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NPTEL (https://swayam.gov.in/explorer?ncCode=NPTEL) » Measure Theory (course)

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Unit 6 - Lebesgue measure and positive Borel measures on locally compact spaces

Week 5 Assessment

The due date for submitting this assignment has passed. **Due on 2020-03-04, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f^{-1}(\{a\})$ is a Lebesgue set for every $a \in \mathbb{R}$. Which of the following are true? ***1 point***

- [ ] f is Lebesgue measurable always
- [ ] f is Borel measurable always
- [ ] $f : (\mathbb{R}, 2^{\mathbb{R}}) \to \mathbb{R}$ is measurable

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f : (\mathbb{R}, 2^{\mathbb{R}}) \to \mathbb{R}$ *is measurable*

2) Let $f_n : [0, 1] \to \mathbb{R}$ be a sequence of Riemann integrable functions such that $\lim_{n\to\infty} f_n(x) = f(x)$ for all $x \in [0, 1]$. Which of the following are always correct? ***1 point***

- [ ] f is Riemann integrable
- [ ] f is Riemann integrable and $\lim_{n\to\infty} \int_0^1 f_n(x)\,dx \to \int_0^1 f(x)\,dx$
- [ ] f is Lebesgue measurable

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is Lebesgue measurable

3) Let $\{f_n\}$ be a sequence of continuous real valued functions defined on $\mathbb{R}$ which converges pointwise to a continuous real valued function f on $\mathbb{R}$. Then which of the following are necessarily true ? ***1 point***

$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

If $0 \leq f_n(x) \leq f(x)\ \forall n \in \mathbb{N}\ and\ x \in \mathbb{R}$ then $\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx$ for all $a, b \in \mathbb{R}$

If $|f_n(x)| \leq e^{-x}\ \forall n \in \mathbb{N}\ and\ x \in \mathbb{R}$ then $\lim_{n\to\infty} \int_a^\infty f_n(x)dx = \int_a^\infty f(x)dx$ for all $a \in \mathbb{R}$

If $|f_n(x)| \leq 1\ \forall n \in \mathbb{N}\ and\ x \in \mathbb{R}$ then $\lim_{n\to\infty} \int_a^\infty f_n(x)dx = \int_a^\infty f(x)dx$ for all $a \in \mathbb{R}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $0 \leq f_n(x) \leq f(x)\ \forall n \in \mathbb{N}\ and\ x \in \mathbb{R}$ then $\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx$ for all $a, b \in \mathbb{R}$
If $|f_n(x)| \leq e^{-x}\ \forall n \in \mathbb{N}\ and\ x \in \mathbb{R}$ then $\lim_{n\to\infty} \int_a^\infty f_n(x)dx = \int_a^\infty f(x)dx$ for all $a \in \mathbb{R}$

4) Let f be integrable over $\mathbb{R}$. Which of the following imply $f = 0\ ae$? ***1 point***

$\int_{\mathbb{R}} f\ dm = 0$

$\int_{\mathbb{R}} |f|\ dm = 0$

$\int_E f\ dm = 0$ for all E measurable.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_{\mathbb{R}} |f|\ dm = 0$
$\int_E f\ dm = 0$ for all E measurable.

5) Let $f : [0, 1] \to \mathbb{R}$ be defined as follows: ***1 point***

$$f(x) = \begin{cases} \sqrt{x} & \text{if} \quad x \quad \text{is irrational} \\ 0 & \text{otherwise} \end{cases}$$

Then which of the following are correct ?

f is measurable.

f is Lebesgue integrable.

f is Riemann integrable.

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No, the answer is incorrect.
Score: 0

Accepted Answers:
f is measurable.
f is Lebesgue integrable.

6) Which of the following are correct? ***1 point***

- $\chi_E, E \subset \mathbb{R}$ is measurable if and only if E is a Lebesgue set
- $\chi_E, E \subset \mathbb{R}$ is Borel measurable if and only if E is a Borel set

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\chi_E, E \subset \mathbb{R}$ is measurable if and only if E is a Lebesgue set
$\chi_E, E \subset \mathbb{R}$ is Borel measurable if and only if E is a Borel set

7) Which of the following are correct? ***1 point***

- If f is Riemann integrable on $[0, 1]$ and $g = f$ except at finitely many points then g is Riemann integrable
- If f is continuous on $[0, 1]$ and $g = f$ except on a countable infinite set then g is Riemann integrable
- If f is continuous on $[0, 1]$ and $g = f$ except on a countable infinite set then g is Lebesgue integrable

No, the answer is incorrect.
Score: 0

Accepted Answers:
If f is Riemann integrable on $[0, 1]$ and $g = f$ except at finitely many points then g is Riemann integrabl
If f is continuous on $[0, 1]$ and $g = f$ except on a countable infinite set then g is Lebesgue integrable

8) Which of the following measures on $[0, 1]$ are $\sigma-$finite? ***1 point***

- Counting measure on $[0, 1]$ where the $\sigma-$algebra is the power set
- Counting measure on $[0, 1]$ where the $\sigma-$algebra is the Borel $\sigma-$algebra of $[0, 1]$
- $\mu(A) =$ number of elements in $A \cap \mathbb{Q}$ where $A \in \mathcal{B}(\mathbb{R})$.
- $\mu(A) =$ number of elements in $A \cap \mathbb{Z}$ where $A \in \mathcal{B}(\mathbb{R})$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mu(A) =$ number of elements in $A \cap \mathbb{Q}$ *where* $A \in \mathcal{B}(\mathbb{R})$.
$\mu(A) =$ number of elements in $A \cap \mathbb{Z}$ *where* $A \in \mathcal{B}(\mathbb{R})$.

9) Which of the following are positive linear functionals ? ***1 point***

- On $C[0, 1]$ define $T(f) = \sum_{n=1}^{\infty} f(\frac{1}{n})$
- On $C_c(\mathbb{R})$ define $T(f) = \sum_n f(n)$

- [ ] On $C_c(0, \infty)$ define $T(f) = \sum_{n=1}^{\infty} f(\frac{1}{n})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
On $C_c(\mathbb{R})$ *define* $T(f) = \sum_n f(n)$
On $C_c(0, \infty)$ *define* $T(f) = \sum_{n=1}^{\infty} f(\frac{1}{n})$

10) Let $X = \mathbb{R} \setminus \{0\}$ and consider $C_c(X)$ Which of the following are positive linear functionals on $C_c(X)$? ***1 point***

- [ ] $T(f) = \sum_{k=1}^{\infty} f(\frac{1}{k}) + \sum_{k=1}^{\infty} f(-\frac{1}{k})$
- [ ] $T(f) = \sum_{k=1}^{\infty} f(-k) + \sum_{k=1}^{\infty} f(k)$
- [ ] $T(f) = \sum_{k=1}^{\infty} f(k + \frac{1}{k})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$T(f) = \sum_{k=1}^{\infty} f(\frac{1}{k}) + \sum_{k=1}^{\infty} f(-\frac{1}{k})$
$T(f) = \sum_{k=1}^{\infty} f(-k) + \sum_{k=1}^{\infty} f(k)$
$T(f) = \sum_{k=1}^{\infty} f(k + \frac{1}{k})$