Unit 5 - Lebesgue measure and its properties

Week 4 Assessment

The due date for submitting this assignment has passed. **Due on 2020-02-26, 23:59 IST.**
As per our records you have not submitted this assignment.

1) Let A be a subset of $[0, 1]$ and m denote the Lebesgue measure on $\mathbb{R}$. Then which of the following are true? ***1 point***

- If A is closed then $m(A) > 0$
- If A is open then $m(A) = m(\bar{A})$, where $\bar{A}$ is the closure of A
- If $m(int(A)) = m(\bar{A})$ then A is (Lebesgue) measurable, where $int(A)$ is the interior of A.
- If $m(int(A)) = m(\bar{A})$ then A need not be measurable

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $m(int(A)) = m(\bar{A})$ then A is (Lebesgue) measurable, where $int(A)$ is the interior of A.

2) Define an equivalence relation in $[1, 2]$ by $x \sim y$ if $x - y$ is rational. Consider the set N consisting of precisely one element from each equivalence class. Then ***1 point***

- N is uncountable
- $[1, 2] \setminus N$ is uncountable
- $m_*(N) = 0$

$E \subset N$ measurable implies $m_*(E) = 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
N is uncountable
$[1, 2] \setminus N$ is uncountable
$E \subset N$ measurable implies $m_*(E) = 0$

3) Let $f: \mathbb{R} \to \mathbb{R}$ be a function. Then which of the following are necessarily true? ***1 point***

If f is measurable, then $\phi \circ f$ is measurable, for any continuous real valued function ϕ

If f^2 is measurable, then f is measurable

If f is differentiable, then f' is measurable

If $g: \mathbb{R} \to \mathbb{R}$ be a measurable function such that $f = g \ a.e$, then f is measurable

No, the answer is incorrect.
Score: 0

Accepted Answers:
If f is measurable, then $\phi \circ f$ is measurable, for any continuous real valued function ϕ
If f is differentiable, then f' is measurable
If $g: R \to R$ be a measurable function such that $f = g \ a.e$, then f is measurable

4) Let $\{f_n\}$ be a sequence of real valued functions defined on $[0, 1]$ which converges pointwise to a continuous real valued function f on $\mathbb{R}$. Then which of the following are necessarily true ? ***1 point***

$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

If $0 \le f_n(x) \le f(x) \ \forall n \in \mathbb{N} \ and \ x \in [0, 1]$ then $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

If $|f_n(x)| \le \frac{1}{\sqrt{x}} \ \forall n \in \mathbb{N} \ and \ x \in [0, 1]$ then $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$

If $|f_n(x)| \le 1 \ \forall n \in \mathbb{N} \ and \ x \in [0, 1]$ then $\lim_{n\to\infty} \int_K f_n(x)dx = \int_K f(x)dx$ for all measurable $K \subset [0, 1]$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $0 \le f_n(x) \le f(x) \ \forall n \in N \ and \ x \in [0, 1]$ then $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$
If $|f_n(x)| \le \frac{1}{\sqrt{x}} \ \forall n \in N \ and \ x \in [0, 1]$ then $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$
If $|f_n(x)| \le 1 \ \forall n \in N \ and \ x \in [0, 1]$ then $\lim_{n\to\infty} \int_K f_n(x)dx = \int_K f(x)dx$ for all measurable $K \subset [0, 1]$

5) Assume $\{f_n\}, \{g_n\}, f, g \in L^1(\mathbb{R}^n)$ be such that $f_n \longrightarrow f$ and $g_n \longrightarrow g$ pointwise a.e., then which of the following are true? ***1 point***

$\int_{\mathbb{R}^n}(f_n + g_n)\, dm \longrightarrow \int_{\mathbb{R}^n}(f + g)\, dm$

- [ ] $|f_n| \leq |f|$ a.e., $|g_n| \leq |g|$ a.e. implies $\int_{\mathbb{R}^n}(f_n + g_n)\, dm \longrightarrow \int_{\mathbb{R}^n}(f + g)\, dm$
- [ ] $|f_n| \leq |g|$ a.e. implies $\int_{\mathbb{R}^n} f_n\, dm \longrightarrow \int_{\mathbb{R}^n} f\, dm$
- [ ] $|f_n| \leq g_n$ a.e. and $\int_{\mathbb{R}^n} g_n\, dm \longrightarrow \int_{\mathbb{R}^n} g\, dm$ implies $\int_{\mathbb{R}^n} f_n\, dm \longrightarrow \int_{\mathbb{R}^n} f\, dm$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$|f_n| \leq |f|$ a.e., $|g_n| \leq |g|$ a.e. implies $\int_{R^n}(f_n + g_n)\, dm \longrightarrow \int_{R^n}(f + g)\, dm$
$|f_n| \leq |g|$ a.e. implies $\int_{R^n} f_n\, dm \longrightarrow \int_{R^n} f\, dm$
$|f_n| \leq g_n$ a.e. and $\int_{R^n} g_n\, dm \longrightarrow \int_{R^n} g\, dm$ implies $\int_{R^n} f_n\, dm \longrightarrow \int_{R^n} f\, dm$

6) Consider the sequence of functions $f_n(x) = e^{-nx^2}$ on $[1, \infty)$. Which of the following are true? ***1 point***

- [ ] $\int_1^\infty f_n(x)dx \to 0$
- [ ] $\sup_n \|f_n\|_1 < \infty$
- [ ] f_n converges in $L^1[1, \infty)$
- [ ] f_n does not converge in $L^p[1, \infty)$ for any $1 \leq p \leq \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_1^\infty f_n(x)dx \to 0$
$\sup_n \|f_n\|_1 < \infty$
f_n converges in $L^1[1, \infty)$

7) Let $\{E_n\}$ be a sequence of measurable sets in $\mathbb{R}$ such that $m(E_n) \to 0$ as $n \to \infty$ and $f \geq 0$ be measurable. Which of the following are true ? ***1 point***

- [ ] $\int_{E_n} f(x)dx \to 0$ as $n \to \infty$
- [ ] If $E_{n+1} \subset E_n,\ \forall n$ then $\int_{E_n} f(x)dx \to 0$ as $n \to \infty$
- [ ] If f is bounded, then $\int_{E_n} f(x)dx \to 0$ as $n \to \infty$
- [ ] If f is integrable and $E_{n+1} \subset E_n,\ \forall n$ then $\int_{E_n} f(x)dx \to 0$ as $n \to \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If f is bounded, then $\int_{E_n} f(x)dx \to 0$ as $n \to \infty$
If f is integrable and $E_{n+1} \subset E_n,\ \forall n$ then $\int_{E_n} f(x)dx \to 0$ as $n \to \infty$

8) Let $f, f_n : (X, \mathrm{F}, \mu) \to \mathbb{R}$ be measurable functions. Then which of the following are true? ***1 point***

- [ ] If $0 \leq f_n$ converges to f uniformly, then $\lim\limits_{n \to \infty} \int_X f_n d\mu = \int_X f d\mu$

- [ ] If $\mu(X)$ is finite and $|f_n(x)| \leq 1$, $\forall x \in X$, and f_n converges to f ae then $\lim_{n\to\infty} \int_X g \circ f_n d\mu = \int_X g \circ f d\mu$, $\forall$ continuous function g on $\mathbb{R}$
- [ ] If $\mu(X) < \infty$ and if f_n are bounded by one, f_n converges to f a.e.(μ), then $\lim_{n\to\infty} \int_X f_n d\mu = \int_X f d\mu$
- [ ] If $f_1 \leq f_2 \leq .... \leq f_n \leq f_{n+1} \leq ....$, and f_n converges to f ae then $\lim_{n\to\infty} \int_X f_n d\mu = \int_X f d\mu$

No, the answer is incorrect.
Score: 0

Accepted Answers:

If $\mu(X)$ is finite and $|f_n(x)| \leq 1$, $\forall x \in X$, and f_n converges to f ae then $\lim_{n\to\infty} \int_X g \circ f_n d\mu = \int_X g \circ f d\mu$, $\forall$ continuous function g on R

If $\mu(X) < \infty$ and if f_n are bounded by one, f_n converges to f a.e.(μ), then $\lim_{n\to\infty} \int_X f_n d\mu = \int_X f d\mu$

9) Let $\{f_n\}$ be a sequence of real valued measurable functions defined on $\mathbb{R}$ which converges uniformly to a real valued function f on $\mathbb{R}$. Then which of the following are necessarily true? **1 point**

- [ ] $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)dx = \int_{-\infty}^{\infty} f(x)dx$
- [ ] $\lim_{n\to\infty} \int_{1}^{\infty} f_n(x)dx = \int_{1}^{\infty} f(x)dx$
- [ ] $\lim_{n\to\infty} \int_{1}^{2} f_n(x)dx = \int_{1}^{2} f(x)dx$
- [ ] $\lim_{n\to\infty} \int_K f_n(x)dx = \int_K f(x)dx$ for any compact set $K \subset \mathbb{R}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\lim_{n\to\infty} \int_{1}^{2} f_n(x)dx = \int_{1}^{2} f(x)dx$

$\lim_{n\to\infty} \int_K f_n(x)dx = \int_K f(x)dx$ *for any compact set* $K \subset R$

10) Let $A \in \mathrm{L}(\mathbb{R}^n)$. Then which of the following are correct ? **1 point**

- [ ] $\delta A \in \mathrm{L}(\mathbb{R}^n)$ for all $\delta > 0$
- [ ] $A + x \in \mathrm{L}(\mathbb{R}^n)$ for all $x \in \mathbb{R}^n$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\delta A \in L(R^n)$ *for all* $\delta > 0$

$A + x \in L(R^n)$ *for all* $x \in R^n$