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NPTEL (https://swayam.gov.in/explorer?ncCode=NPTEL) » Measure Theory (course)

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Unit 12 - Radon-Nikodym theorem and applications

Course outline

Week 11 Assessment

The due date for submitting this assignment has passed. **Due on 2020-04-15, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let $\{f_n\}$ be a sequence in $L^1(\mathbb{R})$ which converges to f in $L^1(\mathbb{R})$. Which of the following are correct? ***1 point***

- $\int_{\mathbb{R}} |f_n(x)|\, dx$ converges to $\int_{\mathbb{R}} |f(x)|\, dx$
- (B) $\{f_n\}$ converges to f almost everywhere on $\mathbb{R}$
- There exists a subsequence of $\{f_n\}$ which converges to f almost everywhere on $\mathbb{R}$
- $\int_{\mathbb{R}} f_n(x)\, g(x)\, dx$ converges to $\int_{\mathbb{R}} f(x)\, g(x)\, dx$ for any $g \in L^\infty(\mathbb{R})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_{\mathbb{R}} |f_n(x)|\, dx$ converges to $\int_{\mathbb{R}} |f(x)|\, dx$
There exists a subsequence of $\{f_n\}$ which converges to f almost everywhere on $\mathbb{R}$
$\int_{\mathbb{R}} f_n(x)\, g(x)\, dx$ converges to $\int_{\mathbb{R}} f(x)\, g(x)\, dx$ for any $g \in L^\infty(\mathbb{R})$

2) Let $\nu(A) = \int_A \frac{1}{1+x^2}\, dx$ for $A \in \mathcal{B}(\mathbb{R})$. Which of the following are correct? ***1 point***

- ν is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$
- Lebesgue measure on $\mathbb{R}$ is absolutely continuous with respect to ν

No, the answer is incorrect.
Score: 0

Accepted Answers:
ν is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$
Lebesgue measure on $\mathbb{R}$ is absolutely continuous with respect to ν

3) Which of the following are correct statements? ***1 point***

- [ ] $T(f) = \int_0^1 f(x)dx$ is a continuous linear functional on $L^1[0,1]$
- [ ] $T(f) = \int_0^1 f(x)dx$ is a continuous linear functional on $L^2(\mathbb{R})$
- [ ] $T(f) = \int_0^1 f(x)dx$ is a continuous linear functional on $L^p[0,1]$ for all $1 \leq p \leq \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$T(f) = \int_0^1 f(x)dx$ is a continuous linear functional on $L^1[0,1]$
$T(f) = \int_0^1 f(x)dx$ is a continuous linear functional on $L^2(\mathbb{R})$
$T(f) = \int_0^1 f(x)dx$ is a continuous linear functional on $L^p[0,1]$ for all $1 \leq p \leq \infty$

4) Let T be an $n \times n$ invertible real matrix. Let μ be the measure defined by $\mu(A) = m(TA)$ where m is the Lebesgue measure on $\mathbb{R}^n$. Which of the following are correct? ***1 point***

- [ ] μ is absolutely continuous with respect to m
- [ ] m is absolutely continuous with respect to μ

No, the answer is incorrect.
Score: 0

Accepted Answers:
μ is absolutely continuous with respect to m
m is absolutely continuous with respect to μ

5) For $A \in \mathcal{B}(\mathbb{R}^2)$ define $A_{\mathbb{R}} = \{(x,0) \in A : x \in \mathbb{R}\} = A \cap (\mathbb{R} \times \{0\})$. Define a measure ν on $\mathcal{B}(\mathbb{R}^2)$ by $\nu(A) = \int_{A_{\mathbb{R}}} e^{-x^2} dx$. Which of the following statements are correct? ***1 point***

- [ ] ν is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}^2$
- [ ] μ is mutually singular with respect to the Lebesgue measure on $\mathbb{R}^2$

No, the answer is incorrect.
Score: 0

Accepted Answers:
μ is mutually singular with respect to the Lebesgue measure on $\mathbb{R}^2$

6) Let μ be a non-zero complex measure on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ which is absolutely continuous with respect to the Lebesgue measure m on $\mathbb{R}$. Let h denote the Radon-Nikodym derivative $\frac{d\mu}{dm}$. Which of the following are possible? ***1 point***

- [ ] h is zero outside $[0,1]$
- [ ] h is zero on irrationals

differentiation theorem

Weekly Feedback forms

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- [ ] h is one on the set $\{\frac{1}{n} : n \in \mathbb{N}\}$ and zero otherwise

No, the answer is incorrect.
Score: 0

Accepted Answers:
h is zero outside $[0, 1]$

7) Let m be the Lebesgue measure on $\mathbb{R}$ and μ be the measure defined by $\mu(A) = m(A) + 1$ if $0 \in A$, $\mu(A) = m(A)$ otherwise. Which of the following are correct? ***1 point***

- [ ] m is not absolutely continuous with respect to μ
- [ ] m is absolutely continuous with respect to μ
- [ ] μ is absolutely continuous with respect to m

No, the answer is incorrect.
Score: 0

Accepted Answers:
m is absolutely continuous with respect to μ

8) Let δ_0 be the measure on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ defined by $\delta_0(A) = 1$ if $0 \in A$ and zero otherwise. Which of the following is correct? ***1 point***

- [ ] $m - \delta_0$ is absolutely continuous with respect to m
- [ ] $m - \delta_0$ is not absolutely continuous with respect to m

No, the answer is incorrect.
Score: 0

Accepted Answers:
$m - \delta_0$ is not absolutely continuous with respect to m

9) Let $f \in L^p[0, 1]$ for some $1 < p < \infty$. Define $T(g) = \int_0^1 f(x)g(x)dx$. Which of the following are correct? ***1 point***

- [ ] T defines a continuous linear functional on $L^\infty[0, 1]$
- [ ] T defines a continuous linear functional on $L^2[0, 1]$
- [ ] T defines a continuous linear functional on $L^q[0, 1]$ for all $q \geq p^*$ where $\frac{1}{p} + \frac{1}{p^*} = 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
T defines a continuous linear functional on $L^\infty[0, 1]$
T defines a continuous linear functional on $L^q[0, 1]$ for all $q \geq p^*$ where $\frac{1}{p} + \frac{1}{p^*} = 1$

10) Let $f_1, f_2 \in L^2(\mathbb{R})$ and let $T(g) = \int_{\mathbb{R}} f_1(x)f_2(x)g(x)dx$. Which of the following is correct? ***1 point***

- [ ] T defines a continuous linear functional on $L^2(\mathbb{R})$
- [ ] T defines a continuous linear functional on $L^1(\mathbb{R})$

- [ ] T defines a continuous linear functional on $L^\infty(\mathbb{R})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
T defines a continuous linear functional on $L^\infty(\mathbb{R})$