Unit 11 - Week 9

Assignment 9

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-10-02, 23:59 IST.

Only Q.3 may have more than one correct answer.

1) Which one of the following indicates 'vortex advection' in $\mathbf{N}_\omega(\mathbf{k})$ term of the dynamical equation of vorticity (in Fourier space)? *1 point*

- $i\sum_{\mathbf{p}}\{\mathbf{k}\cdot\mathbf{u}(\mathbf{p})\}\boldsymbol{\omega}(\mathbf{q})$
- $i\sum_{\mathbf{p}}\{\mathbf{p}\cdot\boldsymbol{\omega}(\mathbf{q})\}\mathbf{u}(\mathbf{k})$
- $i\sum_{\mathbf{p}}\{\mathbf{k}\cdot\mathbf{u}(\mathbf{q})\}\boldsymbol{\omega}(\mathbf{p})$
- $i\sum_{\mathbf{p}}\{\mathbf{k}\cdot\boldsymbol{\omega}(\mathbf{q})\}\mathbf{u}(\mathbf{p})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$i\sum_{\mathbf{p}}\{\mathbf{k}\cdot\mathbf{u}(\mathbf{q})\}\boldsymbol{\omega}(\mathbf{p})$

2) Which one of the following represents the evolution equation (in terms of combined enstrophy transfers) of modal enstrophy $E_\omega(\mathbf{k}')$? *1 point*

- $\frac{d}{dt}E_\omega(\mathbf{k}') = S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q})$
- $\frac{d}{dt}E_\omega(\mathbf{k}') = -S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) - S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q})$
- $\frac{d}{dt}E_\omega(\mathbf{k}') = -S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q})$
- $\frac{d}{dt}E_\omega(\mathbf{k}') = S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) - S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q})$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{d}{dt}E_\omega(\mathbf{k}') = S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q})$

3) Identify the correct expression/expressions about combined enstrophy transfer/ transfers. *1 point*

- $S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega u}(\mathbf{p}|\mathbf{k}',\mathbf{q}) + S^{\omega u}(\mathbf{q}|\mathbf{k}',\mathbf{q}) = 0$
- $S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega u}(\mathbf{p}|\mathbf{k}',\mathbf{q}) + S^{\omega u}(\mathbf{q}|\mathbf{k}',\mathbf{q}) \neq 0$
- $S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega\omega}(\mathbf{p}|\mathbf{k}',\mathbf{q}) + S^{\omega\omega}(\mathbf{q}|\mathbf{k}',\mathbf{q}) = 0$
- $S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega\omega}(\mathbf{p}|\mathbf{k}',\mathbf{q}) + S^{\omega\omega}(\mathbf{q}|\mathbf{k}',\mathbf{q}) \neq 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$S^{\omega u}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega u}(\mathbf{p}|\mathbf{k}',\mathbf{q}) + S^{\omega u}(\mathbf{q}|\mathbf{k}',\mathbf{q}) \neq 0$
$S^{\omega\omega}(\mathbf{k}'|\mathbf{p},\mathbf{q}) + S^{\omega\omega}(\mathbf{p}|\mathbf{k}',\mathbf{q}) + S^{\omega\omega}(\mathbf{q}|\mathbf{k}',\mathbf{q}) = 0$

4) Consider the two-dimensional velocity field: *1 point*

$$\mathbf{u} = (-2\sin y - \sqrt{2}\sin(x+y),\ 2\sin x + \sqrt{2}\sin(x+y)),$$

and the triad combination $\mathbf{k}' = (-1,-1)$, $\mathbf{p} = (1,0)$ and $\mathbf{q} = (0,1)$. Calculate the amplitude of the voriticty Fourier mode for the wavenumber $\mathbf{k}' = (-1,-1)$.

- $(0, 0, \sqrt{2})$
- $(0, 0, -\sqrt{2})$
- $(1, 0, \sqrt{2})$
- none of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(0, 0, \sqrt{2})$

5) Calculate mode to mode enstrophy transfer $S^{\omega u}(\mathbf{k}'|\mathbf{p}|\mathbf{q})$ for Q.4. *1 point*

- $-\sqrt{2}$
- $-\frac{1}{\sqrt{2}}$
- $\sqrt{2}$
- 0

No, the answer is incorrect.
Score: 0
Accepted Answers:
0

6) For three-dimensional hydrodynamic turbulence, enstrophy spectrum $E_\omega(k) =$: *1 point*
[Note: $E_u(k)$ is kinetic energy spectrum.]

- $\frac{E_u(k)}{k}$
- $k^2 E_u(k)$
- $\frac{E_u(k)}{k^2}$
- $k E_u(k)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$k^2 E_u(k)$

7) According to Kraichnan's theory of two-dimensional hydrodynamic turbulence, in $k < k_f$ regime, kinetic energy spectrum $E_u(k) \sim$: *1 point*

- $k^{-5/3}$
- $k^{-1/3}$
- k^{-3}
- none of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
$k^{-5/3}$

8) According to Kraichnan's theory of two-dimensional hydrodynamic turbulence, in $k > k_f$ regime, enstrophy spectrum $E_\omega(k) \sim$: *1 point*

- $k^{-5/3}$
- k^{-3}
- k^{-1}
- none of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
k^{-1}

9) In $k > k_f$ regime, enstrophy dissipation wavenumber k_{d2D} is: *1 point*
[Note: ϵ_ω and ϵ_u are the enstrophy and energy dissipation rates]

- $\left(\frac{\epsilon_u}{\nu^3}\right)^{\frac{1}{4}}$
- $\left(\frac{\epsilon_u}{\nu^3}\right)^{-\frac{1}{4}}$
- $\frac{\epsilon_\omega^{\frac{1}{6}}}{\sqrt{\nu}}$
- $\left(\frac{\epsilon_\omega}{\sqrt{\nu}}\right)^{\frac{1}{6}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{\epsilon_\omega^{\frac{1}{6}}}{\sqrt{\nu}}$

10) In 3D hydrodynamics, enstrophy is expected to increase. Which term of the dynamical equation of vorticity (in real space) is mainly responsible for this? *1 point*

- $(\boldsymbol{\omega}\cdot\nabla)\mathbf{u}$
- $(\mathbf{u}\cdot\nabla)\mathbf{u}$
- $(\mathbf{u}\cdot\nabla)\boldsymbol{\omega}$
- $\nabla^2\mathbf{u}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(\boldsymbol{\omega}\cdot\nabla)\mathbf{u}$