Mathematical Methods in Engineering and Science

IIT Kanpur

Assignment Policy of the Course

- Assignments of this course are of the traditional problem-solving type. Not only the result, but also the approach will be considered for evaluation.
- Submissions are invariably to be in the PDF format. PDF version of scanned copy of neat handwritten work is acceptable. Brevity and to the point solutions will be rewarded. Illegible writing, sloppy work and beating around the bush will be penalized.
- For many segments of the course, a programming background will be useful, though it is not essential.
- For some problems in the assignments, programming will be needed to get the complete solution. But, the programme does not need to be submitted.
  Those students who do not have a programming background may conduct the intial few (2 to 5) steps or iterations manually and indicate through what repititive and automated procedure a programme would need to proceed in order to get the final complete solution. Such solutions will be considered for partial credit.
- There may also be some problems, for which programming may not be needed as such, but may be useful in solving the problem systematically. Students having a programming background are encouraged to take advantage of this.

MMES (2017) Assignment 1

(Full marks = 100)

1. Given that
$$\begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 2 & 2 \\ 4 & 2 & 3 \end{bmatrix},$$
find out the values of a, b, c, d, e and f. (For a square root, select the positive value.)

2. A linear transformation maps the vectors $[2 \;\; 3]^T$ and $[4 \;\; 5]^T$ to vectors $[2 \;\; 0 \;\; 1]^T$ and $[1 \;\; 6 \;\; 0]^T$, respectively.

   (a) What are the domain and co-domain of the transformation?

   (b) How would you determine or describe the range and null space of the transformation?

   (c) Develop the *matrix* representation of the transformation.

   (d) What will be the image of the vector $[2 \;\; 1]^T$ by the transformation?

   (e) If the frame of reference in the domain is rotated by an angle around its origin, then what will be the effect on the *matrix* representation of the transformation?

3. For the linear transformation $\mathbf{A}$, distinct vectors $\mathbf{x}_0$ and $\mathbf{x}_1$ are found to be pre-images of a given vector $\mathbf{y}_0$. What can be said about

   (a) the null space of $\mathbf{A}$, and

   (b) the set of pre-images of $\mathbf{y}_0$?

4. Consider three vectors $\mathbf{u}_1 = [2\ 0\ -1\ 1]^T$, $\mathbf{u}_2 = [1\ 2\ 0\ 3]^T$ and $\mathbf{u}_3 = [3\ 0\ -1\ 2]^T$.

   (a) Find the unit vector $\mathbf{v}_1$ along $\mathbf{u}_1$.

   (b) From $\mathbf{u}_2$, subtract its component along $\mathbf{v}_1$ (which will have magnitude $\mathbf{v}_1^T\mathbf{u}_2$) and hence find the unit vector $\mathbf{v}_2$ such that vectors $\mathbf{v}_1$, $\mathbf{v}_2$ form an orthonormal basis for the subspace spanned by $\mathbf{u}_1$, $\mathbf{u}_2$.

   (c) Similarly, find a vector $\mathbf{v}_3$ which, together with $\mathbf{v}_1$ and $\mathbf{v}_2$, forms an orthonormal basis for the subspace spanned by all the three vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$.

   (d) Find a vector $\mathbf{v}_4$ to complete this basis for the entire space R^4.

   (e) Write a generalized algorithm for building up the vectors $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_l$, $l \le m$, when the given m vectors $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_m$ are in R^n, $m < n$.

5. A surveyor reaches a remote valley to prepare records of land holdings. The valley is a narrow strip of plain land between a mountain ridge and sea, and local people use a local and antiquated system of measures. They have two distant landmarks: the lighthouse and the high peak. To mention the location of any place, they typically instruct: *so many* bans *towards the lighthouse*

and so many kos *towards the high peak.* Upon careful measurement, the surveyor and his assistants found that (a) one *bans* is roughly 200 m, (b) one *kos* is around 15 km, (c) the lighthouse is 10 degrees south of east, and (d) the high peak is 5 degress west of north. The surveyor's team, obviously, uses the standard system, with unit distances of 1 km along east and along north. Now, to convert the local documents into standard system and to make sense to the locals about their intended locations, work out

(a) a conversion formula from valley system to standard system, and

(b) another conversion formula from standard system to valley system.

6. Find the elementary matrices corresponding to the row operation $R_3 \leftarrow R_3 + aR_2$ and column operation $C_2 \leftarrow C_2 - aC_3$ for a 3×4 matrix. Apply these elementary transformations on the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 2 \\ 2 & -4 & -1 & 0 \\ 3 & -1 & 0 & 3 \end{bmatrix}$$

and verify that these are equivalent to pre-multiplication and post-multiplication by the elementary matrices.

7. Find the rank, nullity, range and null space of the matrix

$$\mathbf{Q} = \begin{bmatrix} 3 & 2 & -1 & 4 & 1 \\ 0 & 1 & -1 & 2 & 2 \\ 1 & -3 & 0 & 1 & 2 \\ 4 & -3 & 1 & 1 & -1 \end{bmatrix}.$$

Hence, determine the complete solution of the system $\mathbf{Q}\,\mathbf{x} = [15 \;\; 11 \;\; 11 \;\; 4]^T$.

8. Given that

$$\begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ c & f & 1 \end{bmatrix} \begin{bmatrix} a & d & g \\ 0 & e & h \\ 0 & 0 & i \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 10 & 6 & 1 \\ 5 & -4 & 7 \end{bmatrix},$$

find out the values of a, b, c, d, e, f, g, h and i.

9. Solve the following system of equations for x_3 and x_4 by partitioning and block operations:

$$\begin{aligned} 2x_1 + 3x_3 + x_4 &= 5, \\ x_2 + x_3 + x_4 &= 2, \\ 5x_1 + 2x_2 + x_3 + x_4 &= 3, \\ 2x_1 - x_2 + 2x_3 + 2x_4 &= 5. \end{aligned}$$

10. For $n \times n$ matrices $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{A}$, consider the matrix multiplication $\mathbf{QR} = \mathbf{A}$ columnwise and observe that $r_{1,k}\mathbf{q}_1 + r_{2,k}\mathbf{q}_2 + r_{3,k}\mathbf{q}_3 + \cdots + r_{n,k}\mathbf{q}_n = \mathbf{a}_k$. For the matrix

$$\mathbf{A} = \begin{bmatrix} 6 & 5 & -1 & 0 \\ 6 & 5 & -1 & 6 \\ 6 & 1 & 1 & 0 \\ 6 & 1 & 1 & 2 \end{bmatrix},$$

write out the column equations one by one and determine the *corresponding* columns of an *orthogonal* $\mathbf{Q}$ and an *upper triangular* $\mathbf{R}$.