Unit 7 - Lebesgue measure and invariance properties

Week 6 Assessment

The due date for submitting this assignment has passed. **Due on 2020-03-11, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Which of the following sets are of Lebesgue measure zero? ***1 point***

- Any countable subset of $\mathbb{R}$
- Cantor set
- The set of irrationals in the Cantor set
- $k-$dimensional subspace of $\mathbb{R}^n$, where $k < n$

No, the answer is incorrect.
Score: 0

Accepted Answers:
Any countable subset of $\mathbb{R}$
Cantor set
The set of irrationals in the Cantor set
$k-$*dimensional subspace of* $\mathbb{R}^n$, *where* $k < n$

2) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be an invertible linear map. Which of the following are correct? ***1 point***

- If E is a Lebesgue set then $T(E)$ is a Lebesgue set
- If E is a Borel set then $T(E)$ is a Borel set

No, the answer is incorrect.
Score: 0

Accepted Answers:

If E is a Lebesgue set then $T(E)$ is a Lebesgue set
If E is a Borel set then $T(E)$ is a Borel set

3) Which of the following are true? ***1 point***

- [ ] The outer measure m_* on $\mathbb{R}$ is translation invariant
- [ ] The Lebesgue measure m on $\mathbb{R}$ is translation invariant
- [ ] If μ is any Borel measure on $\mathbb{R}$ and $\mu(K) < \infty$ for all compact set K, then μ is a constant multiple of the Lebesgue measure

No, the answer is incorrect.
Score: 0

Accepted Answers:
The outer measure m_* on $\mathbb{R}$ is translation invariant
The Lebesgue measure m on $\mathbb{R}$ is translation invariant

4) Let μ be a Borel measure on $\mathbb{R}$ such that $\mu(A+n) = \mu(A)$ for all Borel sets A and $n \in \mathbb{Z}$. Then which of the following are always correct? ***1 point***

- [ ] μ is the zero measure
- [ ] μ is a constant multiple of the Lebesgue measure
- [ ] Counting measure on $\mathbb{R}$ satisfies the property $\mu(A+n) = \mu(A)$ for all A and $n \in \mathbb{Z}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Counting measure on $\mathbb{R}$ satisfies the property $\mu(A+n) = \mu(A)$ for all A and $n \in \mathbb{Z}$.

5) Let $A : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Which of the following are correct? ***1 point***

- [ ] If A is singular, then $A(E)$ is a Lebesgue set for all $E \subset \mathbb{R}^n$
- [ ] If A is invertible, then $A(E)$ is a Lebesgue set for all $E \subset \mathbb{R}^n$
- [ ] If A is invertible, then $A(E)$ is a Lebesgue set for all Lebesgue sets $E \subset \mathbb{R}^n$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If A is singular, then $A(E)$ is a Lebesgue set for all $E \subset \mathbb{R}^n$
If A is invertible, then $A(E)$ is a Lebesgue set for all Lebesgue sets $E \subset \mathbb{R}^n$

6) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a positive measurable function and let m denote the Lebesgue measure on $\mathbb{R}$. Which of the following are true? ***1 point***

- [ ] $\int_{E+x_0} f \, dm = \int_E f \, dm$ for all $E \in \mathcal{L}(R^n)$ and $x_0 \in \mathbb{R}^n$
- [ ] $\int_{\mathbb{R}^n} f(x+x_0) \, dx = \int_{\mathbb{R}^n} f \, dm$ for all $x_0 \in \mathbb{R}^n$
- [ ] $\int_{\mathbb{R}^n} f(Ax) \, dx = \int_{\mathbb{R}^n} f(x) \, dx$ for all linear maps $A : \mathbb{R}^n \to \mathbb{R}^n$

differentiation theorem

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No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_{\mathbb{R}^n} f(x+x_0)\,dx = \int_{\mathbb{R}^n} f\,dm$ *for all* $x_0 \in \mathbb{R}^n$

7) Which of the following are true? ***1 point***

- [ ] If $O \subset [0,1]$ is a dense open set then $m(O) = 1$
- [ ] If $O \subset [0,1]$ is open then $m(O) > 0$
- [ ] If $F \subset [0,1]$ is closed and has no interior then $m(F) = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $O \subset [0,1]$ *is open then* $m(O) > 0$

8) Which of the following sets are of Lebesgue measure zero? ***1 point***

- [ ] $C \times \mathbb{R} \subset \mathbb{R}^2$ where $C \subset [0,1]$ is the Cantor set
- [ ] $\mathbb{Q} \times \mathbb{R} \subset \mathbb{R}^2$
- [ ] Countable union of lines passing through the origin in $\mathbb{R}^2$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$C \times \mathbb{R} \subset \mathbb{R}^2$ *where* $C \subset [0,1]$ *is the Cantor set*
$\mathbb{Q} \times \mathbb{R} \subset \mathbb{R}^2$
Countable union of lines passing through the origin in $\mathbb{R}^2$

9) Which of the following sets have positive Lebesgue measure? ***1 point***

- [ ] Any unbounded set in $\mathbb{R}^2$
- [ ] Any unbounded closed set in $\mathbb{R}^2$
- [ ] $(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{R} \subset \mathbb{R}^2$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{R} \subset \mathbb{R}^2$

10)Which of the following are correct? ***1 point***

- [ ] The Cantor set is a closed set
- [ ] The Cantor set is a perfect set
- [ ] The complement of Cantor set in $[0,1]$ has positive Lebesgue measure
- [ ] The Cantor set is uncountable

No, the answer is incorrect.
Score: 0

Accepted Answers:

The Cantor set is a closed set
The Cantor set is a perfect set
The complement of Cantor set in $[0, 1]$ *has positive Lebesgue measure*
The Cantor set is uncountable