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[Announcements \(announcements\)](#)
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[Progress \(student/home\)](#) [Mentor \(student/mentor\)](#)

Unit 3 - Integration and convergence theorems

Course outline

How does an NPTEL online course work?

Sigma algebras, Measures and Integration

Integration and convergence theorems

- Some properties of integrals of positive simple functions (unit? unit=24&lesson=25)
- Monotone convergence theorem and Fatou's lemma (unit? unit=24&lesson=26)
- Integration of complex valued measurable functions (unit? unit=24&lesson=27)

Week 2 Assessment

The due date for submitting this assignment has passed. **Due on 2020-02-12, 23:59 IST.**
As per our records you have not submitted this assignment.

1) $X = \mathbb{R}$, $\mathcal{F} = \{A \subset \mathbb{R} \mid A \text{ is countable or } A^c \text{ is countable}\}$. Let

1 point

$$\mu(A) = \begin{cases} 1 & \text{if } A^c \text{ is countable} \\ 0 & \text{if } A \text{ is countable} \end{cases}$$

Let $f : (X, \mathcal{F}, \mu) \rightarrow \mathbb{R}$ be measurable. Which of the following are always true?

- f is constant ae. (μ)
- f is a non-constant continuous function
- f is a non constant polynomial
- $f(x) = 0 \forall x \in \mathbb{R}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

f is constant ae. (μ)

2) Let (X, \mathcal{F}, μ) be a measure space and let E be a proper subset of X , $E \in \mathcal{F}$ and $0 < \mu(E) < \mu(X)$.

1 point

$$f_n = \begin{cases} \chi_E & \text{if } n \text{ is odd} \\ 1 - \chi_E & \text{if } n \text{ is even} \end{cases}$$

Dominated convergence theorem (unit? unit=24&lesson=28)

Sets of measure zero and completion (unit? unit=24&lesson=29)

Quiz : Week 2 Assessment (assessment? name=94)

Outer measure

Lebesgue measure and its properties

Lebesgue measure and positive Borel measures on locally compact spaces

Lebesgue measure and invariance properties

L^p spaces and completeness

Product spaces and Fubini's theorem

Applications of Fubini's theorem and complex measures

Complex measures and Radon-Nikodym theorem

Radon-Nikodym theorem and applications

Riesz representation theorem and Lebesgue

$\int_X \liminf f_n d\mu < \liminf \int_X f_n d\mu$

$\int_X \liminf f_n d\mu = \liminf \int_X f_n d\mu$

$\int_X \limsup f_n d\mu < \limsup \int_X f_n d\mu$

$\int_X \limsup f_n d\mu = \limsup \int_X f_n d\mu$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\int_X \liminf f_n d\mu < \liminf \int_X f_n d\mu$

3) Consider the space \mathbb{N} with power set sigma algebra and counting measure μ . Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be measurable and zero ae(μ). Which of the following are always true? 1 point

$f(n) = 0 \forall n \in \mathbb{N}$

$f(1) \neq 0, f(n) = 0 \forall n > 1$

$f(n) = 0$ except for finitely many $n \in \mathbb{N}$

$f(n) = 0$ only when n is a prime number

No, the answer is incorrect.
Score: 0

Accepted Answers:

$f(n) = 0 \forall n \in \mathbb{N}$

4) Let X be a non empty set and $A \subset X$ be a proper subset. Consider the sigma algebra $\mathcal{F} = \{\emptyset, X, A, A^c\}$. Let $f : (X, \mathcal{F}) \rightarrow \mathbb{R}$ be measurable. Which of the following are always true? 1 point

$f = \alpha \chi_A + \beta \chi_{A^c}$ for some $\alpha, \beta \in \mathbb{R}$

$f = \alpha \chi_A$ for some $\alpha \in \mathbb{R}$

$f = \beta \chi_{A^c}$ for some $\beta \in \mathbb{R}$

$f \equiv 0$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$f = \alpha \chi_A + \beta \chi_{A^c}$ for some $\alpha, \beta \in \mathbb{R}$

5) Let (X, \mathcal{F}, μ) be a measure space. Let $A_n \in \mathcal{F}$ be such that $A_1 \subset A_2 \subset A_3 \subset \dots$ and $\bigcup_{n=1}^{\infty} A_n = X$. Let $f : (X, \mathcal{F}, \mu) \rightarrow \mathbb{R}$ be a measurable function and $f(x) \geq 0$ ae(μ). Which of the following are always true? 1 point

$f \chi_{A_n} \uparrow f$ ae

$\int_{A_n} f d\mu \uparrow \int_X f d\mu$

differentiation
theorem

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$$\int_{A_n} f d\mu \downarrow \int_X f d\mu$$

$$f \chi_{A_n} \downarrow f$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$f \chi_{A_n} \uparrow f \text{ a.e.}$$

$$\int_{A_n} f d\mu \uparrow \int_X f d\mu$$

6) Let (X, \mathcal{F}, μ) be a measure space and μ be a probability measure, that is $\mu(X) = 1$. Let $\{A_n\}$ be a sequence in \mathcal{F} . Which of the following are true? **1 point**

$$\mu(\limsup A_n) \geq \limsup \mu(A_n)$$

$$\mu(\limsup A_n) \leq \limsup \mu(A_n)$$

$$\mu(\liminf A_n) \geq \liminf \mu(A_n)$$

$$\mu(\liminf A_n) \leq \liminf \mu(A_n)$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\mu(\limsup A_n) \geq \limsup \mu(A_n)$$

$$\mu(\liminf A_n) \leq \liminf \mu(A_n)$$

7) Let (X, \mathcal{F}, μ) be a measure space. Let $f_n : X \rightarrow \mathbb{R}$ be measurable, $A = \{x \in X \mid \lim f_n(x) \text{ exists}\}$. Then, **1 point**

$$A \in \mathcal{F}$$

$$A = \phi$$

$$A = X$$

$$A^c \in \mathcal{F}$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$A \in \mathcal{F}$$

$$A^c \in \mathcal{F}$$

8) Let (X, \mathcal{F}, μ) be a measure space and $A_n \in \mathcal{F}$. Suppose $\mu(X) = 1, \sum \mu(A_n) < \infty$. Then which of the following are true? **1 point**

$$\mu(\limsup A_n) = 0$$

$$\mu(\liminf A_n) = 0$$

$$\mu(\limsup A_n) = 1$$

$$\mu(\liminf A_n) = 1$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\mu(\limsup A_n) = 0$$

$$\mu(\liminf A_n) = 0$$

9) (X, \mathcal{F}, μ) be a probability measure space. Suppose $f_n : X \rightarrow \mathbb{R}$ are measurable and $|f_n| \leq 1$ $ae(\mu) \forall n$. Suppose $f_n \rightarrow 1$ $ae(\mu)$. Then, **1 point**

$$\int_X f_n d\mu \rightarrow 1$$

$$\int_X f_n d\mu \rightarrow 0$$

$$\int_X f_n d\mu \rightarrow \infty$$

$$\int_X f_n d\mu \text{ does not converge}$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\int_X f_n d\mu \rightarrow 1$$

10) Let (X, \mathcal{F}, μ) be a measure space and $A_n \in \mathcal{F}$ such that $\mu(A_n) = 0 \forall n$. Which of the following are true? **1 point**

$$\mu(\bigcup_{n=1}^{\infty} A_n) = 0$$

$$\mu(\bigcup_{n=1}^{\infty} A_n) = 1$$

$$\mu(\bigcup_{n=1}^{\infty} A_n) = \infty$$

$$\mu(\bigcup_{n=1}^{\infty} A_n) > 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\mu(\bigcup_{n=1}^{\infty} A_n) = 0$$

