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NPTEL (https://swayam.gov.in/explorer?ncCode=NPTEL) » **Measure Theory (course)**

Announcements (announcements)

About the Course (https://swayam.gov.in/nd1_noc20_ma02/preview) Ask a Question (forum)

Progress (student/home) Mentor (student/mentor)

Unit 9 - Product spaces and Fubini's theorem

Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

Week 8 Assessment

The due date for submitting this assignment has passed. **Due on 2020-03-25, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Which of the following are correct? ***1 point***

- $\chi_{|x|\leq 1}(x)\ |x|^a\ (1-|x|)^b \in L^1(\mathbb{R}^n)$ iff $a > -n$ and $b > -1$
- $\chi_{|x|\leq 1}(x)\ |x|^a\ (1-|x|)^b \in L^1(\mathbb{R}^n)$ iff $a > -n$ and $b > 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\chi_{|x|\leq 1}(x)\ |x|^a\ (1-|x|)^b \in L^1(\mathbb{R}^n)$ *iff* $a > -n$ *and* $b > -1$

2) Which of the following are correct? ***1 point***

- $\mathcal{B}(\mathbb{R}) \times \mathcal{B}(\mathbb{R}) = \mathcal{B}(\mathbb{R}^2)$
- $\mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}) = \mathcal{L}(\mathbb{R}^2)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mathcal{B}(\mathbb{R}) \times \mathcal{B}(\mathbb{R}) = \mathcal{B}(\mathbb{R}^2)$

3) Which of the following are correct? ***1 point***

- The sigma algebra generated by $\{[a_1, b_1] \times \cdots \times [a_n, b_n] : a_j, b_j \in \mathbb{Q}\}$ is $\mathcal{B}(\mathbb{R}^n)$

L^p spaces and completeness

Product spaces and Fubini's theorem

- Product sigma algebra (unit?unit=62&lesson=63)
- Product measures I (unit?unit=62&lesson=64)
- Product measures II (unit?unit=62&lesson=65)
- Fubini's theorem I (unit?unit=62&lesson=66)
- Fubini's theorem II (unit?unit=62&lesson=67)
- **Quiz : Week 8 Assessment (assessment?name=104)**

Applications of Fubini's theorem and complex measures

Complex measures and Radon-Nikodym theorem

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue differentiation theorem

Weekly Feedback forms

Video download

- [ ] The sigma algebra generated by $\{[a_1, b_1] \times \cdots \times [a_n, b_n] : a_j, b_j \in \mathbb{Q}\}$ is $\mathcal{L}(\mathbb{R}^n)$
- [ ] If $E \in \mathcal{L}(\mathbb{R}^2)$ then $E_x \in \mathcal{L}(\mathbb{R})$ for all $x \in \mathbb{R}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
The sigma algebra generated by $\{[a_1, b_1] \times \cdots \times [a_n, b_n] : a_j, b_j \in \mathbb{Q}\}$ *is* $\mathcal{B}(\mathbb{R}^n)$

4) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be Lebesgue measurable. Which of the following are correct? ***1 point***

- [ ] For each $x \in \mathbb{R}$, $f_x : \mathbb{R} \to \mathbb{R}$ (defined by $f_x(y) = f(x, y)$ is Lebesgue measurable
- [ ] If f is Borel measurable, then for each $x \in \mathbb{R}$, f_x is Borel measurable

No, the answer is incorrect.
Score: 0
Accepted Answers:
If f is Borel measurable, then for each $x \in \mathbb{R}$, f_x is Borel measurable

5) Consider the measure spaces $(\mathbb{R}, \mathcal{B}(\mathbb{R}), m)$ where m is the Lebesgue measure on $\mathbb{R}$ and $(\mathbb{R}, \mathcal{B}(\mathbb{R}), \mu)$ where μ is the measure defined by $\mu(A) = \text{number of rationals in } A \cap [0, 1]$. Let $m \times \mu$ be the corresponding product measure on $(\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2))$. Let $D = \{(x, x) : 0 \leq x \leq 1\} \subset \mathbb{R}^2$, which of the following is correct? ***1 point***

- [ ] $m \times \mu(D) = 0$
- [ ] $m \times \mu(D) = \infty$
- [ ] $m \times \mu(D) = 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$m \times \mu(D) = 0$

6) Consider the measure space $(\mathbb{R}, 2^{\mathbb{R}}, \mu)$ where μ is the measure defined by $\mu(A) = \text{number of rationals in } A$. Let $\mu \times \mu$ be the corresponding product measure on $\mathbb{R}^2$. Let $D = \{(x, x) : 0 \leq x \leq 1\}$. Which of the following is correct? ***1 point***

- [ ] $\mu \times \mu(D) = 0$
- [ ] $\mu \times \mu(D) = \infty$
- [ ] $\mu \times \mu(D) = 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mu \times \mu(D) = \infty$

7) Let $X = Y = [0, 1]$, m Lebesgue measure on $[0, 1]$, μ counting measure on Y. Put $f(x, y) = 1$ if $x = y$ and zero otherwise. Which of the following are correct? ***1 point***

- [ ] $\int_X f(x, y)\, dm(x) = 0$ for all $y \in Y$

- [ ] $\int_Y f(x,y)\,d\mu(y) = 1$ for all $x \in X$
- [ ] $\int_X \int_Y f(x,y)\,d\mu(y)\,dm(x) = \int_Y \int_X f(x,y)\,dm(x)\,d\mu(y)$
- [ ] μ is not $\sigma-$finite so the iterated integrals are not same

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_X f(x,y)\,dm(x) = 0$ for all $y \in Y$
$\int_Y f(x,y)\,d\mu(y) = 1$ for all $x \in X$
μ is not $\sigma-$finite so the iterated integrals are not same

8) Let $f : \mathbb{R} \to \mathbb{R}$ be a Borel measurable non-negative function. Let $A(f) = \{(x,y) : 0 < y < f(x)\}$. Which of the following are correct? ***1 point***

- [ ] $A(f) \in \mathcal{B}(\mathbb{R}^2)$
- [ ] Lebesgue measure of $A(f)$ equals $\int_{\mathbb{R}} f(x)\,dx$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$A(f) \in \mathcal{B}(\mathbb{R}^2)$
Lebesgue measure of $A(f)$ equals $\int_{\mathbb{R}} f(x)\,dx$

9) $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Which of the following are correct? ***1 point***

- [ ] The graph of f, $G(f) = \{(x, f(x)) : x \in \mathbb{R}\}$ is a Borel set in $\mathbb{R}^2$
- [ ] Lebesgue measure of $G(f)$ is zero
- [ ] Lebesgue measure of $G(f)$ is infinity

No, the answer is incorrect.
Score: 0

Accepted Answers:
The graph of f, $G(f) = \{(x, f(x)) : x \in \mathbb{R}\}$ is a Borel set in $\mathbb{R}^2$
Lebesgue measure of $G(f)$ is zero

10) Let $(X, \mathcal{F}, \mu)$ be a σ-finite measure space and f be a positive measurable function on X. For $t \geq 0$ define $F_f(t) = \mu\{x \in X : f(x) > t\}$. Which of the following are correct? ***1 point***

- [ ] F_f is non-increasing and hence Borel measurable
- [ ] $\int_X f\,d\mu = \int_0^\infty F_f(t)\,dt$

No, the answer is incorrect.
Score: 0

Accepted Answers:
F_f is non-increasing and hence Borel measurable
$\int_X f\,d\mu = \int_0^\infty F_f(t)\,dt$