

X


<https://swayam.gov.in>

https://swayam.gov.in/nc_details/NPTEL

reviewer4@nptel.iitm.ac.in ▾

[NPTEL \(https://swayam.gov.in/explorer?ncCode=NPTEL\)](https://swayam.gov.in/explorer?ncCode=NPTEL) » [Measure Theory \(course\)](#)
[Announcements \(announcements\)](#)
[About the Course \(https://swayam.gov.in/nd1_noc20_ma02/preview\)](https://swayam.gov.in/nd1_noc20_ma02/preview) [Ask a Question \(forum\)](#)
[Progress \(student/home\)](#) [Mentor \(student/mentor\)](#)

Unit 10 - Applications of Fubini's theorem and complex measures

Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and

Week 9 Assessment

The due date for submitting this assignment has passed. **Due on 2020-04-01, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Which of the following are complete measure spaces?

1 point

$(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2), m)$ where m is the Lebesgue measure on \mathbb{R}^2

$(\mathbb{R}^2, \mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}), m)$ where m is the Lebesgue measure on \mathbb{R}^2

$(\mathbb{R}^2, \mathcal{L}(\mathbb{R}^2), m_2)$ where m_2 is the Lebesgue measure on \mathbb{R}^2

No, the answer is incorrect.

Score: 0

Accepted Answers:

$(\mathbb{R}^2, \mathcal{L}(\mathbb{R}^2), m_2)$ where m_2 is the Lebesgue measure on \mathbb{R}^2

2) Suppose f is integrable on $[0, b]$, and $g(x) = \int_x^b \frac{f(t)}{t} dt$ for $0 < x \leq b$. Then,

1 point

g is not integrable on $[0, b]$

g is integrable on $[0, b]$ and $\int_0^b g(x) dx = \int_0^b f(t) dt$

No, the answer is incorrect.

Score: 0

Accepted Answers:

invariance properties

L^p spaces and completeness

Product spaces and Fubini's theorem

Applications of Fubini's theorem and complex measures

Completeness of product measures (unit? unit=69&lesson=70)

Polar coordinates (unit? unit=69&lesson=71)

Applications of Fubini's theorem (unit? unit=69&lesson=72)

Complex measures I (unit? unit=69&lesson=73)

Quiz : Week 9 Assessment (assessment? name=105)

Complex measures and Radon-Nikodym theorem

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue differentiation theorem

Weekly Feedback forms

Video download

g is integrable on $[0, b]$ and $\int_0^b g(x) dx = \int_0^b f(t) dt$

3) Let f be the function defined on \mathbb{R}^n by $f(x) = |x|^a$ for $|x| \leq 1$ and zero otherwise. Which **1 point** of the following are correct?

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > -n$$

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a < -n$$

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > -n$$

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > 0$$

4) Let f be the function defined on \mathbb{R}^n by $f(x) = |x|^a$ for $|x| \geq 1$ and zero otherwise. Which **1 point** of the following are correct?

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > -n$$

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a < -n$$

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a < -n$$

5) Let f be the function defined on \mathbb{R}^n by $f(x) = (1 - |x|^2)^a$ for $|x| \leq 1$ and zero otherwise. Which of the following are always correct? **1 point**

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > -n$$

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\int_{\mathbb{R}^n} f(x) dx < \infty \text{ if } a > 0$$

6) Let $f \in L^1(\mathbb{R}^n)$ and let $d_f(\lambda) = m\{x : |f(x)| > \lambda\}$ $\lambda > 0$ where m is the Lebesgue measure on \mathbb{R}^n . Which of the following are correct? **1 point**

d_f is non-increasing and is continuous from right

$$\lambda d_f(\lambda) \rightarrow 0 \text{ as } \lambda \rightarrow \infty$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

d_f is non-increasing and is continuous from right

$$\lambda d_f(\lambda) \rightarrow 0 \text{ as } \lambda \rightarrow \infty$$

7) Let $g \in L^1(\mathbb{R})$ and let F be the function defined by $F(x) = \int_x^{x+1} g(y) dy$. Then which **1 point** of the following are correct?

$$F \in L^1(\mathbb{R})$$

F is bounded

$$\lim_{|x| \rightarrow \infty} F(x) = 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$F \in L^1(\mathbb{R})$$

F is bounded

$$\lim_{|x| \rightarrow \infty} F(x) = 0$$

8) Let $f \in L^1(\mathbb{R})$, define $F(x) = \sum_{k=-\infty}^{\infty} f(x+k)$ $x \in [0, 1]$. Which of the following **1 point** are correct?

$$F \in L^1[0, 1]$$

If $F \in L^1[0, 1]$ implies f is zero a.e.

$F(x)$ is finite almost everywhere

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$F \in L^1[0, 1]$$

$F(x)$ is finite almost everywhere

9) Let $\int_{\mathbb{R}} |f_k - f| dm \rightarrow 0$ as $k \rightarrow \infty$. Which of the following are correct? **1 point**

$f_k \rightarrow f$ almost everywhere

A subsequence of f_k converges to f almost everywhere

For any $\varepsilon > 0$, $m\{x : |f_k(x) - f(x)| > \varepsilon\} \rightarrow 0$ as $k \rightarrow \infty$

No, the answer is incorrect.

Score: 0

Accepted Answers:

A subsequence of f_k converges to f almost everywhere

For any $\varepsilon > 0$, $m\{x : |f_k(x) - f(x)| > \varepsilon\} \rightarrow 0$ as $k \rightarrow \infty$

10) Let f and g be in $L^1(\mathbb{R}^n)$ and additionally assume that g is bounded. Which of the following **1 point** are correct?

$$f * g \in L^1(\mathbb{R}^n)$$

$f * g$ is bounded

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$f * g \in L^1(\mathbb{R}^n)$$

*f * g is bounded*