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Courses » Integral Equations, calculus of variations and its applications

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## Unit 11 - Week 10

## Course outline

## How to access the portal

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## Week 10

- Euler's equation-II
- Functions of several independent variables
- Variational problems in parametric form
- Variational problems of general type
- Variational derivative and invariance of Euler's equation
- Quiz : Assignment 10
- Solutions of assignment-10

## Week 11

## Week 12

## WEEKLY FEEDBACK

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## Assignment 10

The due date for submitting this assignment has passed.  
As per our records you have not submitted this assignment.

Due on 2018-10-10, 23:59 IST.

1) The extremal of the functional  $\int_0^1 (y'^2 + y''^2) dx$  subject to the boundary conditions  $y(0) = 0$ ,  $y(1) = \sinh 1$ ,  $y'(0) = 1$  and  $y'(1) = \cosh 1$ , is **1 point**

$y = \sinh x(1 - \cos \frac{\pi x}{2})$

$y = 1 - \cosh x$

$y = \sinh x$

does not exist

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $y = \sinh x$

2) The extremal of the functional  $\int_0^1 \frac{y'^2}{2} dx$  satisfying the boundary conditions  $y(0) = 0$ ,  $y(1) = \frac{1}{2}$ ,  $y'(0) = 0$  and  $y'(1) = 1$  is **1 point**

$y = \frac{x + \sin(x^2 - x)}{2}$

$y = \frac{1 - \cos(x^2 - x)}{2}$

$y = \frac{x^2}{2}$

does not exist

No, the answer is incorrect.  
Score: 0

Accepted Answers:  
 $y = \frac{x^2}{2}$

3) The extremal of the functional  $\int_0^{\frac{\pi}{2}} (2xy - 2x^2 + \dot{x}^2 - \dot{y}^2) dt$  subject to the boundary conditions  $x(0) = 1$ ,  $y(0) = 0$ ,  $x(\frac{\pi}{2}) = \frac{\pi}{4}$ ,  $y(\frac{\pi}{2}) = \frac{\pi}{2}$ , is given by **1 point**

$x = \left(1 - \frac{\pi}{8} t\right) \cos t + \left(\frac{\pi}{2} - \frac{t}{2}\right) \sin t$

$y = \left(1 - \frac{\pi}{8} t\right) \cos t + \left(\frac{\pi}{2} - \frac{t}{2}\right) \sin t + \frac{\pi}{4} \sin t - \cos t$

$x = \left(1 - \frac{\pi}{4} t\right) \cos t + \left(\frac{\pi}{2} - t\right) \sin t,$

$y = \left(1 - \frac{\pi}{4} t\right) \cos t + \left(\frac{\pi}{2} - t\right) \sin t + \frac{\pi}{4} \sin t - \cos t$

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$$x = \left(1 - \frac{\pi}{8}t\right) \cos t + \left(\frac{\pi}{2} - \frac{t}{2}\right) \sin t,$$

$$y = \left(1 - \frac{\pi}{8}t\right) \cos t + \left(\frac{\pi}{2} - \frac{t}{2}\right) \sin t + \frac{\pi}{2} \sin t - \frac{\pi}{4} \cos t$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$x = \left(1 - \frac{\pi}{8}t\right) \cos t + \left(\frac{\pi}{2} - \frac{t}{2}\right) \sin t$$

$$y = \left(1 - \frac{\pi}{8}t\right) \cos t + \left(\frac{\pi}{2} - \frac{t}{2}\right) \sin t + \frac{\pi}{4} \sin t - \cos t$$

- 4) The extremal of the functional  $\int_0^1 \left\{2x + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} dt$  1 point  
satisfying the boundary conditions  $x(0) = 0$ ,  $y(0) = 1$ ,  $x(1) = 1$ ,  $y(1) = -1$  is

$$x = t, y = -2t + 1$$
$$x = \frac{t(t+1)}{2}, y = -t$$
$$x = t, y = -t$$
$$x = \frac{t(t+1)}{2}, y = -2t + 1$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$x = \frac{t(t+1)}{2}, y = -2t + 1$$

- 5) The equation  $y'' = f(x, y, y')$  is a Euler equation for some functional  $I[y(x)] = \int_a^b F(x, y, y') dx$  1 point  
where  $F(x, y, y')$  is the solution of the partial differential equation

$$F_{y'y'x} + y'F_{y'y'y} + fF_{y'y'y'} + f_{y'}F_{y'y'} = 0$$
$$F_{y'y'x} + y'F_{y'y'y} + fF_{y'y'y'} + F_{y'y'} = 0$$
$$F_{y'y'x} + F_{y'y'y} + fF_{y'y'y'} + f_{y'}F_{y'y'} = 0$$
$$F_{y'y'x} + y'F_{y'y'y} + F_{y'y'y'} + f_{y'}F_{y'y'} = 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$F_{y'y'x} + y'F_{y'y'y} + fF_{y'y'y'} + f_{y'}F_{y'y'} = 0$$

- 6) Consider the functional 1 point

$$I[\phi] = \int_{-\infty}^{\infty} [p(x)(\phi'(x))^2 + 2\phi(x+1)\phi(x-1) - \phi^2(x) - 2\phi(x)f(x)] dx$$

then the differential difference equation for the argument function  $\phi(x)$  is

$$(p\phi') - \phi(x+2) - \phi(x-2) + \phi(x) + f(x) = 0$$
$$(p\phi')' - \phi(x+2) - \phi(x-2) + \phi(x) + f(x) = 0$$
$$(p\phi')' - \phi(x+2) - \phi(x-2) + f(x) = 0$$
$$(p\phi') + \phi(x+2) + \phi(x-2) - \phi(x) + f(x) = 0$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$(p\phi')' - \phi(x+2) - \phi(x-2) + \phi(x) + f(x) = 0$$

- 7) The extremal of the functional 1 point

$$I[z(x, y)] = \iint_D \left[ z_y^2 \left( \frac{xy}{2} \right) - z_x \left( \frac{x^2y^2}{2} \right) \right] dx dy$$

is



$$z = \frac{y^3}{3} + c_1(y) \ln y + c_2(x)$$

$$z = \frac{y^3}{9} + c_1(x) \ln x + c_2(x)$$

$$z = \frac{y^3}{9} + c_1(y) \ln x + c_2(y)$$

$$z = \frac{y^3}{9} + c_1(x) \ln y + c_2(x)$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$z = \frac{y^3}{9} + c_1(x) \ln y + c_2(x)$$

8) The extremal of the functional

1 point

$$I[z(x, y)] = \iint_D (xyz + xp^2) dx dy$$

is

$$z(x, y) = x^2 y + c_1(y) \ln x + c_2(x)$$

$$z(x, y) = x^2 y + c_1(x) \ln y + c_2(y)$$

$$z(x, y) = \frac{x^2 y}{8} + c_1(y) \ln x + c_2(y)$$

none of these.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$z(x, y) = \frac{x^2 y}{8} + c_1(y) \ln x + c_2(y)$$

9)

1 point

Assuming the second order partial derivatives of  $z(x, y)$  continuous, the Euler – Ostrogradski equation for the

$$I[z(x, y)] = \iint_D \sqrt{1 + z_x^2 + z_y^2} dx dy$$

is

$$z_{xx}(1 + z_y^2) + 2z_x z_y z_{xy} - z_{yy}(1 + z_x^2) = 0$$

$$z_{xx}(1 + z_y^2) - 2z_x z_y z_{xy} + z_{yy}(1 + z_x^2) = 0$$

$$z_{xx}(1 + z_x^2) - 2z_x z_y z_{xy} - z_{yy}(1 + z_y^2) = 0$$

none of these.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$z_{xx}(1 + z_y^2) - 2z_x z_y z_{xy} + z_{yy}(1 + z_x^2) = 0$$

10) The variational derivative  $\frac{\delta J}{\delta y}$  of the functional  $J[y(x)] = y(1)$ , defined on  $C'[0, 1]$ ,

1 point

is given by

$$\delta(x - 1)$$

$$\delta(x + 1)$$

$$\delta(x)$$

can not be defined.

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$\delta(x - 1)$$

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