Week 5: Assignment 5

The due date for submitting this assignment has passed.

Due on 2021-09-01, 23:59 IST.

As per our records you have not submitted this assignment.

1) Which of the following are consequences of the mean-value theorem in several variables? ***2 points***

- [ ] If $f : U \to \mathbb{R}$ is a C^1 -smooth function on a convex open susbset $U \subset \mathbb{R}^n$ such that the partial derivatives are all bounded on U . Then f satisfies a Lipschitz condition.
- [ ] Any C^1 -smooth function whose partial derivatives all vanish identically on an open con- nected set is constant.
- [ ] Clairut's theorem on equality of mixed partial derivatives.
- [ ] Symmetry of the second derivative.

No, the answer is incorrect.
Score: 0

Accepted Answers:

If $f : U \to \mathbb{R}$ is a C^1 -smooth function on a convex open susbset $U \subset \mathbb{R}^n$ such that the partial derivatives are all bounded on U . Then f satisfies a Lipschitz condition.

Any C^1 -smooth function whose partial derivatives all vanish identically on an open con- nected set is constant.

2) Let us consider the det function on the space of all 2 × 2-matrices. Which of the following can be used to prove that the det is a differentiable function? ***2 points***

- [ ] The fact that the determinant is a polynomial in the entries.
- [ ] The fact that a matrix is invertible iff the determinant is non-zero.
- [ ] The fact that det is linear in each column.
- [ ] The fact that the map $A \mapsto A^{-1}$ is differentiable on the space of matrices with non-zero determinant.

No, the answer is incorrect.
Score: 0

Accepted Answers:

The fact that the determinant is a polynomial in the entries.

The fact that det is linear in each column.

3) Let $f : U \to F$ be differentiable at $x \in U$. Fix $v \in \mathbb{R}^n$. Let $\gamma_1, \gamma_2 : (-1, 1) \to U$ be two differentiable curves such that $\gamma_1(0) = \gamma_2(0) = x$ and $\gamma_1'(0) = \gamma_2'(0) = v$. Then ***2 points***

- [ ] $f \circ \gamma_1$ and $f \circ \gamma_2$ have the same derivative at 0.
- [ ] $f \circ \gamma_1$ and $f \circ \gamma_2$ have the same derivative at 0 unless $v = 0$ in which case they might be unequal.
- [ ] $f \circ \gamma_1$ and $f \circ \gamma_2$ might not be differentiable curves.
- [ ] If $\gamma_3, \gamma_4 : (-1, 1) \to U$ are two differentiable curves such that $\gamma_3(0) = \gamma_4(0) = x$ and $f \circ \gamma_3$ and $f \circ \gamma_4$ have the same derivative at 0 for all differentiable functions f then $\gamma_3'(0) = \gamma_4'(0)$.

No, the answer is incorrect.
Score: 0

Accepted Answers:

$f \circ \gamma_1$ and $f \circ \gamma_2$ have the same derivative at 0.

If $\gamma_3, \gamma_4 : (-1, 1) \to U$ are two differentiable curves such that $\gamma_3(0) = \gamma_4(0) = x$ and $f \circ \gamma_3$ and $f \circ \gamma_4$ have the same derivative at 0 for all differentiable functions f then $\gamma_3'(0) = \gamma_4'(0)$.

4) Let : $\mathbb{R}^n \to \mathbb{R}$ be differentiable function such $f(tx) = tf(x) \forall t \in \mathbb{R}, \forall x \in \mathbb{R}^n$ then ***0 points***

- [ ] f is identically constant.
- [ ] f is a linear functional.
- [ ] $f(x) = \nabla f(0) \cdot x$.
- [ ] There does not exists a function with the property that $f(tx) = tf(x) \forall t \in \mathbb{R}, \forall x \in \mathbb{R}^n$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$f(x) = \nabla f(0) \cdot x$.