r_{\max}}{a r_{\min}}}$$

$$\text{Similarly, } v_{\min} = \sqrt{\frac{GM r_{\min}}{a r_{\max}}}$$

$$\text{We know, } T = 2\pi \sqrt{\frac{a^3}{GM}}$$

$$\therefore v_{\max} \cdot v_{\min} = \frac{GM}{a} = \left(\frac{2\pi a}{T} \right)^2$$

$$\Rightarrow \boxed{a = \frac{T}{2\pi} \sqrt{v_{\max} \cdot v_{\min}}}$$

⑧ We know, from Kepler's law, $T = 2\pi \sqrt{\frac{r^3}{GM}}$

Here, $r = a + c + b = a + b + c$

$$\therefore T^2 = \frac{4\pi^2}{GM} (a+b+c)^3$$

$$\Rightarrow \boxed{M = \frac{4\pi^2 (a+b+c)^3}{GT^2}}$$

⑨ For an elliptic orbit, $v = \sqrt{GM \left(\frac{2}{r} - \frac{1}{a} \right)}$

$$r = R = 6400 \text{ km} = 6400 \times 10^3 \text{ m}$$

~~$$r = 8000 \text{ km} = 8000 \times 10^3 \text{ m}$$~~

$$2a = 80,000 \text{ km}, \quad M = 6 \times 10^{24} \text{ kg}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$$

$$\therefore v = \sqrt{(6.67 \times 10^{-11}) \times (6 \times 10^{24}) \left(\frac{2}{6400 \times 10^3} - \frac{1}{40000 \times 10^3} \right)} \text{ m/s}$$

$$\approx \frac{\cancel{10,926} \text{ m/s}}{\approx 10,726 \text{ m/s}} \approx \underline{\underline{10 \text{ km/s}}}$$

⑩ Angular momentum conservation, $L_p = L_a$

$$m v_p r_p = m v_a r_a$$

$$2) v_a r_a = v_p r_p = 10.25 \times 6570 = \underline{\underline{67342.5 \text{ kg-m}^2/\text{sec}}}$$