DEPARTMENT OF PHYSICS
Indian Institute of Technology Kharagpur
Classical Mechanics-I
Assignment-3: Motion under a central force-2

1. For a motion under the central force $-\frac{k}{r^3}$. If it starts on the $+ve$ X-axis at a distance a away from the origin and moves with speed v_0 in direction making an angle α with X-axis, the differential equation can be written as
(i) $\frac{d^2r}{dt^2} = -\frac{k - ma^2v_0^2\sin^2\alpha}{mr^3}$
(ii) $\frac{d^2r}{dt^2} = \frac{k - ma^2v_0^2\sin^2\alpha}{mr^3}$
(iii) $\frac{d^2r}{dt^2} = -\frac{k + ma^2v_0^2\sin^2\alpha}{mr^3}$
(iv) $\frac{d^2r}{dt^2} = \frac{k + ma^2v_0^2\sin^2\alpha}{mr^3}$

2. A particle is described by an attractive central force moves in an orbit given by $r = acos(\theta)$, the law of force is proportional to
(i) $\frac{1}{r^2}$
(ii) $\frac{1}{r^3}$
(iii) $\frac{1}{r^4}$
(iv) $\frac{1}{r^5}$

3. A particle describes an equiangular spiral $r = ae^{\theta}$ in such a manner that its acceleration has no radial component. Then
(i) angular velocity is zero
(ii) angular velocity is constant and magnitude of velocity is proportional to r
(iii) angular velocity is constant and magnitude of velocity is proportional to $\frac{1}{r}$
(iv) angular velocity and magnitude of velocity is proportional to r.

4. For attractive inverse square force field $f(R) = -\frac{k}{r^2}$, show that the velocity at any point of the for an hyperbolic path may be given as
(i) $v^2 = \frac{k}{m}[\frac{2}{r} - \frac{1}{a}]$
(ii) $v^2 = \frac{k}{m}[\frac{2}{r} + \frac{1}{a}]$
(iii) $v^2 = \frac{m}{k}[\frac{2}{r} - \frac{1}{a}]$
(iv) $v^2 = \frac{m}{k}[\frac{2}{r} + \frac{1}{a}]$

5. A small satellite revolves around a planet in an orbit of radius slightly greater than the radius of the planet, which is spherical. If the average density of the planet is ρ, the period of revolution of satellite.
(a) independent of R of the planet
(b) depends on R^2 of the planet
(c) depends on R^3 of the planet
(d) depends on R^4 of the planet

6. The central force necessary to make a particle describe the lemniscate $r^2 = a^2\ cos\ 2\theta$ is
   (i) proportional to r^7
   (ii) inversely proportional to r
   (iii) proportional to r
   (iv) inversely proportional to r^7

7. If a particle describes a elliptic orbit under the influence of an attractive central force $(= -\frac{k}{r^2})$, then the period of revolution of the particle is
   (i) $2\pi a^{3/2}\sqrt{\frac{m}{k}}$
   (ii) $2\pi a^{3/2}\sqrt{\frac{k}{m}}$
   (iii) $\pi a^{3/2}\sqrt{\frac{m}{k}}$
   (iv) $\pi a^{3/2}\sqrt{\frac{k}{m}}$

8. Find the law of force to the pole when the orbit described by the cardioid $r = a(1 - cos\ \theta)$
   (i) $\propto$ to r^{-1}
   (ii) $\propto$ to r^{-2}
   (iii) $\propto$ to r^{-3}
   (iv) $\propto$ to r^{-4}

9. Which one is the correct expression of areal velocity
   (i) $\frac{1}{2}r^2\dot{\theta}$
   (ii) $r^2\dot{\theta}$
   (iii) $\frac{1}{2}r^2\dot{\theta}^2$
   (iv) $\frac{1}{2}\dot{r}^2\dot{\theta}$

10. On the earth surface g can be expressed as
   (i) $\frac{\sqrt{GM}}{R}$
   (ii) $\frac{GM}{R}$
   (iii) $\frac{GM}{R^2}$
   (iv) $\sqrt{\frac{GM}{R}}$

End