Unit 2 - Review of linear algebra and multivariable calculus

Course outline

- How does an NPTEL online course work?
- Review of linear algebra and multivariable calculus
  - Basic linear algebra
  - Multivariable calculus - 1
  - Multivariable calculus - 2
  - The derivative map
  - Inverse Function Theorem
  - Constant Rank Theorem
  - Smooth functions with compact support
  - Quiz : Assessment 1
- Smooth manifolds and Smooth maps
- Tangent spaces
- Tangent Spaces (Contd.)
- Submanifolds
- Vector Fields and Lie Groups
- Flows of vector fields
- Lie Brackets and Lie Algebras
- Differential forms and Symmetric tensors.
- Alternating tensors
- Differential Forms
- Orientation on manifolds.
- Weekly Feedback forms
- Video download

Assessment 1

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-02-12, 23:59 IST.

Each question may have more than one correct option.

1) Consider the map, $f : M(n,\mathbb{R}) \to M(n,\mathbb{R})$ which satisfies $[f(A)]_{ij} = ([A]_{ij})^2$, where $[A]_{ij}$ denotes the ij^{th} entry of the matrix A. Which of the following statements are true? ***1 point***

- The directional derivative of f along the direction H is $AH + HA$
- The derivative of f is represented by an $n \times n$ diagonal matrix with respect to the standard basis on $M(n,\mathbb{R})$
- The derivative of f is represented by an $n^2 \times n^2$ diagonal matrix with respect to the standard basis on $M(n,\mathbb{R})$
- The directional derivative of f along the direction H is the matrix whose ij^{th} entry is $2 \cdot [A]_{ij} \cdot [H]_{ij}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
The derivative of f is represented by an $n^2 \times n^2$ diagonal matrix with respect to the standard basis on $M(n,\mathbb{R})$
The directional derivative of f along the direction H is the matrix whose ij^{th} entry is $2 \cdot [A]_{ij} \cdot [H]_{ij}$

2) Which of the following is true about $S(n,\mathbb{R})$, the set of symmetric $n \times n$ matrices? ***1 point***

- $S(n,\mathbb{R})$ is not a subspace of $M(n,\mathbb{R})$
- $S(n,\mathbb{R})$ is the kernel of a surjective linear map from $M(n,\mathbb{R})$ to $\mathbb{R}^{\frac{n(n-1)}{2}}$
- $S(n,\mathbb{R})$ is the kernel of a surjective linear map from $M(n,\mathbb{R})$ to $\mathbb{R}^{\frac{n(n+1)}{2}}$
- None of the other statements are true

No, the answer is incorrect.
Score: 0

Accepted Answers:
$S(n,\mathbb{R})$ is the kernel of a surjective linear map from $M(n,\mathbb{R})$ to $\mathbb{R}^{\frac{n(n-1)}{2}}$

3) Let $A \in M_n(\mathbb{R})$. Let T be the linear map from $\mathbb{R}^n$ to itself represented by A with respect to the standard basis on $\mathbb{R}^n$. Consider the map, $S = T \circ T : \mathbb{R}^n \to \mathbb{R}^n$. Then, ***1 point***

- S is not a linear map.
- S is a linear map and it's derivative is the linear map represented by $2A$ in the standard basis on $\mathbb{R}^n$
- S is a linear map and it's derivative is the linear map represented by A^2 in the standard basis on $\mathbb{R}^n$
- S is a linear map but it's derivative is not a linear map.

No, the answer is incorrect.
Score: 0

Accepted Answers:
S is a linear map and it's derivative is the linear map represented by A^2 in the standard basis on $\mathbb{R}^n$

4) Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous map such that all its partial derivatives exist on $\mathbb{R}^n$ and are continuous. If f is invertible, what can be said about the Jacobian matrix of f ? ***1 point***

- The Jacobian matrix must have determinant 1 or -1 at every point.
- The Jacobian matrix must have non-zero determinant at every point.
- The Jacobian matrix has to be symmetric.
- None of the other options are correct.

No, the answer is incorrect.
Score: 0

Accepted Answers:
None of the other options are correct.

5) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \frac{xy}{x^2+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Which of the following statements are correct? ***1 point***

- $\frac{\delta f}{\delta x}$ and $\frac{\delta f}{\delta y}$ don't exist at $(0,0)$.
- $\frac{\delta f}{\delta x}$ and $\frac{\delta f}{\delta y}$ exist but f is not differentiable at $(0,0)$.
- f is differentiable at $(0,0)$.
- f is continuous at $(0,0)$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{\delta f}{\delta x}$ and $\frac{\delta f}{\delta y}$ exist but f is not differentiable at $(0,0)$.

6) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function and $a(t) = (t + t^3, 0)$, $b(t) = (sin(t), cos(t))$ and $c(t) = (cos(t), sin(t))$. Which of the following statements are true? ***1 point***

- $\frac{d(f \circ a)}{dt}(0) > \frac{d(f \circ c)}{dt}(0)$
- $\frac{d(f \circ a)}{dt}(0) < \frac{d(f \circ b)}{dt}(0)$
- $\frac{d(f \circ a)}{dt}(0) = \frac{d(f \circ b)}{dt}(0)$
- The relation between $\frac{d(f \circ a)}{dt}(0)$ and $\frac{d(f \circ c)}{dt}(0)$ depends on the function f.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{d(f \circ a)}{dt}(0) = \frac{d(f \circ b)}{dt}(0)$
The relation between $\frac{d(f \circ a)}{dt}(0)$ and $\frac{d(f \circ c)}{dt}(0)$ depends on the function f.

7) Let $m \neq n$ and $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^n$ be two differentiable maps such that $g \circ f : \mathbb{R}^n \to \mathbb{R}^n$ is an invertible map . Which of the following statements are true? ***1 point***

- $n < m$ and $\mathrm{rank}(Df(a)) \leq n$ for all $a \in \mathbb{R}^n$.
- $n > m$ and $\mathrm{rank}(Df(a)) \leq m$ for all $a \in \mathbb{R}^n$.
- $n > m$ and $\mathrm{rank}(Df(a)) \leq n$ for all $a \in \mathbb{R}^n$.
- $n < m$ and $\mathrm{rank}(Df(a)) \leq m$ for all $a \in \mathbb{R}^n$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
$n < m$ and $\mathrm{rank}(Df(a)) \leq n$ for all $a \in \mathbb{R}^n$.
$n < m$ and $\mathrm{rank}(Df(a)) \leq m$ for all $a \in \mathbb{R}^n$.

8) Let $f, g : \mathbb{R}^n \to \mathbb{R}^n$ be two maps such that $f \circ g$ is differentiable. Which of the following statements are true? ***1 point***

- Both f and g must be differentiable maps.
- Only f must be differentiable.
- Only g must be differentiable.
- Both f and g can be non-differentiable maps.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Both f and g can be non-differentiable maps.

9) Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a differentiable map such that its Jacobian matrix is invertible at each point of $\mathbb{R}^n$. Which of the following statements are true? ***1 point***

- f is invertible.
- f is locally injective map.
- f is an injective map.
- f is surjective.

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is locally injective map.

10) Let $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f_1(x,y) = (x + y^2, x^2 + y)$ and $f_2(x,y) = (xy, x + y)$. Which of the following statements are true? ***1 point***

- Only f_1 is invertible in some neighbourhood of $(0,0)$.
- Only f_2 is invertible in some neighbourhood of $(0,0)$.
- Both f_1 and f_2 are invertible in some neighbourhood of $(0,0)$.
- Neither f_1 nor f_2 are invertible in any neighbourhood of $(0,0)$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Only f_1 is invertible in some neighbourhood of $(0,0)$.

You were allowed to submit this assignment only once.