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Unit 5 - Lebesgue measure and its properties

Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

Outer measure

Lebesgue measure and its properties

- Lebesgue sigma algebra (unit? unit=36&lesson=37)
- Lebesgue measure (unit? unit=36&lesson=38)
- Fine properties of measurable sets (unit? unit=36&lesson=39)

Week 4 Assessment

The due date for submitting this assignment has passed. **Due on 2020-02-26, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let A be a subset of $[0, 1]$ and m denote the Lebesgue measure on \mathbb{R} . Then which of the following are true? **1 point**

If A is closed then $m(A) > 0$

If A is open then $m(A) = m(\bar{A})$, where \bar{A} is the closure of A

If $m(\text{int}(A)) = m(\bar{A})$ then A is (Lebesgue) measurable, where $\text{int}(A)$ is the interior of A .

If $m(\text{int}(A)) = m(\bar{A})$ then A need not be measurable

No, the answer is incorrect.

Score: 0

Accepted Answers:

If $m(\text{int}(A)) = m(\bar{A})$ then A is (Lebesgue) measurable, where $\text{int}(A)$ is the interior of A .

2) Define an equivalence relation in $[1, 2]$ by $x \sim y$ if $x - y$ is rational. Consider the set N consisting of precisely one element from each equivalence class. Then **1 point**

N is uncountable

$[1, 2] \setminus N$ is uncountable

$m_*(N) = 0$

Invariance properties of Lebesgue measure (unit? unit=36&lesson=40)

Non measurable set (unit? unit=36&lesson=41)

Quiz : Week 4 Assessment (assessment? name=100)

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

L^p spaces and completeness

Product spaces and Fubini's theorem

Applications of Fubini's theorem and complex measures

Complex measures and Radon-Nikodym theorem

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue differentiation theorem

Weekly Feedback forms

Video download

$E \subset \mathbb{N}$ measurable implies $m_*(E) = 0$

No, the answer is incorrect.
 Score: 0

Accepted Answers:
 \mathbb{N} is uncountable
 $[1, 2] \setminus \mathbb{N}$ is uncountable
 $E \subset \mathbb{N}$ measurable implies $m_*(E) = 0$

3) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Then which of the following are necessarily true? **1 point**

- If f is measurable, then $\phi \circ f$ is measurable, for any continuous real valued function ϕ
- If f^2 is measurable, then f is measurable
- If f is differentiable, then f' is measurable
- If $g: \mathbb{R} \rightarrow \mathbb{R}$ be a measurable function such that $f = g$ a. e., then f is measurable

No, the answer is incorrect.
 Score: 0

Accepted Answers:
 If f is measurable, then $\phi \circ f$ is measurable, for any continuous real valued function ϕ
 If f is differentiable, then f' is measurable
 If $g: \mathbb{R} \rightarrow \mathbb{R}$ be a measurable function such that $f = g$ a. e., then f is measurable

4) Let $\{f_n\}$ be a sequence of real valued functions defined on $[0, 1]$ which converges pointwise to a continuous real valued function f on \mathbb{R} . Then which of the following are necessarily true? **1 point**

- $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$
- If $0 \leq f_n(x) \leq f(x) \forall n \in \mathbb{N}$ and $x \in [0, 1]$ then $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$
- If $|f_n(x)| \leq \frac{1}{\sqrt{x}} \forall n \in \mathbb{N}$ and $x \in [0, 1]$ then $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$
- If $|f_n(x)| \leq 1 \forall n \in \mathbb{N}$ and $x \in [0, 1]$ then $\lim_{n \rightarrow \infty} \int_K f_n(x) dx = \int_K f(x) dx$ for all measurable $K \subset [0, 1]$

No, the answer is incorrect.
 Score: 0

Accepted Answers:
 If $0 \leq f_n(x) \leq f(x) \forall n \in \mathbb{N}$ and $x \in [0, 1]$ then $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$
 If $|f_n(x)| \leq \frac{1}{\sqrt{x}} \forall n \in \mathbb{N}$ and $x \in [0, 1]$ then $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$
 If $|f_n(x)| \leq 1 \forall n \in \mathbb{N}$ and $x \in [0, 1]$ then $\lim_{n \rightarrow \infty} \int_K f_n(x) dx = \int_K f(x) dx$ for all measurable $K \subset [0, 1]$

5) Assume $\{f_n\}, \{g_n\}, f, g \in L^1(\mathbb{R}^n)$ be such that $f_n \rightarrow f$ and $g_n \rightarrow g$ pointwise a.e., then which of the following are true? **1 point**

$\int_{\mathbb{R}^n} (f_n + g_n) dm \rightarrow \int_{\mathbb{R}^n} (f + g) dm$

$|f_n| \leq |f|$ a.e., $|g_n| \leq |g|$ a.e. implies $\int_{\mathbb{R}^n} (f_n + g_n) dm \rightarrow \int_{\mathbb{R}^n} (f + g) dm$

$|f_n| \leq |g|$ a.e. implies $\int_{\mathbb{R}^n} f_n dm \rightarrow \int_{\mathbb{R}^n} f dm$

$|f_n| \leq g_n$ a.e. and $\int_{\mathbb{R}^n} g_n dm \rightarrow \int_{\mathbb{R}^n} g dm$ implies $\int_{\mathbb{R}^n} f_n dm \rightarrow \int_{\mathbb{R}^n} f dm$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$|f_n| \leq |f|$ a.e., $|g_n| \leq |g|$ a.e. implies $\int_{\mathbb{R}^n} (f_n + g_n) dm \rightarrow \int_{\mathbb{R}^n} (f + g) dm$

$|f_n| \leq |g|$ a.e. implies $\int_{\mathbb{R}^n} f_n dm \rightarrow \int_{\mathbb{R}^n} f dm$

$|f_n| \leq g_n$ a.e. and $\int_{\mathbb{R}^n} g_n dm \rightarrow \int_{\mathbb{R}^n} g dm$ implies $\int_{\mathbb{R}^n} f_n dm \rightarrow \int_{\mathbb{R}^n} f dm$

6) Consider the sequence of functions $f_n(x) = e^{-nx^2}$ on $[1, \infty)$. Which of the following are true? **1 point**

$\int_1^\infty f_n(x) dx \rightarrow 0$

$\sup_n \|f_n\|_1 < \infty$

f_n converges in $L^1[1, \infty)$

f_n does not converge in $L^p[1, \infty)$ for any $1 \leq p < \infty$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\int_1^\infty f_n(x) dx \rightarrow 0$

$\sup_n \|f_n\|_1 < \infty$

f_n converges in $L^1[1, \infty)$

7) Let $\{E_n\}$ be a sequence of measurable sets in \mathbb{R} such that $m(E_n) \rightarrow 0$ as $n \rightarrow \infty$ and $f \geq 0$ be measurable. Which of the following are true? **1 point**

$\int_{E_n} f(x) dx \rightarrow 0$ as $n \rightarrow \infty$

If $E_{n+1} \subset E_n$, $\forall n$ then $\int_{E_n} f(x) dx \rightarrow 0$ as $n \rightarrow \infty$

If f is bounded, then $\int_{E_n} f(x) dx \rightarrow 0$ as $n \rightarrow \infty$

If f is integrable and $E_{n+1} \subset E_n$, $\forall n$ then $\int_{E_n} f(x) dx \rightarrow 0$ as $n \rightarrow \infty$

No, the answer is incorrect.

Score: 0

Accepted Answers:

If f is bounded, then $\int_{E_n} f(x) dx \rightarrow 0$ as $n \rightarrow \infty$

If f is integrable and $E_{n+1} \subset E_n$, $\forall n$ then $\int_{E_n} f(x) dx \rightarrow 0$ as $n \rightarrow \infty$

8) Let $f, f_n: (X, \mathcal{F}, \mu) \rightarrow \mathbb{R}$ be measurable functions. Then which of the following are true? **1 point**

If $0 \leq f_n$ converges to f uniformly, then $\lim_{n \rightarrow \infty} \int_X f_n d\mu = \int_X f d\mu$

If $\mu(X)$ is finite and $|f_n(x)| \leq 1, \forall x \in X$, and f_n converges to f a.e then $\lim_{n \rightarrow \infty} \int_X g \circ f_n d\mu = \int_X g \circ f d\mu, \forall$

continuous function g on \mathbb{R}

If $\mu(X) < \infty$ and if f_n are bounded by one, f_n converges to f a.e. (μ), then $\lim_{n \rightarrow \infty} \int_X f_n d\mu = \int_X f d\mu$

If $f_1 \leq f_2 \leq \dots \leq f_n \leq f_{n+1} \leq \dots$, and f_n converges to f a.e then $\lim_{n \rightarrow \infty} \int_X f_n d\mu = \int_X f d\mu$

No, the answer is incorrect.

Score: 0

Accepted Answers:

If $\mu(X)$ is finite and $|f_n(x)| \leq 1, \forall x \in X$, and f_n converges to f a.e then $\lim_{n \rightarrow \infty} \int_X g \circ f_n d\mu = \int_X g \circ f d\mu, \forall$

continuous function g on \mathbb{R}

If $\mu(X) < \infty$ and if f_n are bounded by one, f_n converges to f a.e. (μ), then $\lim_{n \rightarrow \infty} \int_X f_n d\mu = \int_X f d\mu$

9) Let $\{f_n\}$ be a sequence of real valued measurable functions defined on \mathbb{R} which converges uniformly to a real valued function f on \mathbb{R} . Then which of the following are necessarily true? **1 point**

$$\lim_{n \rightarrow \infty} \int_{-\infty}^{\infty} f_n(x) dx = \int_{-\infty}^{\infty} f(x) dx$$

$$\lim_{n \rightarrow \infty} \int_1^{\infty} f_n(x) dx = \int_1^{\infty} f(x) dx$$

$$\lim_{n \rightarrow \infty} \int_1^2 f_n(x) dx = \int_1^2 f(x) dx$$

$$\lim_{n \rightarrow \infty} \int_K f_n(x) dx = \int_K f(x) dx \text{ for any compact set } K \subset \mathbb{R}$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\lim_{n \rightarrow \infty} \int_1^2 f_n(x) dx = \int_1^2 f(x) dx$$

$$\lim_{n \rightarrow \infty} \int_K f_n(x) dx = \int_K f(x) dx \text{ for any compact set } K \subset \mathbb{R}$$

10) Let $A \in L(\mathbb{R}^n)$. Then which of the following are correct ? **1 point**

$\delta A \in L(\mathbb{R}^n)$ for all $\delta > 0$

$A + x \in L(\mathbb{R}^n)$ for all $x \in \mathbb{R}^n$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$\delta A \in L(\mathbb{R}^n)$ for all $\delta > 0$

$A + x \in L(\mathbb{R}^n)$ for all $x \in \mathbb{R}^n$

