X

(https://swayam.gov.in) (https://swayam.gov.in/nc_details/NPTEL)

reviewer4@nptel.iitm.ac.in

NPTEL (https://swayam.gov.in/explorer?ncCode=NPTEL) » Measure Theory (course)

Announcements (announcements)

About the Course (https://swayam.gov.in/nd1_noc20_ma02/preview) Ask a Question (forum)

Progress (student/home) Mentor (student/mentor)

Unit 10 - Applications of Fubini's theorem and complex measures

Week 9 Assessment

The due date for submitting this assignment has passed. **Due on 2020-04-01, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Which of the following are complete measure spaces? ***1 point***

$(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2), m)$ where m is the Lebesgue measure on $\mathbb{R}^2$

$(\mathbb{R}^2, \mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}), m)$ where m is the Lebesgue measure on $\mathbb{R}^2$

$(\mathbb{R}^2, \mathcal{L}(\mathbb{R}^2), m_2)$ where m_2 is the Lebesgue measure on $\mathbb{R}^2$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$(\mathbb{R}^2, \mathcal{L}(\mathbb{R}^2), m_2)$ *where* m_2 *is the Lebesgue measure on* $\mathbb{R}^2$

2) Suppose f is integrable on $[0, b]$, and $g(x) = \int_x^b \frac{f(t)}{t}\, dt$ for $0 < x \leq b$. Then, ***1 point***

g is not integrable on $[0, b]$

g is integrable on $[0, b]$ and $\int_0^b g(x)\, dx = \int_0^b f(t)\, dt$

No, the answer is incorrect.
Score: 0

Accepted Answers:

g is integrable on $[0, b]$ and $\int_0^b g(x)\,dx = \int_0^b f(t)\,dt$

3) Let f be the function defined on $\mathbb{R}^n$ by $f(x) = |x|^a$ for $|x| \leq 1$ and zero otherwise. Which of the following are correct? ***1 point***

- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > -n$
- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a < -n$
- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > -n$
$\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > 0$

4) Let f be the function defined on $\mathbb{R}^n$ by $f(x) = |x|^a$ for $|x| \geq 1$ and zero otherwise. Which of the following are correct? ***1 point***

- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > -n$
- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a < -n$
- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a < -n$

5) Let f be the function defined on $\mathbb{R}^n$ by $f(x) = (1 - |x|^2)^a$ for $|x| \leq 1$ and zero otherwise. Which of the following are always correct? ***1 point***

- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > -n$
- $\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_{\mathbb{R}^n} f(x)\,dx < \infty$ if $a > 0$

6) Let $f \in L^1(\mathbb{R}^n)$ and let $d_f(\lambda) = m\{x : \ |f(x)| > \lambda\} \quad \lambda > 0$ where m is the Lebesgue measure on $\mathbb{R}^n$. Which of the following are correct? ***1 point***

- d_f is non-increasing and is continuous from right
- $\lambda d_f(\lambda) \to 0$ as $\lambda \to \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
d_f is non-increasing and is continuous from right

$\lambda d_f(\lambda) \to 0$ as $\lambda \to \infty$

7) Let $g \in L^1(\mathbb{R})$ and let F be the function defined by $F(x) = \int_x^{x+1} g(y)\, dy$. Then which of the following are correct? ***1 point***

- [ ] $F \in L^1(\mathbb{R})$
- [ ] F is bounded
- [ ] $\lim_{|x| \to \infty} F(x) = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$F \in L^1(\mathbb{R})$
F *is bounded*
$\lim_{|x| \to \infty} F(x) = 0$

8) Let $f \in L^1(\mathbb{R})$, define $F(x) = \sum_{k=-\infty}^{\infty} f(x+k) \quad x \in [0,1]$. Which of the following are correct? ***1 point***

- [ ] $F \in L^1[0,1]$
- [ ] If $F \in L^1[0,1]$ implies f is zero a.e.
- [ ] $F(x)$ is finite almost everywhere

No, the answer is incorrect.
Score: 0

Accepted Answers:
$F \in L^1[0,1]$
$F(x)$ *is finite almost everywhere*

9) Let $\int_{\mathbb{R}} |f_k - f|\, dm \to 0$ as $k \to \infty$. Which of the following are correct? ***1 point***

- [ ] $f_k \to f$ almost everywhere
- [ ] A subsequence of f_k converges to f almost everywhere
- [ ] For any $\varepsilon > 0, m\{x : |f_k(x) - f(x)| > \varepsilon\} \to 0$ as $k \to \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
A subsequence of f_k *converges to* f *almost everywhere*
For any $\varepsilon > 0, m\{x : |f_k(x) - f(x)| > \varepsilon\} \to 0$ *as* $k \to \infty$

10) Let f and g be in $L^1(\mathbb{R}^n)$ and additionally assume that g is bounded. Which of the following are correct? ***1 point***

- [ ] $f * g \in L^1(\mathbb{R}^n)$
- [ ] $f * g$ is bounded

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f * g \in L^1(\mathbb{R}^n)$
$f * g$ *is bounded*