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## Unit 12 - Radon-Nikodym theorem and applications

### Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

## Week 11 Assessment

The due date for submitting this assignment has passed. **Due on 2020-04-15, 23:59 IST.**  
As per our records you have not submitted this assignment.

1) Let  $\{f_n\}$  be a sequence in  $L^1(\mathbb{R})$  which converges to  $f$  in  $L^1(\mathbb{R})$ . Which of the following are correct? **1 point**

$\int_{\mathbb{R}} |f_n(x)| dx$  converges to  $\int_{\mathbb{R}} |f(x)| dx$

(B)  $\{f_n\}$  converges to  $f$  almost everywhere on  $\mathbb{R}$

There exists a subsequence of  $\{f_n\}$  which converges to  $f$  almost everywhere on  $\mathbb{R}$

$\int_{\mathbb{R}} f_n(x) g(x) dx$  converges to  $\int_{\mathbb{R}} f(x) g(x) dx$  for any  $g \in L^\infty(\mathbb{R})$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\int_{\mathbb{R}} |f_n(x)| dx$  converges to  $\int_{\mathbb{R}} |f(x)| dx$

There exists a subsequence of  $\{f_n\}$  which converges to  $f$  almost everywhere on  $\mathbb{R}$

$\int_{\mathbb{R}} f_n(x) g(x) dx$  converges to  $\int_{\mathbb{R}} f(x) g(x) dx$  for any  $g \in L^\infty(\mathbb{R})$

2) Let  $\nu(A) = \int_A \frac{1}{1+x^2} dx$  for  $A \in \mathcal{B}(\mathbb{R})$ . Which of the following are correct? **1 point**

$\nu$  is absolutely continuous with respect to the Lebesgue measure on  $\mathbb{R}$

Lebesgue measure on  $\mathbb{R}$  is absolutely continuous with respect to  $\nu$

### **L<sup>p</sup> spaces and completeness**

### **Product spaces and Fubini's theorem**

### **Applications of Fubini's theorem and complex measures**

### **Complex measures and Radon-Nikodym theorem**

### **Radon-Nikodym theorem and applications**

- Radon Nikodym theorem II (unit? unit=80&lesson=81)
- Consequences of Radon-Nikodym theorem I (unit? unit=80&lesson=82)
- Consequences of Radon-Nikodym theorem II (unit? unit=80&lesson=83)
- Continuous linear functionals on L<sup>p</sup> spaces I (unit? unit=80&lesson=84)
- Continuous linear functionals on L<sup>p</sup> spaces II (unit? unit=80&lesson=85)
- Quiz : Week 11 Assessment (assessment? name=107)**

### **Riesz representation theorem and Lebesgue**

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\nu$  is absolutely continuous with respect to the Lebesgue measure on  $\mathbb{R}$   
 Lebesgue measure on  $\mathbb{R}$  is absolutely continuous with respect to  $\nu$

3) Which of the following are correct statements?

**1 point**

$T(f) = \int_0^1 f(x)dx$  is a continuous linear functional on  $L^1[0, 1]$

$T(f) = \int_0^1 f(x)dx$  is a continuous linear functional on  $L^2(\mathbb{R})$

$T(f) = \int_0^1 f(x)dx$  is a continuous linear functional on  $L^p[0, 1]$  for all  $1 \leq p \leq \infty$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$T(f) = \int_0^1 f(x)dx$  is a continuous linear functional on  $L^1[0, 1]$

$T(f) = \int_0^1 f(x)dx$  is a continuous linear functional on  $L^2(\mathbb{R})$

$T(f) = \int_0^1 f(x)dx$  is a continuous linear functional on  $L^p[0, 1]$  for all  $1 \leq p \leq \infty$

4) Let  $T$  be an  $n \times n$  invertible real matrix. Let  $\mu$  be the measure defined by  $\mu(A) = m(TA)$  where  $m$  is the Lebesgue measure on  $\mathbb{R}^n$ . Which of the following are correct? **1 point**

$\mu$  is absolutely continuous with respect to  $m$

$m$  is absolutely continuous with respect to  $\mu$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\mu$  is absolutely continuous with respect to  $m$

$m$  is absolutely continuous with respect to  $\mu$

5) For  $A \in \mathcal{B}(\mathbb{R}^2)$  define  $A_{\mathbb{R}} = \{(x, 0) \in A : x \in \mathbb{R}\} = A \cap (\mathbb{R} \times \{0\})$ . Define a measure  $\nu$  on  $\mathcal{B}(\mathbb{R}^2)$  by  $\nu(A) = \int_{A_{\mathbb{R}}} e^{-x^2} dx$ . Which of the following statements are correct? **1 point**

$\nu$  is absolutely continuous with respect to the Lebesgue measure on  $\mathbb{R}^2$

$\mu$  is mutually singular with respect to the Lebesgue measure on  $\mathbb{R}^2$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\mu$  is mutually singular with respect to the Lebesgue measure on  $\mathbb{R}^2$

6) Let  $\mu$  be a non-zero complex measure on  $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$  which is absolutely continuous with respect to the Lebesgue measure  $m$  on  $\mathbb{R}$ . Let  $h$  denote the Radon-Nikodym derivative  $\frac{d\mu}{dm}$ . Which of the following are possible? **1 point**

$h$  is zero outside  $[0, 1]$

$h$  is zero on irrationals

**differentiation  
theorem**

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$h$  is one on the set  $\{\frac{1}{n} : n \in \mathbb{N}\}$  and zero otherwise

No, the answer is incorrect.

Score: 0

Accepted Answers:

$h$  is zero outside  $[0, 1]$

7) Let  $m$  be the Lebesgue measure on  $\mathbb{R}$  and  $\mu$  be the measure defined by  $\mu(A) = m(A) + 1$  **1 point** if  $0 \in A$ ,  $\mu(A) = m(A)$  otherwise. Which of the following are correct?

$m$  is not absolutely continuous with respect to  $\mu$

$m$  is absolutely continuous with respect to  $\mu$

$\mu$  is absolutely continuous with respect to  $m$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$m$  is absolutely continuous with respect to  $\mu$

8) Let  $\delta_0$  be the measure on  $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$  defined by  $\delta_0(A) = 1$  if  $0 \in A$  and zero otherwise. **1 point** Which of the following is correct?

$m - \delta_0$  is absolutely continuous with respect to  $m$

$m - \delta_0$  is not absolutely continuous with respect to  $m$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$m - \delta_0$  is not absolutely continuous with respect to  $m$

9) Let  $f \in L^p[0, 1]$  for some  $1 < p < \infty$ . Define  $T(g) = \int_0^1 f(x)g(x)dx$ . Which of the following are correct? **1 point**

$T$  defines a continuous linear functional on  $L^\infty[0, 1]$

$T$  defines a continuous linear functional on  $L^2[0, 1]$

$T$  defines a continuous linear functional on  $L^q[0, 1]$  for all  $q \geq p^*$  where  $\frac{1}{p} + \frac{1}{p^*} = 1$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$T$  defines a continuous linear functional on  $L^\infty[0, 1]$

$T$  defines a continuous linear functional on  $L^q[0, 1]$  for all  $q \geq p^*$  where  $\frac{1}{p} + \frac{1}{p^*} = 1$

10) Let  $f_1, f_2 \in L^2(\mathbb{R})$  and let  $T(g) = \int_{\mathbb{R}} f_1(x)f_2(x)g(x)dx$ . Which of the following is correct? **1 point**

$T$  defines a continuous linear functional on  $L^2(\mathbb{R})$

$T$  defines a continuous linear functional on  $L^1(\mathbb{R})$



$T$  defines a continuous linear functional on  $L^\infty(\mathbb{R})$

No, the answer is incorrect.

Score: 0

Accepted Answers:

*$T$  defines a continuous linear functional on  $L^\infty(\mathbb{R})$*