NPTEL course offered by IIT Madras

Risk and Reliability of Offshore structures

<u>Tutorial 5: Reliability methods</u>

Answer all questions **Total marks: 25**

1. Explain First-Order Second Moment (FOSM) method of Reliability analysis

In the FOSM, a first order Taylor series approximation of the limit state function is used and only second moment statistics of the random variables are employed to obtain the probability of failure. In its original form, two random variables are used to derive the method. The limit state function is defined as follows:

Z=R-S

Assuming that R and S are statistically independent normally distributed random variables, Z is also normally distributed. Its mean and covariance are obtained as

$\mu_Z = \mu_R - \mu_S$ and $\sigma_Z^2 = \sigma_R^2 + \sigma_S^2$

The probability of failure is given by:

$P_f = P[Z < 0] = P[(R - S) < 0]$

If Z is a normal variant, then it may be easily shown that P_f is given by

$\mathrm{P_f} = \Phi\left(-\frac{\mu_Z}{\sigma_Z}\right)$

in which Φ is the cumulative distribution function for a standard normal variable. Substituting for μ_Z and σ_Z, P_f can be rewritten as

$$P_f = 1 - \Phi\left(\frac{\boldsymbol{\mu_R - \mu_S}}{\sqrt{\boldsymbol{\sigma_R^2 + \sigma_S^2}}}\right)$$

The ratio of $\frac{\mu_Z}{\sigma_Z}$ is denoted by β and is well known as reliability index (safety index) in the theory of reliability. Then, P_f is also popularly expressed as

$P_f = \Phi(-\beta)$

If the variables R and S are log normally distributed, then the limit state function is defined as:

$Z = ln\left(\frac{R}{S}\right)$

Z is again a normal variable, and the probability of failure can be expressed as below:

$$P_f = 1 - \Phi\left\{\frac{ln\frac{\mu_R}{\mu_S}\sqrt{\frac{1+\delta_S^2}{1+\delta_R^2}}}{\sqrt{ln(1+\delta_R^2)ln(1+\delta_S^2)}}\right\}$$

Where, δ_R and δ_S are the coefficient variations of *R* and *S*.

The above formulation may be generalized to many random variables, denoted by a vector **X**. Let the performance function be written as below:

$$Z = G(X)$$

The Taylor series expansion of the performance function about the mean values gives

$$Z = G(\bar{X}) + \sum_{i=1}^{N}\frac{\partial G}{\partial x_i}(x_i - \bar{x}_i) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^2 G}{\partial x_i \partial x_j}(x_i - \bar{x}_i)(x_j - \bar{x}_j)$$

In which $\bar{x}_i$ is the mean of the variable x_i. Truncating the series at the linear terms, the first order approximation of mean and variance of Z are obtained as

$$\bar{Z} = G(\bar{X})$$

$$\sigma_Z^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 G}{\partial x_i \partial x_j}cov(x_i x_j)$$

In which $cov(x_i, x_j)$ is the covariance of x_i andx_j. If the variables are assumed to be statistically independent, then

$$\sigma_Z^2 = \sum_{i=1}^{n}\left(\frac{\partial G}{\partial x_i}\right)^2 \sigma_{x_i}^2$$

Partial derivatives of the above expression are obtained at the mean values.

2. Explain Hazofer-Lind method of Reliability analysis

Hasofer-Lind method is an improvisation over the FOSM method. The method is centred on the computation of the design point or the minimum distance of the performance function from the origin. This minimum distance is shown to be the safety index β for the case of normal random variables with linear performance function.

The method uses reduced variables defined as

$$x_i' = \frac{x_i - \bar{x}_i}{\sigma_{x_i}} \quad (i = \ 1, \ldots\ldots, n)$$

Thus, the reduced variable has zero mean and unit standard direction. With the help of the reduced variable, the original limit state or performance function $G(X) = 0$ is converted to $(X') = 0$. The minimum distance, called β_{HL}, can be expressed as

$\beta_{HL} = \left[(X'_d)\, (X'_d)^T \right]^{\frac{1}{2}}$

In which X'_d is the minimum distance point on the limit state function and is called design point or checking point.

The importance of finding β_{HL}can be explained with the help of the linear limit state function of two variables. Consider the limit state function as

$Z = R - S = 0$

The reduced variables are then defined as

$R' = \frac{R-\mu_R}{\sigma_R}$

$S' = \frac{S-\mu_S}{\sigma_S}$

Substituting for *R* and *S*, the limit state equation may be expressed in terms of R' and S' as

$\sigma_R R' - \sigma_S S' + \mu_R - \mu_s$

In the space of reduced variable, the limit state function can be plotted as shown in Figure 2.8. It is apparent from Figure 2.3 that if the limit state line is near to the origin, the failure region is larger and the probability of failure is more. Converse is the case for less probability of failure. The minimum distance of the line from the origin is computed as below:

$$\beta_{HL} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

Note that $\beta_{HL}$is the same as βdefined for normal variables *R* and *S*. Thus, β_{HL}can be regarded as a measure of the safety index.

3. Write a brief note on Second-order Reliability method

Limit state function could be nonlinear because of many reasons such as (i) nonlinear relationship between random variables and the limit state function, (ii) transformation of non-normal variables to standard normal variables, and (iii) transformation from correlated to uncorrelated variables. If the limit state function is highly nonlinear and the joint probability density function does not decay rapidly as it moves away from the minimum distance point, then higher order approximation is required for the failure probability computations. There are many second order reliability methods proposed by various researchers, in which P_f is obtained using different assumptions and approximations. One of the commonly used method is use the curvature of the limit state function around the minimum distance point, by approximating it as a quadratic function. A closed form solution for the probability of failure of a region bounded by a quadratic limit state is given by:

$$p_f \approx \emptyset(-\beta)\Pi_{i=1}^{n-1} 1 + \beta K_i^{\frac{-1}{2}}$$

Where, k_i denotes the i^{th} main curvature of the limit state function at the minimum distance point. The main curvature k_i is the i^{th} Eigen value of the second derivative matrix **A** of the limit state surface at the design point in rotated normal space. The elements of the matrix **A** is given by:

$$a_{ij} = \frac{(RDR^T)}{|\nabla G(Y^*)|} \qquad (i,j = 1,2,\dots\dots,n-1)$$

Where, R is the orthogonal transformation matrix for which nth row is selected to be $\frac{Y^*}{(Y^{*T}Y^*)^{\frac{1}{2}}}$

A standard Gram-Schmidt algorithm may be used to determine R; D is the $n \times n$ second derivative matrix of the limit state surface in the standard normal space evaluated at the design point; $\nabla G(Y^*)$ is the gradient vector in the standard normal space.

4. Explain series systems in the context of system reliability

A system is an assemblage of components, each having a limit state function associated with it that defines the performance of that specific component. In the context of component reliability problem, only structural elements with two performance states (safe and failure) are considered, which is similar in case of system reliability as well. But it is important to note that there exists a possibility of having components and systems with multiple performance states.

Let us Consider a system consisting of n components. For the i^{th} component variable is defined as follows:

$a_i = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ If component i does not function (has failed)

similarly, system variable is defined as follows:

$$az = \begin{cases} & 1\ if\ the\ system\ function \\ 0 & if\ the\ system\ doesnot\ function \end{cases}$$

The random variable a_i is an indicator function that specifies the performance state of the i^{th} component. It should be apparent that the probability that $a_1 = 0$ can be estimated by a component reliability analysis. The indicator function a Which specifies the performance state of the system, is a function of the performance of its components.

$$a_Z = \Psi(\text{a})$$

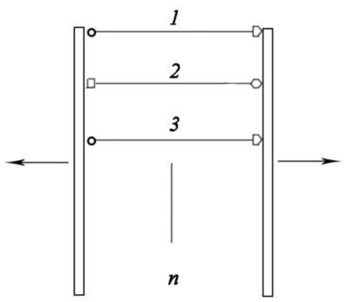
Where $a = [a_1, a_2, a_3, \dots \dots . a_n]^T$ $\Psi(\quad)$ denotes the system Auction. While the performance criterion of an individual element is usually easy to define and quantify through a limit state friction, the performance criteria for a system can be complex. An important and often difficult step in system reliability analyses is the identification of all the combinations of component failures that constitute a failure of the system. Systems are often idealized in the following ways. A system that fails if any of its components fails is called a series system. Such systems are often called weakest link systems clue to the analogy that *a* chain is only as strong as its weakest link. System function for a series system is given by:

$a_z = \Psi(a) = min(a_1, a_2, \ldots\ldots a_n) = \Pi_{i=1}^{n} a_i$

Clearly, if any component fails, then the product term in the above equation is equal to zero.

5. Explain parallel systems in the context of system reliability

A system that fails only if all of its components fail is called a parallel system Such systems are represented schematically as shown in Figure below:



The system function for a parallel system is

$a_z = \Psi(a) = max(a_1, a_2, \ldots\ldots a_n) = 1 - \Pi_{i=1}^{n}(1 - a_j)$

Note that the above expression will result in zero only when $a_i = 0$, i=1,2,…., n i.e.. When all the components which have been failed. If even one component functions, then the product term in the above expression is equal to zero and the system function is equal to one.