Unit 3 - Integration and convergence theorems

Week 2 Assesment

The due date for submitting this assignment has passed. **Due on 2020-02-12, 23:59 IST.**
As per our records you have not submitted this assignment.

1) $X = \mathbb{R}, \mathcal{F} = \{A \subset \mathbb{R} \mid A \text{ is countable or } A^c \text{ is countable}\}$. Let ***1 point***

$$\mu(A) = \begin{cases} 1 & \text{if } A^c \text{ is countable} \\ 0 & \text{if } A \text{ is countable} \end{cases}$$

Let $f : (X, \mathcal{F}, \mu) \to \mathbb{R}$ be measurable. Which of the following are always true?

- [ ] f is constant ae.(μ)
- [ ] f is a non-constant continuous function
- [ ] f is a non constant polynomial
- [ ] $f(x) = 0 \ \forall \ x \in \mathbb{R}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is constant ae.(μ)

2) Let $(X, \mathcal{F}, \mu)$ be a measure space and let E be a proper subset of X, $E \in \mathcal{F}$ and $0 < \mu(E) < \mu(X)$. ***1 point***

$$f_n = \begin{cases} \chi_E & \text{if } n \text{ is odd} \\ 1 - \chi_E & \text{if } n \text{ is even} \end{cases}$$

- [ ] $\int_X \liminf f_n d\mu \ < \ \liminf \int_X f_n d\mu$
- [ ] $\int_X \liminf f_n d\mu \ = \ \liminf \int_X f_n d\mu$
- [ ] $\int_X \limsup f_n d\mu \ < \ \limsup \int_X f_n d\mu$
- [ ] $\int_X \limsup f_n d\mu \ = \ \limsup \int_X f_n d\mu$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_X \liminf f_n d\mu \ < \ \liminf \int_X f_n d\mu$

3) Consider the space $\mathbb{N}$ with power set sigma algebra and counting measure μ. Let $f : \mathbb{N} \to \mathbb{R}$ be measurable and zero ae(μ). Which of the following are always true? ***1 point***

- [ ] $f(n) = 0 \ \forall n \in \mathbb{N}$
- [ ] $f(1) \neq 0, f(n) = 0 \ \forall n > 1$
- [ ] $f(n) = 0$ except for finitely many $n \in \mathbb{N}$
- [ ] $f(n) = 0$ only when n is a prime number

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f(n) = 0 \ \forall n \in \mathbb{N}$

4) Let X be a non empty set and $A \subset X$ be a proper subset. Consider the sigma algebra $\mathcal{F} = \{\phi, X, A, A^c\}$. Let $f : (X, \mathcal{F}) \to \mathbb{R}$ be measurable. Which of the following are always true? ***1 point***

- [ ] $f = \alpha\chi_A + \beta\chi_{A^c}$ for some $\alpha, \beta \in \mathbb{R}$
- [ ] $f = \alpha\chi_A$ for some $\alpha \in \mathbb{R}$
- [ ] $f = \beta\chi_{A^c}$ for some $\beta \in \mathbb{R}$
- [ ] $f \equiv 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f = \alpha\chi_A + \beta\chi_{A^c}$ for some $\alpha, \beta \in \mathbb{R}$

5) Let $(X, \mathcal{F}, \mu)$ be a measure space. Let $A_n \in \mathcal{F}$ be such that $A_1 \subset A_2 \subset A_3 \subset \cdots$ and $\cup_{n=1}^{\infty} A_n = X$. Let $f : (X, \mathcal{F}, \mu) \to \mathbb{R}$ be a measurable function and $f(x) \geq 0$ ae(μ). Which of the following are always true? ***1 point***

- [ ] $f\chi_{A_n} \uparrow f$ ae
- [ ] $\int_{A_n} f d\mu \uparrow \int_X f d\mu$
- [ ]

differentiation theorem

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$\int_{A_n} f d\mu \downarrow \int_X f d\mu$

$f \chi_{A_n} \downarrow f$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f \chi_{A_n} \uparrow f$ *ae*
$\int_{A_n} f d\mu \uparrow \int_X f d\mu$

6) Let $(X, \mathcal{F}, \mu)$ be a measure space and μ be a probability measure, that is $\mu(X) = 1$. Let $\{A_n\}$ be a sequence in $\mathcal{F}$. Which of the following are true? ***1 point***

$\mu(\limsup A_n) \geq \limsup \mu(A_n)$

$\mu(\limsup A_n) \leq \limsup \mu(A_n)$

$\mu(\liminf A_n) \geq \liminf \mu(A_n)$

$\mu(\liminf A_n) \leq \liminf \mu(A_n)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mu(\limsup A_n) \geq \limsup \mu(A_n)$
$\mu(\liminf A_n) \leq \liminf \mu(A_n)$

7) Let $(X, \mathcal{F}, \mu)$ be a measure space. Let $f_n : X \to \mathbb{R}$ be measurable, $A = \{x \in X \mid \lim f_n(x)$ exists $\}$. Then, ***1 point***

$A \in \mathcal{F}$

$A = \phi$

$A = X$

$A^c \in \mathcal{F}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$A \in \mathcal{F}$
$A^c \in \mathcal{F}$

8) Let $(X, \mathcal{F}, \mu)$ be a measure space and $A_n \in \mathcal{F}$. Suppose $\mu(X) = 1, \sum \mu(A_n) < \infty$. Then which of the following are true? ***1 point***

$\mu(\limsup A_n) = 0$

$\mu(\liminf A_n) = 0$

$\mu(\limsup A_n) = 1$

$\mu(\liminf A_n) = 1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mu(\limsup A_n) = 0$
$\mu(\liminf A_n) = 0$

9) $(X, \mathcal{F}, \mu)$ be a probability measure space. Suppose $f_n : X \to \mathbb{R}$ are measurable and $|f_n| \leq 1 \ ae(\mu) \ \forall n$. Suppose $f_n \to 1 \ ae(\mu)$. Then, **_1 point_**

- [ ] $\int_X f_n d\mu \to 1$
- [ ] $\int_X f_n d\mu \to 0$
- [ ] $\int_X f_n d\mu \to \infty$
- [ ] $\int_X f_n d\mu$ does not converge

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\int_X f_n d\mu \to 1$

10) Let $(X, \mathcal{F}, \mu)$ be a measure space and $A_n \in \mathcal{F}$ such that $\mu(A_n) = 0 \ \forall n$. Which of the following are true? **_1 point_**

- [ ] $\mu(\cup_{n=1}^{\infty} A_n) = 0$
- [ ] $\mu(\cup_{n=1}^{\infty} A_n) = 1$
- [ ] $\mu(\cup_{n=1}^{\infty} A_n) = \infty$
- [ ] $\mu(\cup_{n=1}^{\infty} A_n) > 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\mu(\cup_{n=1}^{\infty} A_n) = 0$