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[Announcements \(announcements\)](#)
[About the Course \(https://swayam.gov.in/nd1_noc20_ma02/preview\)](https://swayam.gov.in/nd1_noc20_ma02/preview) [Ask a Question \(forum\)](#)
[Progress \(student/home\)](#) [Mentor \(student/mentor\)](#)

Unit 6 - Lebesgue measure and positive Borel measures on locally compact spaces

Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

● Measurable functions (unit?)

Week 5 Assessment

The due date for submitting this assignment has passed. **Due on 2020-03-04, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that $f^{-1}(\{a\})$ is a Lebesgue set for every $a \in \mathbb{R}$. Which of the following are true? **1 point**

f is Lebesgue measurable always

f is Borel measurable always

$f : (\mathbb{R}, 2^{\mathbb{R}}) \rightarrow \mathbb{R}$ is measurable

No, the answer is incorrect.

Score: 0

Accepted Answers:

$f : (\mathbb{R}, 2^{\mathbb{R}}) \rightarrow \mathbb{R}$ is measurable

2) Let $f_n : [0, 1] \rightarrow \mathbb{R}$ be a sequence of Riemann integrable functions such that $\lim_{n \rightarrow \infty} f_n(x) = f(x)$ for all $x \in [0, 1]$. Which of the following are always correct? **1 point**

f is Riemann integrable

f is Riemann integrable and $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \rightarrow \int_0^1 f(x) dx$

f is Lebesgue measurable

unit=42&lesson=43)

- Riemann and Lebesgue integrals (unit? unit=42&lesson=44)
- Locally compact Hausdorff spaces (unit? unit=42&lesson=45)
- Riesz representation theorem (unit? unit=42&lesson=46)
- Positive Borel measures (unit? unit=42&lesson=47)
- Quiz : Week 5 Assessment (assessment? name=101)**

Lebesgue measure and invariance properties

L^p spaces and completeness

Product spaces and Fubini's theorem

Applications of Fubini's theorem and complex measures

Complex measures and Radon-Nikodym theorem

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue differentiation theorem

Weekly Feedback forms

No, the answer is incorrect.
Score: 0

Accepted Answers:
 f is Lebesgue measurable

3) Let $\{f_n\}$ be a sequence of continuous real valued functions defined on \mathbb{R} which converges **1 point** pointwise to a continuous real valued function f on \mathbb{R} . Then which of the following are necessarily true ?

$$\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$$

If $0 \leq f_n(x) \leq f(x) \forall n \in \mathbb{N}$ and $x \in \mathbb{R}$ then $\lim_{n \rightarrow \infty} \int_a^b f_n(x) dx = \int_a^b f(x) dx$ for all $a, b \in \mathbb{R}$

If $|f_n(x)| \leq e^{-x} \forall n \in \mathbb{N}$ and $x \in \mathbb{R}$ then $\lim_{n \rightarrow \infty} \int_a^\infty f_n(x) dx = \int_a^\infty f(x) dx$ for all $a \in \mathbb{R}$

If $|f_n(x)| \leq 1 \forall n \in \mathbb{N}$ and $x \in \mathbb{R}$ then $\lim_{n \rightarrow \infty} \int_a^\infty f_n(x) dx = \int_a^\infty f(x) dx$ for all $a \in \mathbb{R}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

If $0 \leq f_n(x) \leq f(x) \forall n \in \mathbb{N}$ and $x \in \mathbb{R}$ then $\lim_{n \rightarrow \infty} \int_a^b f_n(x) dx = \int_a^b f(x) dx$ for all $a, b \in \mathbb{R}$

If $|f_n(x)| \leq e^{-x} \forall n \in \mathbb{N}$ and $x \in \mathbb{R}$ then $\lim_{n \rightarrow \infty} \int_a^\infty f_n(x) dx = \int_a^\infty f(x) dx$ for all $a \in \mathbb{R}$

4) Let f be integrable over \mathbb{R} . Which of the following imply $\int f = 0$ ae? **1 point**

$$\int_{\mathbb{R}} f dm = 0$$

$$\int_{\mathbb{R}} |f| dm = 0$$

$$\int_E f dm = 0 \text{ for all } E \text{ measurable.}$$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$\int_{\mathbb{R}} |f| dm = 0$$

$$\int_E f dm = 0 \text{ for all } E \text{ measurable.}$$

5) Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined as follows: **1 point**

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \text{ is irrational} \\ 0 & \text{otherwise} \end{cases}$$

Then which of the following are correct ?

f is measurable.

f is Lebesgue integrable.

f is Riemann integrable.

Video download

No, the answer is incorrect.
Score: 0

Accepted Answers:

f is measurable.

f is Lebesgue integrable.

6) Which of the following are correct?

1 point

$\chi_E, E \subset \mathbb{R}$ is measurable if and only if E is a Lebesgue set

$\chi_E, E \subset \mathbb{R}$ is Borel measurable if and only if E is a Borel set

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\chi_E, E \subset \mathbb{R}$ is measurable if and only if E is a Lebesgue set

$\chi_E, E \subset \mathbb{R}$ is Borel measurable if and only if E is a Borel set

7) Which of the following are correct?

1 point

If f is Riemann integrable on $[0, 1]$ and $g = f$ except at finitely many points then g is Riemann integrable

If f is continuous on $[0, 1]$ and $g = f$ except on a countable infinite set then g is Riemann integrable

If f is continuous on $[0, 1]$ and $g = f$ except on a countable infinite set then g is Lebesgue integrable

No, the answer is incorrect.

Score: 0

Accepted Answers:

If f is Riemann integrable on $[0, 1]$ and $g = f$ except at finitely many points then g is Riemann integrable

If f is continuous on $[0, 1]$ and $g = f$ except on a countable infinite set then g is Lebesgue integrable

8) Which of the following measures on $[0, 1]$ are σ -finite?

1 point

Counting measure on $[0, 1]$ where the σ -algebra is the power set

Counting measure on $[0, 1]$ where the σ -algebra is the Borel σ -algebra of $[0, 1]$

$\mu(A) =$ number of elements in $A \cap \mathbb{Q}$ where $A \in \mathcal{B}(\mathbb{R})$.

$\mu(A) =$ number of elements in $A \cap \mathbb{Z}$ where $A \in \mathcal{B}(\mathbb{R})$.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\mu(A) =$ number of elements in $A \cap \mathbb{Q}$ where $A \in \mathcal{B}(\mathbb{R})$.

$\mu(A) =$ number of elements in $A \cap \mathbb{Z}$ where $A \in \mathcal{B}(\mathbb{R})$.

9) Which of the following are positive linear functionals ?

1 point

On $C[0, 1]$ define $T(f) = \sum_{n=1}^{\infty} f(\frac{1}{n})$

On $C_c(\mathbb{R})$ define $T(f) = \sum_n f(n)$



On $C_c(0, \infty)$ define $T(f) = \sum_{n=1}^{\infty} f(\frac{1}{n})$

No, the answer is incorrect.

Score: 0

Accepted Answers:

On $C_c(\mathbb{R})$ define $T(f) = \sum_n f(n)$

On $C_c(0, \infty)$ define $T(f) = \sum_{n=1}^{\infty} f(\frac{1}{n})$

10) Let $X = \mathbb{R} \setminus \{0\}$ and consider $C_c(X)$ Which of the following are positive linear functionals **1 point** on $C_c(X)$?



$$T(f) = \sum_{k=1}^{\infty} f(\frac{1}{k}) + \sum_{k=1}^{\infty} f(-\frac{1}{k})$$



$$T(f) = \sum_{k=1}^{\infty} f(-k) + \sum_{k=1}^{\infty} f(k)$$



$$T(f) = \sum_{k=1}^{\infty} f(k + \frac{1}{k})$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$T(f) = \sum_{k=1}^{\infty} f(\frac{1}{k}) + \sum_{k=1}^{\infty} f(-\frac{1}{k})$$

$$T(f) = \sum_{k=1}^{\infty} f(-k) + \sum_{k=1}^{\infty} f(k)$$

$$T(f) = \sum_{k=1}^{\infty} f(k + \frac{1}{k})$$