Unit 3 - Smooth manifolds and Smooth maps

Assessment 2

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-02-12, 23:59 IST.

Each question may have more than one correct option.

1) Consider $\mathbb{R}$ with the usual topology and the atlas $\{(\mathbb{R}, \varphi_n)\}_n$, where, $n \in \mathbb{Z}, n \geq 0$ and $\varphi_n(x) = x^{2n+1}$. Then, *1 point*

- $\mathbb{R}$ with the given atlas is neither a topological manifold nor a smooth manifold.
- $\mathbb{R}$ with the given atlas is a topological manifold but not a smooth manifold.
- $\mathbb{R}$ with the given atlas is a topological manifold and a smooth manifold.
- $\mathbb{R}$ with the given atlas is a smooth manifold but not a topological manifold.

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\mathbb{R}$ with the given atlas is a topological manifold but not a smooth manifold.

2) Let $f : U \to V$ be a smooth map, where U and V are open subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively. Then, *1 point*

- If f is surjective, then f is a submersion.
- If f is injective and Df has constant rank, then f is an immersion.
- If f is injective, then f is an immersion.
- If f is an immersion, then f is injective.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If f is injective and Df has constant rank, then f is an immersion.

3) Let $f : U \to V$ be a smooth map where U and V are open subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. $f(U)$ is necessarily open if *1 point*

- $m \geq n$, Df has constant rank k, where $m \geq k \geq n$
- $m \geq n$, Df has constant rank k, where $k < n$
- $n \geq m$, Df has constant rank k, where $n \geq k \geq m$
- $n \geq m$, Df has constant rank k where $k < m$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$n \geq m$, Df has constant rank k, where $n \geq k \geq m$

4) Which of the following subsets of $\mathbb{R}^2$ are 1-dimensional topological manifolds? *1 point*

- $\{(x, y) \in \mathbb{R}^2 : |x| = |y|\}$
- $\{(x, y) \in \mathbb{R}^2 : |x| = |y|, y \geq 0\}$
- $\{(x, y) \in \mathbb{R}^2 : |x| = |y|, y > 0\}$
- $\{(x, y) \in \mathbb{R}^2 : |x| = |y|, y \neq 0\}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\{(x, y) \in \mathbb{R}^2 : |x| = |y|, y \geq 0\}$
$\{(x, y) \in \mathbb{R}^2 : |x| = |y|, y > 0\}$
$\{(x, y) \in \mathbb{R}^2 : |x| = |y|, y \neq 0\}$

5) Which of the following maps from $\mathbb{R}$ to is a diffeomorphism? *1 point*
$\mathbb{R}$

- $f(t) = t^5$
- any bijective immersion
- any bijective submersion
- $f(t) = e^t$

No, the answer is incorrect.
Score: 0
Accepted Answers:
any bijective immersion
any bijective submersion

6) Let U and V be open subsets of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. Which of the following statements are true? *1 point*

- If $n = m$ and $f : U \to V$ is a submersion, then f is an open map.
- If $n = m$ and $f : U \to V$ is an immersion, then f is an open map.
- If $n > m$ and $f : U \to V$ is a submersion, then f is an open map.
- If $n < m$ and $f : U \to V$ is an immersion, then f is an open map.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $n = m$ and $f : U \to V$ is a submersion, then f is an open map.
If $n = m$ and $f : U \to V$ is an immersion, then f is an open map.
If $n > m$ and $f : U \to V$ is a submersion, then f is an open map.

7) Let $f : (-1, 1) \to \mathbb{R}$ be defined by $f(x) = tan(\frac{\pi x}{2})$. Then which of the following statements are false? *1 point*

- Support(f) is closed in $(-1, 1)$.
- Support(f)=$[-1, 1]$.
- Support(f)=$(-1, 1)$.
- Support(f) is compact.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Support(f) is closed in $(-1, 1)$.
Support(f)=$(-1, 1)$.

8) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a smooth map and $U, V \subset \mathbb{R}^n$ be such that $U = D(0, r_1)$ and $V = D(0, r_2)$ where $r_1 < r_2$. Which of the following statements are true? *1 point*

- There exist a smooth map $g : \mathbb{R}^n \to \mathbb{R}$ such that $f(t)g(t) = 1$ for all $t \in U$ and $f(t)g(t) = f(t)$ for all $t \in \mathbb{R}^n \setminus V$.
- There exist a smooth map $g : \mathbb{R}^n \to \mathbb{R}$ such that $f(t)g(t) = 1$ for all $t \in U$ and $f(t)g(t) = 0$ for all $t \in \mathbb{R}^n \setminus V$.
- There exist a smooth map $g : \mathbb{R}^n \to \mathbb{R}$ such that $f(t)g(t) = 0$ for all $t \in U$ and $f(t)g(t) = f(t)$ for all $t \in \mathbb{R}^n \setminus V$.
- There exist a smooth map $g : \mathbb{R}^n \to \mathbb{R}$ such that $f(t)g(t) = f(t)$ for all $t \in U$ and $f(t)g(t) = 0$ for all $t \in \mathbb{R}^n \setminus V$.

No, the answer is incorrect.
Score: 0
Accepted Answers:
There exist a smooth map $g : \mathbb{R}^n \to \mathbb{R}$ such that $f(t)g(t) = 0$ for all $t \in U$ and $f(t)g(t) = f(t)$ for all $t \in \mathbb{R}^n \setminus V$.
There exist a smooth map $g : \mathbb{R}^n \to \mathbb{R}$ such that $f(t)g(t) = f(t)$ for all $t \in U$ and $f(t)g(t) = 0$ for all $t \in \mathbb{R}^n \setminus V$.

9) Let M be a smooth manifold of dimension n. Which of the following statements are true? *1 point*

- Every connected component of M is a manifold.
- Every open subset $U \subset M$ is a manifold.
- A subset $S \subset M$ is a manifold if and only if it is open in M.
- For any $m \leq n$, there exist a subset $S \subset M$ which is a manifold of dimension m.

No, the answer is incorrect.
Score: 0
Accepted Answers:
Every connected component of M is a manifold.
Every open subset $U \subset M$ is a manifold.
For any $m \leq n$, there exist a subset $S \subset M$ which is a manifold of dimension m.

10) Which of the following statements are true? *1 point*

- If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$ are two immersions, then $g \circ f$ is also an immersion.
- If $f : \mathbb{R}^n \to \mathbb{R}^m$ is an immersion and $g : \mathbb{R}^m \to \mathbb{R}^k$ is a submersion, then $g \circ f$ is also an immersion.
- If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$ are two submersions, then $g \circ f$ is also a submersion.
- If $f : \mathbb{R}^n \to \mathbb{R}^m$ is an immersion and $g : \mathbb{R}^m \to \mathbb{R}^k$ is a submersion, then $g \circ f$ is also a submersion.

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$ are two immersions, then $g \circ f$ is also an immersion.
If $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^k$ are two submersions, then $g \circ f$ is also a submersion.

You were allowed to submit this assignment only once.