## Courses » Integral Equations,calculus of variations and its applications

## Unit 5 - Week 4

## Course outline

How to access
the portal

## Week-1

## Week 2

Week 3

Week 4
Green's
function for self adjoint linear differential equations

- Green's
function for non-homogeneous boundary value problem
- Fredholm alternative theorem-I
- Fredholm
alternative
theorem-II
- Fredholm
method of
solutions
Quiz
Assignment 4
Solution of Assignment 4


## Assignment 4

The due date for submitting this assignment has passed
As per our records you have not submitted this assignment.

1) A Green function for the BVP

1 point $\frac{d}{d x}\left(\left(1-x^{2}\right) \frac{d y}{d x}\right)=0, y(0)=0, \mathrm{y}(1)$ is finite, is given by
$G(x, \xi)= \begin{cases}\frac{1}{2} \ln \frac{1-x}{1+x}, & 0 \leq x \leq \xi \\ \frac{1}{2} \ln \frac{1-\xi}{1+\xi}, & \xi \leq x \leq 1\end{cases}$
$G(x, \xi)= \begin{cases}x \ln \frac{1-\xi}{1+\xi}, & 0 \leq x \leq \xi \\ (x-1) \ln \frac{1-\xi}{1+\xi}, & \xi \leq x \leq 1\end{cases}$
$G(x, \xi)= \begin{cases}\ln \frac{1+x}{1-x}, & 0 \leq x \leq \xi \\ \ln \frac{1+\xi}{1-\xi}, & \xi \leq x \leq 1\end{cases}$

None of these.
No, the answer is incorrect.
Score: 0
Accepted Answers:
$G(x, \xi)= \begin{cases}\frac{1}{2} \ln \frac{1-x}{1+x}, & 0 \leq x \leq \xi ; \\ \frac{1}{2} \ln \frac{1-\xi}{1+\xi}, & \xi \leq x \leq 1 .\end{cases}$
2) Consider the following IVP $y^{\prime}(x)=f(x), x \geq 0, y(0)=0$.

Then Green function $G(\xi, x)$ for this IVP
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## Week 8

## Week 9

## Week 10

## Week 11

Week 12

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is $H(\xi-x)+2$.
No, the answer is incorrect.
Score: 0
Accepted Answers:
exists but not continuous
3) A Green function for the $B V P$
$x y^{\prime \prime}+y^{\prime}=0, y(l)=0, \mathrm{y}(0)$ is bounded, is given by

$$
G(x, \xi)= \begin{cases}x \ln \frac{\xi}{l}, & 0 \leq x \leq \xi ; \\ \xi \ln \frac{x}{l}, & \xi \leq x \leq l .\end{cases}
$$

$$
G(x, \xi)= \begin{cases}x \ln \xi l, & 0 \leq x \leq \xi ; \\ \xi \ln x l, & \xi \leq x \leq l .\end{cases}
$$

$$
G(x, \xi)= \begin{cases}\ln \frac{\xi}{l}, & 0 \leq x \leq \xi ; \\ \ln \frac{x}{l}, & \xi \leq x \leq l .\end{cases}
$$

$$
G(x, \xi)= \begin{cases}\frac{1}{2} \ln \xi l, & 0 \leq x \leq \xi \\ \frac{1}{2} \ln x l, & \xi \leq x \leq l\end{cases}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$G(x, \xi)= \begin{cases}\ln \frac{\xi}{l}, & 0 \leq x \leq \xi ; \\ \ln \frac{x}{l}, & \xi \leq x \leq l .\end{cases}$
4) A Green function for the BVP
$y^{i v}=0, y(0)=y^{\prime}(0)=y^{\prime \prime}(1)=y^{\prime \prime \prime}(1)=0$ is given by
$G(x, \xi)= \begin{cases}x^{2}(3 \xi-x), & 0 \leq x \leq \xi ; \\ \xi^{2}(3 x-\xi), & \xi \leq x \leq 1 .\end{cases}$
$G(x, \xi)= \begin{cases}x(3 \xi-x)^{2}, & 0 \leq x \leq \xi ; \\ \xi(3 x-\xi)^{2}, & \xi \leq x \leq 1 .\end{cases}$
$G(x, \xi)= \begin{cases}\frac{x^{2}}{6}(3 \xi-x), & 0 \leq x \leq \xi ; \\ \frac{\xi^{2}}{6}(3 x-\xi), & \xi \leq x \leq 1 .\end{cases}$

None of these
No, the answer is incorrect.
Score: 0
Accepted Answers:
$G(x, \xi)= \begin{cases}\frac{x^{2}}{6}(3 \xi-x), & 0 \leq x \leq \xi \\ \frac{\xi^{2}}{6}(3 x-\xi), & \xi \leq x \leq 1\end{cases}$
5) Using Green function the solution of the BVP

$$
y^{i v}=1, y(0)=y^{\prime}(0)=y^{\prime \prime}(1)=y^{\prime \prime \prime}(1)=0 \text { is given by }
$$

$$
\begin{aligned}
& y(x)=\frac{x^{2}}{24}\left(x^{2}-4 x-6\right) \\
& y(x)=x^{2}\left(\frac{x^{2}}{24}-4 x+6\right) \\
& y(x)=\frac{x^{2}}{24}\left(x^{2}+2 x-6\right) \\
& y(x)=x^{2}\left(\frac{x^{2}}{24}+4 x-6\right) .
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$y(x)=\frac{x^{2}}{24}\left(x^{2}-4 x-6\right)$
6)

The integral equation corresponding to the BVP $x y^{\prime \prime}+y^{\prime}=x+\lambda y, y(1)=y(e)=($ $y(x)=f(x)+\lambda \int_{1}^{e} G(x, \xi) y(\xi) d \xi$, where $f(x)$ is given by

$$
\begin{aligned}
& f(x)=\frac{1}{4}\left(e^{2} \ln x+x^{2}-1\right) \\
& \left.f(x)=\frac{1}{4}\left[\left(1-e^{2}\right) \ln x+1-x^{2}\right)\right] \\
& \left.f(x)=\frac{1}{4}\left[\left(e^{2}-1\right) \ln x+1-x^{2}\right)\right] \\
& \left.f(x)=\frac{1}{4}\left[\left(1-e^{2}\right) \ln x+x^{2}-1\right)\right] .
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:

$$
\left.f(x)=\frac{1}{4}\left[\left(1-e^{2}\right) \ln x+x^{2}-1\right)\right] .
$$

7) 1 the associated integral equation for the $B V P y^{\prime \prime}=\lambda y+e^{x}, y(0)=y(1)=0 i$

If the associated integral equation for the BVP $y^{\prime \prime}=\lambda y+e^{x}, y(0)=y(1)=0 i$ $s$ given by $y(x)=\lambda \int_{0}^{1} G(x, \xi) y(t) d t+f(x)$, then $f(x)$ is given by

$$
\begin{aligned}
& f(x)=e^{x}+e x-x-1 \\
& f(x)=e^{x}-e x+x-1 \\
& f(x)=e^{x}+e x+x+1 \\
& f(x)=e^{x}-e x-x-1
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f(x)=e^{x}-e x+x-1$
8) Consider the Fredholm integral equation $y(x)=\lambda \int_{0}^{\pi} \cos x \cos t y(t) d t . \quad 1$ point The root of $D_{3}(1)$, the resolvent determinant of size $3 \times 3$ is

$$
\begin{aligned}
& f(x)=\frac{1}{\pi} \\
& f(x)=\frac{4}{\pi} \\
& f(x)=\frac{\pi}{2} \\
& f(x)=\frac{2}{\pi}
\end{aligned}
$$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$f(x)=\frac{2}{\pi}$
9)

1 point
Consider the Fredholm integral equation $y(x)=f(x)+\lambda \int_{0}^{2 \pi}(\sin x-\sin t) y(t) d t$. $U$ sing Fredholm alternative, the resolving kernel $\Gamma(x, t ; \lambda)$ is given by


No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{(\sin x-\sin t)-\lambda \pi(1+2 \sin x \sin t)}{1+2 \pi^{2} \lambda^{2}}$
10)

1 point
Consider the Fredholm integral equation $y(x)=f(x)+\lambda \int_{-1}^{1}(x+t+1) y(t) d t$.Usir Fredholm alternative, the resolving kernel $\Gamma(x, t ; \lambda)$ is

$$
\begin{aligned}
& \frac{(x+t+1)-2 \lambda(3 x t+1)}{3+6 \lambda-4 \lambda^{2}} \\
& \frac{3(x+t+1)-2 \lambda(3 x t+1)}{3-6 \lambda+4 \lambda^{2}} \\
& \frac{3(x+t+1)+2 \lambda(3 x t+1)}{3-6 \lambda-4 \lambda^{2}} \\
& \text { None of these. }
\end{aligned}
$$

No, the answer is incorrect.
Score: 0

