1

$$v^2 = \mu\left(\frac{a}{x} - 1\right)$$

$$\Rightarrow 2v.\frac{dv}{dx} = -\mu\frac{a}{x^2}$$

$$\Rightarrow a = v.\frac{dv}{dx} = -\mu\frac{a}{2x^2}$$

$\therefore \boxed{F(x) \propto -\frac{1}{x^2}} \Rightarrow$ Towards the fixed point.

(2) $\left.\frac{dx}{dt}\right|_{t=5} = v_5 = 30-20 = 10$ cm/s

$\left.\frac{dx}{dt}\right|_{t=5.01} = v_{5.01} = 30-20.04 = 9.96$ cm/s

Ave. vel. $= \frac{10+9.96}{2} = \frac{19.96}{2} = 9.98$ cm/s

(3) (i) Acceleration, $v\frac{dv}{dx} = -\frac{k}{m}v^3$

$\Rightarrow \frac{v\,dv}{v^3} = -\frac{k}{m}dx$

$\Rightarrow \int_u^v \frac{dv}{v^2} = -\frac{k}{m}\int_0^x dx$

$\Rightarrow \cancel{-}\left[\frac{1}{v}\right]_u^v = \cancel{-}\frac{k}{m}x$

$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{k}{m}x$

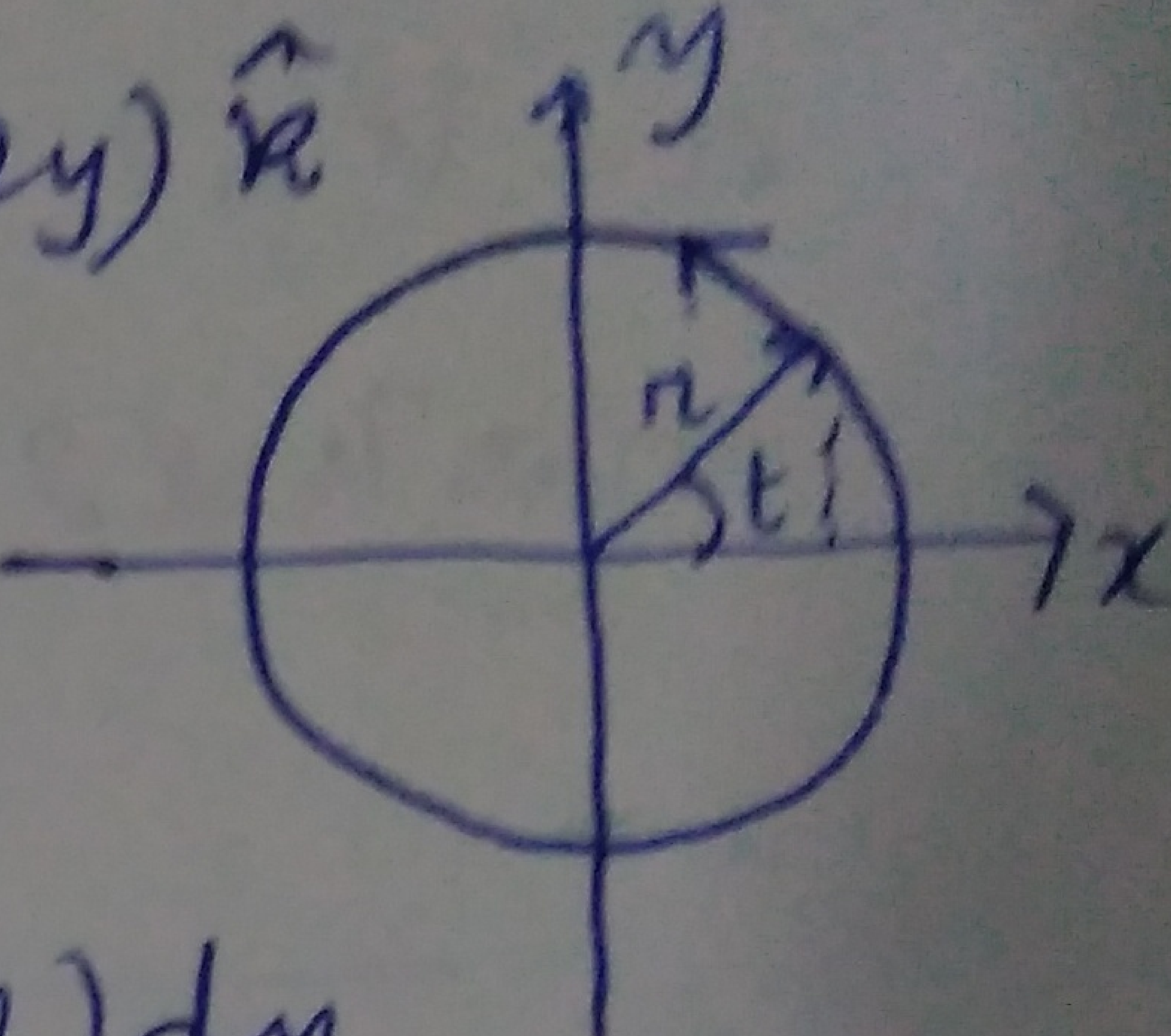
$\Rightarrow \frac{u-v}{vu} = \frac{k}{m}x$

$\Rightarrow u = v + \frac{k}{m}xvu = v\left(1+\frac{k}{m}xu\right)$

$\Rightarrow \boxed{v = \frac{mu}{m+kxu}}$ (Proved)

④ For xy-plane, $z = 0$

$$\therefore \vec{F} = (2x-y)\hat{i} + (x+y)\hat{j} + (3x-2y)\hat{k}$$



$$\vec{r} = \hat{i}x + \hat{j}y$$

$$\therefore d\vec{r} = \hat{i}\,dx + \hat{j}\,dy$$

$$\therefore W = \int_C \vec{F}\cdot d\vec{r} = \int_C (2x-y)\,dx + (x+y)\,dy$$

as radius $r = 3$, let, $\left.\begin{array}{l} x = 3\cos t \\ y = 3\sin t \end{array}\right\}$ Parametric eqn, $t = 0$ to 2π (from fig)

$$\therefore W = \int_0^{2\pi} [6\cos t - 3\sin t][-3\sin t]\,dt + [3\cos t + 3\sin t][3\cos t]\,dt$$

$$= \int_0^{2\pi} [9 - 9\cos t \cdot \sin t]\,dt = 9\int_0^{2\pi} dt - \underbrace{9\int_0^{2\pi} \sin t\, d(\sin t)}_{=0} = 9\times 2\pi$$

$$= 18\pi$$

(5) $\vec{F} = -\vec{\nabla} V$

$\Rightarrow \hat{i} F_x + \hat{j} F_y + \hat{k} F_z = -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]$

$\Rightarrow F_x = -\frac{\partial V}{\partial x} \qquad F_y = -\frac{\partial V}{\partial y} \qquad F_z = -\frac{\partial V}{\partial z}$

$\therefore F_x = -\frac{\partial V}{\partial x}$

$\Rightarrow -\frac{\partial V}{\partial x} = 2xy + z^3$

$\Rightarrow V = -x^2 y - xz^3 + c_1$

$\therefore F_y = -\frac{\partial V}{\partial y}$

$\Rightarrow -\frac{\partial V}{\partial y} = x^2$

$\Rightarrow V = -x^2 y + c_2$

$\therefore F_z = -\frac{\partial V}{\partial z}$

$\Rightarrow -\frac{\partial V}{\partial z} = 3xz^2$

$\Rightarrow V = -xz^3 + c_3$

$\therefore \boxed{V(x,y,z) = -x^2 y - xz^3 + \text{Const.}}$

⑥ (i) $m = 2$

$V = x^2 + y^2$

$\vec{F} = -\vec{\nabla} V = -2(\hat{i}x + \hat{j}y)$

$\Rightarrow m\frac{d\vec{v}}{dt} = -2(\hat{i}x + \hat{j}y)$

$\Rightarrow \hat{i}\frac{d^2x}{dt^2} + \hat{j}\frac{d^2y}{dt^2} = -(\hat{i}x + \hat{j}y)$

Equ^n of motions are

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= -x \Rightarrow \frac{d^2x}{dt^2} + x = 0 \\ \text{and} \quad \frac{d^2y}{dt^2} &= -y \Rightarrow \frac{d^2y}{dt^2} + y = 0 \end{aligned}\right\}$$

(ii) $\frac{d^2x}{dt^2} + x = 0$

Comparing $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$x(t) = A\cos t \pm B\sin t$

Similarly, $y(t) = C\cos t \pm D\sin t$

(iii) $v(t) = \dot{x}(t) = \frac{dx}{dt} = -A\sin t \pm B\cos t$

Now, we have to find out the constant A, B, C, D.

B.C $\left.\begin{array}{l} x = 2, \quad \dot{x} = 0 \\ y = 1, \quad \dot{y} = 0 \end{array}\right] \Rightarrow$ at $t = 0$ i.e. motion is started by initial displacement

$\therefore x(0) = 2 = A$ and $\dot{x}(0) = 0 = \pm B \Rightarrow B = 0$

$\therefore \begin{array}{l} A = 2 \\ B = 0 \end{array}$

$y(0) = 1 = C$ and $\dot{y}(0) = 0 = \pm D \Rightarrow D = 0$

$\therefore \begin{array}{l} C = 1 \\ D = 0 \end{array}$

$\therefore x(t) = 2\cos t$

$y(t) = \cos t$

$\dot{x}(t) = -2\sin t$

$\dot{y}(t) = -\sin t$

(7) $F(x) \propto \frac{1}{x^3} \Rightarrow F(x) = -\mu/x^3$, ('-ve' sign as attracted towards a fixed point)

$W = \int \vec{F}(x) \cdot d\vec{x}$

$= -\mu \int_a^b \frac{dx}{x^3}$, $b > a$

$\Rightarrow W = +\frac{\mu}{2}\left[\frac{1}{b^2} - \frac{1}{a^2}\right] =$ '-ve'

P.E. gained by the particle, $\boxed{V = -W = \frac{\mu}{2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}$

Motion under Resistive Medium

(8)

$$m\frac{dv}{dt} = -mg - mkv^2$$

At, $v = u$

$mg = mku^2$

$\Rightarrow g = \frac{k}{m}u^2$

$$\Rightarrow \frac{dv}{dt} = -kU^2 - \frac{k}{\cancel{m}}v^2$$

$$\Rightarrow \int_v^0 \frac{dv}{v^2+U^2} = -k\int_0^t dt$$

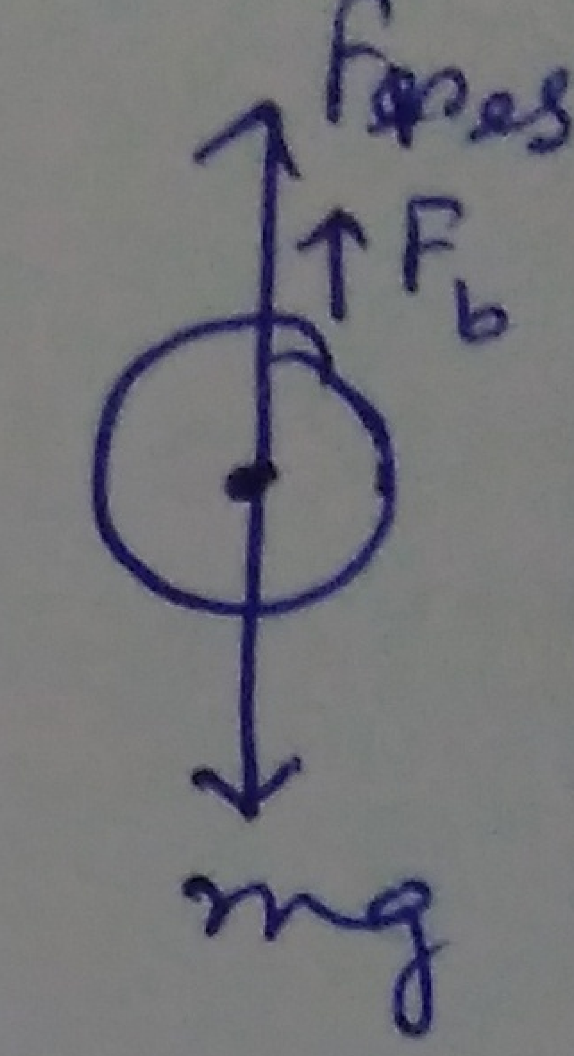
$$\Rightarrow \frac{1}{U}\left[\tan^{-1}\frac{v}{U}\right]_v^0 = -k\,\cancel{dt}\,t$$

$$\Rightarrow t = \frac{1}{kU}\tan^{-1}\frac{v}{U} = \frac{1}{\frac{g}{U^2}u}\tan^{-1}\left(\frac{v}{u}\right)$$

$$\Rightarrow \boxed{t = \frac{u}{g}\tan^{-1}\left(\frac{v}{u}\right)}$$

(9) Eqn of motion of a body, considering buoyancy

$$mg - F_b - F_{res} = ma$$



$$F_b = \left(\frac{4}{3}\pi r^3 \sigma\right) g$$

$$F_{res} = 6\pi\eta r v$$

for terminal velocity ($v = v_t$)

$$mg - F_b - F_{res} = 0$$

$$\Rightarrow \left(\frac{4}{3}\pi r^3 \rho\right) g - \left(\frac{4}{3}\pi r^3 \sigma\right) g - 6\pi\eta r v_t = 0$$

$$\Rightarrow \frac{4}{3}\pi r^3 (\rho - \sigma) g = 6\pi\eta r v_t$$

$$\Rightarrow v_t = \frac{2}{9}\frac{(\rho - \sigma) r^2 g}{\eta} = \frac{2 \times (7.8 - 0.8) \times 10^3 \times (2 \times 10^{-3})^2 \times 9.8}{0.83}$$

$$\approx 0.069 \text{ m/s} \approx 0.07 \text{ m/s}$$

$$\boxed{v_t \approx 0.07 \text{ m/s}}$$

(10) Eqn of motion, (upward direction)

$$v\frac{dv}{dx} = -g - kv^2 = -k(v_t^2 + v^2) \quad [\because v_t = \sqrt{g/k}]$$

$$\Rightarrow \frac{1}{2}\int_{v_t\tan\alpha}^{0} \frac{d(v^2)}{(v_t^2+v^2)} = -k\int_0^h dx$$

$$\Rightarrow \frac{1}{2}\ln\left|v_t^2+v^2\right|_0^{v_t\tan\alpha} = +kh \Rightarrow kh = \frac{1}{2}\ln\left|\frac{v_t^2\sec^2\alpha}{v_t^2}\right|$$

$$\Rightarrow h = \frac{1}{2k}\ln|\sec^2\alpha|$$

Let, particle hits the ground with speed v_f.

For downward motion, $v_f^2 = 0^2 + 2gh = 2g\frac{1}{2k}\ln|\sec^2\alpha|$

$$\Rightarrow v_f^2 = \frac{2g}{k}\ln|\sec\alpha|$$

Here,

$$v\frac{dv}{dx} = -kv^2 + g = -k(v^2 - v_t^2)$$

$$\Rightarrow \frac{1}{2}\int_0^{v_f}\frac{d(v^2)}{v^2 - v_t^2} = -k\int_0^h dx \Rightarrow \frac{1}{2}\ln\left|v^2 - v_t^2\right|_0^{v_f} = -kh$$

$$\Rightarrow \cancel{\frac{1}{2}}\ln\left|\frac{v_t^2 - v_f^2}{v_t^2}\right| = -\cancel{\frac{1}{2}}\ln|\sec^2\alpha| = \ln|\cos^2\alpha|$$

$$\Rightarrow 1 - \frac{v_f^2}{v_t^2} = \cos^2\alpha \Rightarrow \frac{v_f^2}{v_t^2} = 1 - \cos^2\alpha = \sin^2\alpha \Rightarrow \cancel{v_f = v_t\cdot\sin\alpha} ✓$$

$$\Rightarrow \boxed{v_f = v_t\cdot\sin\alpha}$$