

## Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

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Week 4

Week 5

- 13.1 Properties of the derivative map
- 13.2 The mean-value theorem
- 13.3 Differentiating under the integral sign
- 14.1 Higher-order derivatives
- 14.2 Symmetry of the second derivative
- 15.1 Taylor's theorem
- 15.2 Taylor's theorem with remainder
- Quiz: Week 5: Assignment 5
- Week 5 Feedback Form: Real Analysis II
- Lecture materials

Week 6

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## Week 5: Assignment 5

The due date for submitting this assignment has passed.

**Due on 2021-09-01, 23:59 IST.**

As per our records you have not submitted this assignment.

 1) Which of the following are consequences of the mean-value theorem in several variables? **2 points**


 If  $f : U \rightarrow \mathbb{R}$  is a  $C^1$ -smooth function on a convex open subset  $U \subset \mathbb{R}^n$  such that the partial derivatives are all bounded on  $U$ . Then  $f$  satisfies a Lipschitz condition.

 Any  $C^1$ -smooth function whose partial derivatives all vanish identically on an open connected set is constant.

Clairut's theorem on equality of mixed partial derivatives.

Symmetry of the second derivative.

**No, the answer is incorrect.**
**Score: 0**
**Accepted Answers:**

 If  $f : U \rightarrow \mathbb{R}$  is a  $C^1$ -smooth function on a convex open subset  $U \subset \mathbb{R}^n$  such that the partial derivatives are all bounded on  $U$ . Then  $f$  satisfies a Lipschitz condition.

 Any  $C^1$ -smooth function whose partial derivatives all vanish identically on an open connected set is constant.

 2) Let us consider the det function on the space of all  $2 \times 2$ -matrices. Which of the following can be used to prove that the det is a differentiable function? **2 points**


The fact that the determinant is a polynomial in the entries.

The fact that a matrix is invertible iff the determinant is non-zero.

The fact that det is linear in each column.

 The fact that the map  $A \mapsto A^{-1}$  is differentiable on the space of matrices with non-zero determinant.

**No, the answer is incorrect.**
**Score: 0**
**Accepted Answers:**

The fact that the determinant is a polynomial in the entries.

The fact that det is linear in each column.

 3) Let  $f : U \rightarrow F$  be differentiable at  $x \in U$ . Fix  $v \in \mathbb{R}^n$ . Let  $\gamma_1, \gamma_2 : (-1, 1) \rightarrow U$  be two differentiable curves such that  $\gamma_1(0) = \gamma_2(0) = x$  and  $\gamma_1'(0) = \gamma_2'(0) = v$ . Then **2 points**

 $f \circ \gamma_1$  and  $f \circ \gamma_2$  have the same derivative at 0.

 $f \circ \gamma_1$  and  $f \circ \gamma_2$  have the same derivative at 0 unless  $v = 0$  in which case they might be unequal.

 $f \circ \gamma_1$  and  $f \circ \gamma_2$  might not be differentiable curves.

 If  $\gamma_3, \gamma_4 : (-1, 1) \rightarrow U$  are two differentiable curves such that  $\gamma_3(0) = \gamma_4(0) = x$  and  $f \circ \gamma_3$  and  $f \circ \gamma_4$  have the same derivative at 0 for all differentiable functions  $f$  then  $\gamma_3'(0) = \gamma_4'(0)$ .

**No, the answer is incorrect.**
**Score: 0**
**Accepted Answers:**
 $f \circ \gamma_1$  and  $f \circ \gamma_2$  have the same derivative at 0.

 If  $\gamma_3, \gamma_4 : (-1, 1) \rightarrow U$  are two differentiable curves such that  $\gamma_3(0) = \gamma_4(0) = x$  and  $f \circ \gamma_3$  and  $f \circ \gamma_4$  have the same derivative at 0 for all differentiable functions  $f$  then  $\gamma_3'(0) = \gamma_4'(0)$ .

 4) Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  be differentiable function such  $f(tx) = tf(x) \forall t \in \mathbb{R}, \forall x \in \mathbb{R}^n$  then **0 points**

 $f$  is identically constant.

 $f$  is a linear functional.

 $f(x) = \nabla f(0) \cdot x$ .

 There does not exists a function with the property that  $f(tx) = tf(x) \forall t \in \mathbb{R}, \forall x \in \mathbb{R}^n$ 
**No, the answer is incorrect.**
**Score: 0**
**Accepted Answers:**
 $f(x) = \nabla f(0) \cdot x$ .