Mathematical Methods in Engineering and Science

IIT Kanpur

Assignment Policy of the Course

- Assignments of this course are of the traditional problem-solving type. Not only the result, but also the approach will be considered for evaluation.

- Submissions are invariably to be in the PDF format. PDF version of scanned copy of neat handwritten work is acceptable. Brevity and to the point solutions will be rewarded. Illegible writing, sloppy work and beating around the bush will be penalized.

- For many segments of the course, a programming background will be useful, though it is not essential.

- For some problems in the assignments, programming will be needed to get the complete solution. But, the programme does not need to be submitted.
  Those students who do not have a programming background may conduct the intial few (2 to 5) steps or iterations manually and indicate through what repititive and automated procedure a programme would need to proceed in order to get the final complete solution. Such solutions will be considered for partial credit.

- There may also be some problems, for which programming may not be needed as such, but may be useful in solving the problem systematically. Students having a programming background are encouraged to take advantage of this.

MMES (2017) Assignment 11

(Full marks $= 100$)

1. Solve the partial differential equation $xu_{xy} + 2yu = 0$ by the method of separation of variables.

2. Solve the PDE $u_{xx} + 3u_{xy} + 2u_{yy} = x + y$ using the method of characteristics.

3. For a non-integer positive number ϵ, solve the Cauchy problem,

$$\frac{\partial^2 y}{\partial t^2} = 1.44\frac{\partial^2 y}{\partial x^2}, \quad y(0,t) = y(\pi,t) = 0 \text{ for } t \geq 0, \quad y(x,0) = 0, \ y_t(x,0) = \sin(\epsilon x) \text{ for } 0 < x < \pi,$$

   of a vibrating string and plot its shape at time $t = 0.5, 1.0, 1.5, 2.0, 2.5, 3.0,$ with $\epsilon = 0.9$.

4. Solve the elastic membrane equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

   for a square membrane of unit side and wave speed $c = 1$, if the membrane is initially static with the configuration $u(x,y,0) = k\sin\pi x \sin\pi y$ for $0 \leq x \leq 1, 0 \leq y \leq 1$.

5. Starting from the definition

$$\cos z = \frac{1}{2}(e^{iz} + e^{-iz}) \quad \text{and} \quad \sin z = \frac{1}{2i}(e^{iz} - e^{-iz})$$

   as functions of a complex variable, develop their derivatives.

6. (a) Show that e^z is a periodic function.

   (b) Show that $\ln z$ is multi-valued, identify its branch with argument of z in the interval $-\pi < \arg z \leq \pi$ as the principal value $\operatorname{Ln} z$ and explore the possibility of establishing its derivative.

7. (a) Find the linear fractional transformation $w(z)$ that maps the origin, the point $z = 1$ and the point at infinity of the z-plane to the origin, the point at infinity and the point $w = i$ of the w-plane respectively.

   (b) Depict the conformal mapping of rays from the origin of the z-plane in directions $\theta = k\frac{\pi}{4}$ for $k = 0, 1, 2, \cdots, 7$ through this transformation.

8. Integrate $\bar{z}$ along the circular contour with centre at z_0 and radius ρ.

9. Let C be the circle $|z| = \rho e^{i\theta}$. Evaluate $\oint_C z^n dz$ by using the indefinite integral and explain the characteristic case of $n = -1$.

10. Integrate the functions (a) $g(z) = \frac{e^{z^2}}{z^2(z-1-i)}$ and (b) $f(z) = \frac{2z^3-3}{z(z-1-i)^2}$ over contour C, consisting of $|z| = 2$ (counterclockwise) and $|z| = 1$ (clockwise).