Week 4: Assignment 4

The due date for submitting this assignment has passed.

Due on 2021-09-01, 23:59 IST.

As per our records you have not submitted this assignment.

1) Which of the following functions are sublinear on $\mathbb{R}$? *0 points*

- h
- h^2
- $\log(1+|h|)$
- $e^{-h}-1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
h^2
$\log(1+|h|)$
$e^{-h}-1$

2) Consider the function *1 point*

$$f(x,y)=\begin{cases} y & x\neq 0\\ 0 & x=0\end{cases}$$

Then

- f is differentiable at 0.
- f is continuous at 0.
- The partial derivatives of f exist at 0.
- f is continuously differentiable at 0.

No, the answer is incorrect.
Score: 0

Accepted Answers:
f is continuous at 0.
The partial derivatives of f exist at 0.

3) Let $f:\mathbb{R}^n\to\mathbb{R}$ be a differentiable at all points in an open ball centred at 0. Which of the following are correct interpretations of the derivative map at 0? *1 point*

- The slope of the tangent to the graph of f.
- The best linear approximation of f at 0.
- Viewing the matrix of the derivative map under the standard basis as a vector, this vector gives the direction of maximum rate of change of f.
- The vector in the previous part is normal to the graph of f at $(0, f(x))$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The best linear approximation of f at 0.
Viewing the matrix of the derivative map under the standard basis as a vector, this vector gives the direction of maximum rate of change of f.

4) Let $U\subset\mathbb{R}^n$ be open and let $f:U\to\mathbb{R}$ be differentiable and whose derivative map is the 0 linear functional at all points. Then *1 point*

- f is identically 0.
- f is identically constant.
- f is identically zero if f is continuously differentiable.
- Either f is identically constant or U is disconnected.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Either f is identically constant or U is disconnected.

5) We will identify the collection of $n\times n$ matrices as $\mathbb{R}^{n^2}$ for this problem. Which of the following function defined on the space of matrices are differentiable (the codomains are not the same in each option)? *2 points*

- The determinant function.
- The transpose map.
- Fix a $n\times n$ invertible matrix B and consider $A\mapsto BAB^{-1}$.
- The trace of the matrix.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The determinant function.
The transpose map.
Fix a $n\times n$ invertible matrix B and consider $A\mapsto BAB^{-1}$.
The trace of the matrix.

6) In this problem, we identify the set of complex numbers with $\mathbb{R}^2$ in the obvious way. Let $f:\mathbb{C}\to\mathbb{C}$ be differentiable. Then the derivative map $Df(0):\mathbb{R}^2\to\mathbb{R}^2$ can be viewed as a map $T:\mathbb{C}\to\mathbb{C}$ under our identification of $\mathbb{C}$ with $\mathbb{R}^2$. Then *2 points*

- T is automatically complex linear.
- T is never complex linear unless T is the zero map.
- T is complex linear if $\det T>0$.
- Let the matrix of $Df(0)$ be

$$\begin{bmatrix} a & b\\ c & d\end{bmatrix}$$

Then T is complex linear if $a=d$ and $b=-c$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Let the matrix of $Df(0)$ be

$$\begin{bmatrix} a & b\\ c & d\end{bmatrix}$$

Then T is complex linear if $a=d$ and $b=-c$.