## Courses » Theory of groups for physics applications

Announcements Course Ask a Question Progress Mentor FAQ

## Unit 7 - Week

6

## Course outline

How to access
the portal

Week 1

Week 2

Week 3

## Week 4

Week 5

Week 6

- Lecture 21: Orthogonality For
Characters-I
- Lecture 22: Orthogonality
For
Characters-II
- Lecture 23:

Character
Tables \&
Molecular
Applications-I

- Lecture 24:

Character
Tables \&
Molecular
Applications-II

- Week6-Lecture Slides and


## Week 6-Assignment 6-MCQ

The due date for submitting this assignment has passed.
As per our records you have not submitted this
Due on 2018-09-12, 23:59 IST. assignment.

1) Given that in some representation $D$, the following elements of $S_{3}$ are represented as 1 point $\left(\begin{array}{lll}1 & 2 & 3 \\ 2 & 1 & 3\end{array}\right)=\left(\begin{array}{lll}0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1\end{array}\right) \equiv D_{2} ; \quad\left(\begin{array}{ccc}1 & 2 & 3 \\ 2 & 3 & 1\end{array}\right)=\left(\begin{array}{lll}0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0\end{array}\right) \equiv D_{3}$
Find the weightages $W_{i}$ of the irreducible representations $D_{i}$ (i=1,2,3) in $D$ ( $D_{1}$ denotes the identity element).

$$
W_{1}=1, W_{2}=0, W_{3}=1
$$

$$
W_{1}=1, W_{2}=1, W_{3}=1
$$

$$
W_{1}=1, W_{2}=0, W_{3}=2
$$

$$
W_{1}=1, W_{2}=2, W_{3}=1
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$W_{1}=1, W_{2}=0, W_{3}=1$
2) The multiplication table of a group is given in table 1.

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| Quiz : Week |  |
| :--- | :--- |
| 6-Assignment | ce De |
| 6-MCQ |  |
| Week6- |  |
| Assignment6- |  |
| Solutions |  |

## Week 7

Week 8

Week 9

Week 10

Week 11

|  | E | A | B | C | K | L | M | N |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| E | E | A | B | C | K | L | M | N |
| A | A | K | N | B | L | E | C | M |
| B | B | C | K | L | M | N | E | A |
| C | C | M | L | K | N | B | A | E |
| K | K | L | M | N | E | A | B | C |
| L | L | E | C | M | A | K | N | B |
| M | M | N | E | A | B | C | K | L |
| N | N | B | L | E | C | M | A | K |

## Table 1.

Identify all the conjugacy classes.$\{E\},\{K\},\{A, L\},\{C, N\}$$\{E\},\{K\},\{A, L\},\{B, M\}$$\{E\},\{K\},\{A, L\},\{C, N\},\{B, M\}$$\{E\},\{K\},\{A, L\},\{C, N\},\{B, M\},\{A, B, C\}$
No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{E\},\{K\},\{A, L\},\{C, N\},\{B, M\}$
3) Given the great orthogonality theorem for unitary representations $\alpha$ and $\beta$

1 point

$$
\sum_{g \in G} D_{i l}^{(\alpha)}(g) D_{j m}^{(\beta) *}(g)=\frac{|G|}{n_{\alpha}} \delta_{i j} \delta_{m l} \delta^{\alpha \beta}
$$

Obtain a similar statement for the characters of the representations.

$$
\begin{aligned}
& \sum_{g \in G} \chi^{(\alpha) *}(g) \chi^{(\beta) *}(g)=|G| \delta^{\alpha \beta} \\
& \sum_{g \in G} \chi^{(\alpha) *}(g) \chi^{(\beta)}(g)=|G| \delta^{\alpha \beta} \\
& \sum_{g \in G} \chi^{(\alpha)}(g) \chi^{(\beta)}(g)=|G| \delta^{\alpha \beta} \\
& \sum_{g \in G} \chi^{(\alpha)}(g) \chi^{(\beta) *}(g)=|G| \delta^{\alpha \beta}
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\sum_{g \in G} \chi^{(\alpha)}(g) \chi^{(\beta) *}(g)=|G| \delta^{\alpha \beta}$
4) If $\Gamma$ is a $d$ dimensional irreducible representation of a group $G$ and $B$ is a $d \times d$ matrix such that $\Gamma(T) B=B \Gamma(T)$ for every $T \in G$, then $B$ must be a/anNull matrixIdempotent matrix.Multiple of the unit matrixNo conclusions regarding he matrix nature can be drawn
No, the answer is incorrect.

## Score: 0

Accepted Answers:
Multiple of the unit matrix
5) For a specific representation, $\Gamma(E)=\mathbb{I}_{n \times n}$ for the identity $E$ of a group $G$, then the 1 point character $\chi(E)$ will be


No, the answer is incorrect.
Score: 0
Accepted Answers:
$n$
6) "For two representations of a group to be equivalent, they must have identical character 1 point systems" -- This statement isa necessary condition but not sufficient.a sufficient condition but not necessary.both necessary and sufficient.not a valid condition.

No, the answer is incorrect.
Score: 0
Accepted Answers:
a necessary condition but not sufficient.
7) For a finite group $G$, the number of inequivalent irreducible representations is equal to the $\mathbf{1}$ point
number of generator(s) of $G$
number of conjugacy class(es) of $G$
order of group $G$
number of factor groups of $G$
No, the answer is incorrect.
Score: 0
Accepted Answers:
number of conjugacy class(es) of $G$
8) Dimensions $d_{i}$ of the inequivalent irreducible representations of the crystallographic point 1 point group $D_{4}$ are

$$
\begin{aligned}
& d_{1}=d_{2}=d_{3}=d_{4}=2, d_{5}=1 \\
& d_{1}=d_{2}=d_{3}=d_{4}=1, d_{5}=2
\end{aligned}
$$

$$
d_{1}=d_{2}=d_{3}=1, d_{4}=d_{5}=2
$$

- None of the above

No, the answer is incorrect.
Score: 0
Accepted Answers:
$d_{1}=d_{2}=d_{3}=d_{4}=1, d_{5}=2$
9) Consider a group which contains of an element $g$ of order $m$ i.e. such that $g^{m}=I$, when 1 point represented by an $n \times n$ matrix satisfies

$$
\chi(g)=\sum_{k=1}^{n} \lambda_{k}
$$

where $\lambda_{m}$ is $m^{\text {th }}$ root of unity. Using the above property and algebraic properties of character tables, supply the "?" entry.

| Class $\backslash$ Irrep | $\chi^{(1)}$ | $\chi^{(2)}$ | $\chi^{(3)}$ | $\chi^{(4)}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $n_{(1)}=1$ | $n_{(2)}=1$ | $n_{(3)}=1$ | $n_{(4)}=3$ |
| $C_{1}$ | 1 | 1 | 1 | $?$ |
| $C_{2}$ | 1 | $\lambda_{3}$ | $\lambda_{3}^{2}$ | 0 |
| $C_{3}$ | 1 | $\lambda_{3}^{2}$ | $\lambda_{3}$ | 0 |
| $C_{4}$ | 1 | 1 | 1 | -1 |

No, the answer is incorrect.
Score: 0
Accepted Answers:
3
10Consider the set of $3 \times 3$ matrices $M(a)$ with $a \in \mathbb{R}$
1 point

$$
M(a)=\left(\begin{array}{lll}
1 & 0 & 0 \\
a & 1 & 0 \\
a^{2} & 2 a & 1
\end{array}\right)
$$

This representation can be termed asFaithfulUnfaithfulTrivialEquivalent
No, the answer is incorrect.
Score: 0
Accepted Answers:
Faithful

