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## Unit 7 - Lebesgue measure and invariance properties

### Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

## Week 6 Assessment

The due date for submitting this assignment has passed. **Due on 2020-03-11, 23:59 IST.**  
As per our records you have not submitted this assignment.

1) Which of the following sets are of Lebesgue measure zero? **1 point**

Any countable subset of  $\mathbb{R}$

Cantor set

The set of irrationals in the Cantor set

$k$ -dimensional subspace of  $\mathbb{R}^n$ , where  $k < n$

No, the answer is incorrect.

Score: 0

Accepted Answers:

*Any countable subset of  $\mathbb{R}$*

*Cantor set*

*The set of irrationals in the Cantor set*

*$k$ -dimensional subspace of  $\mathbb{R}^n$ , where  $k < n$*

2) Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$  be an invertible linear map. Which of the following are correct? **1 point**

If  $E$  is a Lebesgue set then  $T(E)$  is a Lebesgue set

If  $E$  is a Borel set then  $T(E)$  is a Borel set

No, the answer is incorrect.

Score: 0

Accepted Answers:

- Lebesgue measure via Riesz representation theorem (unit? unit=48&lesson=49)
- Construction of Lebesgue measure (unit? unit=48&lesson=50)
- Invariance properties of Lebesgue measure (unit? unit=48&lesson=51)
- Linear transformations and Lebesgue measure (unit? unit=48&lesson=52)
- Cantor set (unit? unit=48&lesson=53)
- Cantor function (unit? unit=48&lesson=54)
- Quiz : Week 6 Assessment (assessment? name=102)**

**L<sup>p</sup> spaces and completeness**

**Product spaces and Fubini's theorem**

**Applications of Fubini's theorem and complex measures**

**Complex measures and Radon-Nikodym theorem**

**Radon-Nikodym theorem and applications**

**Riesz representation theorem and Lebesgue**

If  $E$  is a Lebesgue set then  $T(E)$  is a Lebesgue set  
 If  $E$  is a Borel set then  $T(E)$  is a Borel set

3) Which of the following are true?

**1 point**

- The outer measure  $m_*$  on  $\mathbb{R}$  is translation invariant
- The Lebesgue measure  $m$  on  $\mathbb{R}$  is translation invariant

If  $\mu$  is any Borel measure on  $\mathbb{R}$  and  $\mu(K) < \infty$  for all compact set  $K$ , then  $\mu$  is a constant multiple of the Lebesgue measure

No, the answer is incorrect.  
 Score: 0

Accepted Answers:

- The outer measure  $m_*$  on  $\mathbb{R}$  is translation invariant*
- The Lebesgue measure  $m$  on  $\mathbb{R}$  is translation invariant*

4) Let  $\mu$  be a Borel measure on  $\mathbb{R}$  such that  $\mu(A + n) = \mu(A)$  for all Borel sets  $A$  and  $n \in \mathbb{Z}$ . Then which of the following are always correct?

**1 point**

- $\mu$  is the zero measure
- $\mu$  is a constant multiple of the Lebesgue measure

Counting measure on  $\mathbb{R}$  satisfies the property  $\mu(A + n) = \mu(A)$  for all  $A$  and  $n \in \mathbb{Z}$ .

No, the answer is incorrect.  
 Score: 0

Accepted Answers:

*Counting measure on  $\mathbb{R}$  satisfies the property  $\mu(A + n) = \mu(A)$  for all  $A$  and  $n \in \mathbb{Z}$ .*

5) Let  $A : \mathbb{R}^n \rightarrow \mathbb{R}^n$  be a linear transformation. Which of the following are correct?

**1 point**

- If  $A$  is singular, then  $A(E)$  is a Lebesgue set for all  $E \subset \mathbb{R}^n$
- If  $A$  is invertible, then  $A(E)$  is a Lebesgue set for all  $E \subset \mathbb{R}^n$
- If  $A$  is invertible, then  $A(E)$  is a Lebesgue set for all Lebesgue sets  $E \subset \mathbb{R}^n$

No, the answer is incorrect.  
 Score: 0

Accepted Answers:

- If  $A$  is singular, then  $A(E)$  is a Lebesgue set for all  $E \subset \mathbb{R}^n$*
- If  $A$  is invertible, then  $A(E)$  is a Lebesgue set for all Lebesgue sets  $E \subset \mathbb{R}^n$*

6) Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  be a positive measurable function and let  $m$  denote the Lebesgue measure on  $\mathbb{R}^n$ . Which of the following are true?

- $\int_{E+x_0} f \, dm = \int_E f \, dm$  for all  $E \in \mathcal{L}(\mathbb{R}^n)$  and  $x_0 \in \mathbb{R}^n$
- $\int_{\mathbb{R}^n} f(x + x_0) \, dx = \int_{\mathbb{R}^n} f \, dm$  for all  $x_0 \in \mathbb{R}^n$
- $\int_{\mathbb{R}^n} f(Ax) \, dx = \int_{\mathbb{R}^n} f(x) \, dx$  for all linear maps  $A : \mathbb{R}^n \rightarrow \mathbb{R}^n$

**differentiation  
theorem**

**Weekly Feedback  
forms**

**Video download**

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\int_{\mathbb{R}^n} f(x + x_0) dx = \int_{\mathbb{R}^n} f dm \text{ for all } x_0 \in \mathbb{R}^n$$

7) Which of the following are true?

1 point

If  $O \subset [0, 1]$  is a dense open set then  $m(O) = 1$

If  $O \subset [0, 1]$  is open then  $m(O) > 0$

If  $F \subset [0, 1]$  is closed and has no interior then  $m(F) = 0$

No, the answer is incorrect.

Score: 0

Accepted Answers:

If  $O \subset [0, 1]$  is open then  $m(O) > 0$

8) Which of the following sets are of Lebesgue measure zero?

1 point

$C \times \mathbb{R} \subset \mathbb{R}^2$  where  $C \subset [0, 1]$  is the Cantor set

$\mathbb{Q} \times \mathbb{R} \subset \mathbb{R}^2$

Countable union of lines passing through the origin in  $\mathbb{R}^2$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$C \times \mathbb{R} \subset \mathbb{R}^2$  where  $C \subset [0, 1]$  is the Cantor set

$\mathbb{Q} \times \mathbb{R} \subset \mathbb{R}^2$

Countable union of lines passing through the origin in  $\mathbb{R}^2$

9) Which of the following sets have positive Lebesgue measure?

1 point

Any unbounded set in  $\mathbb{R}^2$

Any unbounded closed set in  $\mathbb{R}^2$

$(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{R} \subset \mathbb{R}^2$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{R} \subset \mathbb{R}^2$

10) Which of the following are correct?

1 point

The Cantor set is a closed set

The Cantor set is a perfect set

The complement of Cantor set in  $[0, 1]$  has positive Lebesgue measure

The Cantor set is uncountable

No, the answer is incorrect.

Score: 0

Accepted Answers:

*The Cantor set is a closed set*

*The Cantor set is a perfect set*

*The complement of Cantor set in  $[0, 1]$  has positive Lebesgue measure*

*The Cantor set is uncountable*