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reviewer4@nptel.iitm.ac.in

NPTEL (https://swayam.gov.in/explorer?ncCode=NPTEL) » Measure Theory (course)

Announcements (announcements)

About the Course (https://swayam.gov.in/nd1_noc20_ma02/preview) Ask a Question (forum)

Progress (student/home) Mentor (student/mentor)

Unit 11 - Complex measures and Radon-Nikodym theorem

Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

Week 10 Assessment

The due date for submitting this assignment has passed. **Due on 2020-04-08, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let μ be a complex measure on $(X, \mathcal{F})$. For $E \in \mathcal{F}$ define $\nu(E)$ to be the supremum of $\{\sum |\mu(E_j)|\}$ where the supremum is taken over finite measurable partitions $\{E_j\}$ of E. Which of the following are correct? ***1 point***

- [ ] $\nu = |\mu|$
- [ ] There exists $E \in \mathcal{F}$ such that $\nu(E) < |\mu|(E)$
- [ ] ν is not a measure

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\nu = |\mu|$

2) Let μ be a real measure defined on $(\mathbb{N}, 2^{\mathbb{N}})$, $\mu(\{j\}) = a_j$ where $a_j \in \mathbb{R}$ and $\sum |a_j| < \infty$. Which of the following are correct? ***1 point***

- [ ] $\mu^+(A) = \sum_{\{j \in A; a_j \geq 0\}} a_j$
- [ ] $\mu^-(A) = -\sum_{\{j \in A; a_j < 0\}} a_j$
- [ ] $|\mu|(A) = \sum_{j \in A} |a_j|$

L^p spaces and completeness

Product spaces and Fubini's theorem

Applications of Fubini's theorem and complex measures

Complex measures and Radon-Nikodym theorem

- Complex measures II (unit?unit=74&lesson=75)
- Absolutely continuous measures (unit?unit=74&lesson=76)
- L^2 space (unit?unit=74&lesson=77)
- Continuous linear functionals (unit?unit=74&lesson=78)
- Radon-Nikodym theorem I (unit?unit=74&lesson=79)
- **Quiz : Week 10 Assessment (assessment?name=106)**

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue differentiation theorem

Weekly Feedback forms

Video download

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mu^+(A) = \sum_{\{j\in A; a_j\geq 0\}} a_j$
$\mu^-(A) = -\sum_{\{j\in A; a_j< 0\}} a_j$
$|\mu|(A) = \sum_{j\in A} |a_j|$

3) Which of the following are correct? ***1 point***

- [ ] For a complex measure λ, if λ is concentrated on A then $|\lambda|$ is concentrated on A
- [ ] For a complex measure λ, if $|\lambda|$ is concentrated on A then so is λ

No, the answer is incorrect.
Score: 0

Accepted Answers:
For a complex measure λ, if λ is concentrated on A then $|\lambda|$ is concentrated on A
For a complex measure λ, if $|\lambda|$ is concentrated on A then so is λ

4) Let λ be the Borel measure defined by $\lambda(A) = \sum_{n\in\mathbb{Z}\cap A} \frac{(i)^n}{n^2}$, $A \in \mathcal{B}(\mathbb{R})$. Which of the following are correct? ***1 point***

- [ ] λ is concentrated on the set $\{\frac{1}{n^2} : n \in \mathbb{Z}\}$
- [ ] λ is concentrated on $\mathbb{Z}$
- [ ] $|\lambda|$ is concentrated on $\mathbb{Z}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
λ is concentrated on $\mathbb{Z}$
$|\lambda|$ is concentrated on $\mathbb{Z}$

5) Let m be the Lebesgue measure on $\mathbb{R}$ and let μ be the measure defined by $\mu(A) = \text{ number of rationals in A}$, for $A \in \mathcal{B}(\mathbb{R})$. Which of the following is correct? ***1 point***

- [ ] μ is mutually singular to m
- [ ] μ is absolutely continuous with respect to m

No, the answer is incorrect.
Score: 0

Accepted Answers:
μ is mutually singular to m

6) Let $(X, \mathcal{F}, \mu)$ be a positive measure space. Which of the following sets are convex? ***1 point***

- [ ] $\{f \in L^2(\mu) : \int_X |f|^2 \, d\mu \leq 1\}$
- [ ] $\{f \in L^2(\mu) : \int_X |f|^2 \, d\mu = 1\}$
- [ ] $\{f \in L^2(\mu) : 1 \leq \int_X |f|^2 \, d\mu \leq 2\}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\{f \in L^2(\mu) : \int_X |f|^2 \, d\mu \leq 1\}$

7) Let $(X, \mathcal{F}, \mu)$ be a positive measure space and let $f \in L^2(\mu)$. Which of the following are convex sets? **1 point**

- $\{g \in L^2(\mu) : \int_X fg \, d\mu = 2\}$
- $\{g \in L^2(\mu) : \left|\int_X fg\right| \leq 1\}$
- $\{g \in L^2(\mu) : \left|\int_X fg\right| \geq 2\}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\{g \in L^2(\mu) : \int_X fg \, d\mu = 2\}$
$\{g \in L^2(\mu) : \left|\int_X fg\right| \leq 1\}$

8) Let $(X, \mathcal{F}, \mu)$ be a positive measure space and $f \in L^2(\mu)$ be a non-zero function. Let $M = \{g \in L^2(\mu) : \int_X fg d\mu = 0\}$. Which of the following is correct? **1 point**

- M is closed
- M is closed and convex
- The vector with minimal norm in M is f

No, the answer is incorrect.
Score: 0

Accepted Answers:
M is closed
M is closed and convex

9) Which of the following sets have a vector with minimal norm? **1 point**

- $\{\frac{n+1}{n} f_n : f_n \in L^2[0,1], \|f_n\|_2 = 1, \langle f_n, f_m \rangle = 0 \text{ for } n \neq m\}$
- $\{g \in L^2(\mathbb{R}) : \left|\int_0^1 g(t) \, dt\right| > 1\}$
- $\{g \in L^2[0,1] : T(g) = 2\}$ where $T : L^2[0,1] \to \mathbb{C}$ is a continuous linear functional.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\{g \in L^2[0,1] : T(g) = 2\}$ *where $T : L^2[0,1] \to \mathbb{C}$ is a continuous linear functional.*

10) Let $(X, \mathcal{F}, \mu)$ be a positive measure space and $X = \cup_{n=1}^{\infty} A_n$ where $A_n \in \mathcal{F}$ and $A_k \cap A_j = \phi$ if $k \neq j$. Let $\{a_n\}$ be a sequence of complex numbers and consider the map $T : L^2(\mu) \to \mathbb{C}$ defined by $T(f) = \sum_n a_n \int_{A_n} f \, d\mu$. Which of the following are correct? **1 point**

- T is a continuous linear functional if and only if $\sum |a_n| < \infty$
- T is continuous linear functional if and only if $\sum |a_n|^2 < \infty$

- [ ] T is a continuous linear functional if and only if $\sum |a_n|^2 \mu(A_n) < \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
T is a continuous linear functional if and only if $\sum |a_n|^2 \mu(A_n) < \infty$