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NPTEL (https://swayam.gov.in/explorer?ncCode=NPTEL) » **Measure Theory (course)**

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Unit 8 - L^p spaces and completeness

Week 7 Assessment

The due date for submitting this assignment has passed. **Due on 2020-03-18, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let $(X, \mathcal{F}, \mu)$ be a measure space. Then, ***1 point***

- $f, g \in L^1(\mu)$ implies $fg \in L^1(\mu)$
- $f, g \in L^2(\mu)$ implies $fg \in L^2(\mu)$
- $f, g \in L^2(\mu)$ implies $fg \in L^1(\mu)$
- $f \in L^1(\mu)$ and $f \in L^\infty(\mu)$ implies $f \in L^2(\mu)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f, g \in L^2(\mu)$ *implies* $fg \in L^1(\mu)$
$f \in L^1(\mu)$ *and* $f \in L^\infty(\mu)$ *implies* $f \in L^2(\mu)$

2) Which of the following are true? ***1 point***

- $L^1[0, 1] \subset L^2[0, 1]$
- $L^1[0, \infty) \subset L^2[0, \infty)$
- $L^2[0, 1] \subset L^1[0, 1]$

$L^2[0,\infty) \subset L^1[0,\infty)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$L^2[0,1] \subset L^1[0,1]$

3) Let $f = \chi_{[0,\frac{1}{2}]}$. Then, **1 point**

f is continuous almost every where with respect to the Lebesgue measure on $\mathbb{R}$

f can be approximated by continuous functions in the L^∞ norm

There exists a continuous function g such that $f = g$ almost every where

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is continuous almost every where with respect to the Lebesgue measure on $\mathbb{R}$

4) Which of the following are correct? **1 point**

$\chi_{|x|\leq 1}(x)\ |x|^a \in L^1(\mathbb{R}^n)$ iff $a > -n$

$\chi_{|x|\leq 1}(x)\ |x|^a \in L^1(\mathbb{R}^n)$ iff $a < -n$

$\chi_{|x|\geq 1}(x)\ |x|^a \in L^1(\mathbb{R}^n)$ iff $a > -n$

$\chi_{|x|\geq 1}(x)\ |x|^a \in L^1(\mathbb{R}^n)$ iff $a < -n$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\chi_{|x|\leq 1}(x)\ |x|^a \in L^1(\mathbb{R}^n)$ *iff* $a > -n$
$\chi_{|x|\geq 1}(x)\ |x|^a \in L^1(\mathbb{R}^n)$ *iff* $a < -n$

5) Let $f : \mathbb{R} \to \mathbb{R}$ be a measurable function. Then, **1 point**

$f \in L^1(\mathbb{R})$ implies f is bounded

$f \in L^1(\mathbb{R})$ and f is continuous implies f is bounded

$f \in L^1(\mathbb{R})$ and f is continuous implies that $\lim_{|x|\to\infty} f(x) = 0$

$f \in L^1(\mathbb{R})$ and f is uniformly continuous implies that f is bounded

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f \in L^1(\mathbb{R})$ *and* f *is uniformly continuous implies that* f *is bounded*

6) Let $f_n : [0,1] \to \mathbb{R}$ be defined by $f_n(x) = x^n, x \in [0,1]$ for $n = 1, 2, 3, \cdots$. Which of the following are correct? **1 point**

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- [ ] f_n converges to zero uniformly in $[0, 1]$
- [ ] f_n converges to zero in $L^1[0, 1]$
- [ ] f_n converges to zero in $L^p[0, 1]$ for all $1 \leq p < \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
f_n converges to zero in $L^1[0, 1]$
f_n converges to zero in $L^p[0, 1]$ for all $1 \leq p < \infty$

7) Let $f_n : [1, \infty) \to \mathbb{R}$ be defined by $f_n(x) = x^{-n}, x \in [1, \infty)$ for $n = 1, 2, 3, \cdots$. Which of the following are correct? ***1 point***

- [ ] f_n converges to zero uniformly
- [ ] f_n converges to zero in $L^1[1, \infty)$
- [ ] f_n converges to zero in $L^p[1, \infty)$ for all $1 \leq p < \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
f_n converges to zero in $L^1[1, \infty)$
f_n converges to zero in $L^p[1, \infty)$ for all $1 \leq p < \infty$

8) Let $f_n : [2, \infty) \to \mathbb{R}$ be defined by $f_n(x) = x^{-n}, x \in [2, \infty)$ for $n = 1, 2, 3, \cdots$. Which of the following are correct? ***1 point***

- [ ] f_n converges to zero uniformly
- [ ] f_n converges to zero in $L^1[2, \infty)$
- [ ] f_n converges to zero in $L^p[2, \infty)$ for all $1 \leq p \leq \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
f_n converges to zero uniformly
f_n converges to zero in $L^1[2, \infty)$
f_n converges to zero in $L^p[2, \infty)$ for all $1 \leq p \leq \infty$

9) Let $(X, \mathcal{F}, \mu)$ be a measure space and $1 \leq p, r, s \leq \infty$. Which of the following are correct? ***1 point***

- [ ] If $p < r < s$, then $L^p \cap L^s(\mu) \subset L^r(\mu)$
- [ ] If $\mu(X) < \infty$, then $L^p(\mu) \subset L^r(\mu)$ if $r < p$
- [ ] If $\mu(X) < \infty$ and $f \in L^\infty(\mu)$ then $\|f\|_p \to \|f\|_\infty$ as $p \to \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:

If $p < r < s$, *then* $L^p \cap L^s(\mu) \subset L^r(\mu)$
If $\mu(X) < \infty$, *then* $L^p(\mu) \subset L^r(\mu)$ *if* $r < p$
If $\mu(X) < \infty$ *and* $f \in L^\infty(\mu)$ *then* $\|f\|_p \to \|f\|_\infty$ *as* $p \to \infty$

10) Let $(X, \mathcal{F}, \mu)$ be a measure space and let f and g be positive measurable functions such that $fg \geq a$ for some $a > 0$. Then, ***1 point***

- [ ] If $\mu(X) = 1$, $\left(\int_X f d\mu\right)\left(\int_X g\, d\mu\right) \geq a$
- [ ] If $\mu(X) < 1$, $\left(\int_X f d\mu\right)\left(\int_X g\, d\mu\right) \geq a$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $\mu(X) = 1$, $\left(\int_X f d\mu\right)\left(\int_X g\, d\mu\right) \geq a$