Unit 6 - Week 4

Assignment 4

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2019-08-28, 23:59 IST.

1) If we consider thermal convection between two infinite plates at $z = 0$ and $z = \pi$, the basis function for u_z which satisfies the free-slip boundary conditions will be *1 point*

- $u_z(\mathbf{k}) 2\sin(nz)\exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) + c.c.$
- $u_z(\mathbf{k}) 2\cos(nz)\exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) + c.c.$
- $u_z(\mathbf{k}) 2\sin(n\pi z)\exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) + c.c.$
- $u_z(\mathbf{k}) 2\cos(n\pi z)\exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) + c.c.$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$u_z(\mathbf{k}) 2\sin(nz)\exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp) + c.c.$

2) The non-dimensionalized equation for velocity in Rayleigh Benard convection (RBC) is *1 point*

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\nabla\sigma + \mathrm{RaPr}\,\theta\hat{z} + \mathrm{Pr}\nabla^2\mathbf{u}.$$

Which term(s) in the above equation has (have) to be dropped to linearize the equation?

- $(\mathbf{u} \cdot \nabla)\,\mathbf{u}$ and $\mathrm{Pr}\nabla^2\mathbf{u}$
- $(\mathbf{u} \cdot \nabla)\,\mathbf{u}$ and $\mathrm{RaPr}\,\theta\hat{z}$
- $(\mathbf{u} \cdot \nabla)\,\mathbf{u}$ only
- $\mathrm{Pr}\nabla^2\mathbf{u}$ only

No, the answer is incorrect.
Score: 0
Accepted Answers:
$(\mathbf{u} \cdot \nabla)\,\mathbf{u}$ *only*

3) Consider RBC between two infinite plates. The free-slip boundary conditions for u_x and u_z at the bottom and top plates will be: *1 point*

- $u_x = 0; \quad u_z = 0$
- $\frac{\partial u_x}{\partial z} = 0; \quad u_z = 0$
- $u_x = 0; \quad \frac{\partial u_z}{\partial z} = 0$
- $\frac{\partial u_x}{\partial z} = 0; \quad \frac{\partial u_z}{\partial z} = 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{\partial u_x}{\partial z} = 0; \quad u_z = 0$

4) When writing the RBC equations (with incompressibility and without rotation) in Craya-Herring basis, the active modes of $\mathbf{u}(\mathbf{k})$ lie along which basis vectors? *1 point*

- $\hat{e}_1$ and $\hat{e}_2$
- $\hat{e}_2$ and $\hat{e}_3$
- $\hat{e}_1$ and $\hat{e}_3$
- $\hat{e}_1,\ \hat{e}_2$ and $\hat{e}_3$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\hat{e}_1$ and $\hat{e}_2$

5) Consider a matrix equation of the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, where *0 points*

$$\mathbf{A} = \begin{pmatrix} -\mathrm{Pr}\,k^2 & -\mathrm{RaPr}\,k_\perp/k \\ -k_\perp/k & -k^2 \end{pmatrix}.$$

The product of the eigenvalues of $\mathbf{A}$ will be:

- $-(\mathrm{RaPr}+1)k_\perp/k$
- $-(\mathrm{Pr}+1)k^2$
- $\mathrm{RaPr}\,k_\perp^2/k^2$
- $\mathrm{Pr}k^2 - \mathrm{RaPr}\,k_\perp^2/k^2$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathrm{Pr}k^2 - \mathrm{RaPr}\,k_\perp^2/k^2$

6) For the stability analysis of RBC (worked out in the video lecture), what is the wavelength corresponding to $k_\perp$ at the critical Rayleigh number for $n = 2$? *1 point*

- 1
- 2
- $\sqrt{2}$
- $2\sqrt{2}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sqrt{2}$

7) For rotating flows, an additional non-dimensional number, Taylor number, appears in the equation for velocity. This number is defined as the ratio between: *1 point*

- $\frac{\text{Coriolis force}}{\text{Viscous force}}$
- $\frac{\text{Coriolis force}}{\text{Inertial force}}$
- $\frac{\text{Coriolis force}}{\text{Buoyant force}}$
- $\frac{\text{Coriolis force}}{\text{Pressure gradient}}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{\text{Coriolis force}}{\text{Viscous force}}$

8) For rotating flows, which of the following non-dimensional numbers defines the ratio between viscous forces to Coriolis forces? *1 point*

- Ekman number
- Richardson number
- Rossby number
- Taylor number

No, the answer is incorrect.
Score: 0
Accepted Answers:
Ekman number

9) For rotating Rayleigh Benard convection, the critical Rayleigh number, Ra_c, is given by: *1 point*

$$\mathrm{Ra}_c = \frac{Ta}{s} + (n\pi)^4\frac{(1+s)^3}{s},$$

where $s = (k_\perp/n\pi)^2$. For large values of Ta, the minimum of Ra_c at $n = 1$ scales according to:

- $\mathrm{Ra}_c \sim \mathrm{Ta}$
- $\mathrm{Ra}_c \sim \mathrm{Ta}^2$
- $\mathrm{Ra}_c \sim \mathrm{Ta}^{1/3}$
- $\mathrm{Ra}_c \sim \mathrm{Ta}^{2/3}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathrm{Ra}_c \sim \mathrm{Ta}^{2/3}$

10) In Craya-Herring basis, the equations for RBC with rotation are of the form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, where *1 point*

$$\mathbf{A} = \begin{pmatrix} -\mathrm{Pr}\,k^2 & \mathrm{Pr}\sqrt{\mathrm{Ta}}\,cos\zeta & 0 \\ -\mathrm{Pr}\sqrt{\mathrm{Ta}}\,cos\zeta & -\mathrm{Pr}\,k^2 & -\mathrm{RaPr}\sin\zeta \\ 0 & -\sin\zeta & -k^2 \end{pmatrix}.$$

For pure rotation, once the viscous terms have decayed, the above matrix reduces to:

- $\begin{pmatrix} 0 & \mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta \\ -\mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta & 0 \end{pmatrix}$
- $\begin{pmatrix} 1 & \mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta \\ -\mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta & -1 \end{pmatrix}$
- $\begin{pmatrix} \mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta & 0 \\ 0 & -\mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta \end{pmatrix}$
- $\begin{pmatrix} \mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta & 1 \\ -1 & -\mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta \end{pmatrix}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\begin{pmatrix} 0 & \mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta \\ -\mathrm{Pr}\sqrt{\mathrm{Ta}}\cos\zeta & 0 \end{pmatrix}$