

## Course outline

How does an NPTEL online course work?

Week 0

Week 1

Week 2

Week 3

Week 4

10.1 Vector-valued functions

10.2 Scalar-valued functions of a vector variable

10.3 Directional derivatives and the gradient

11.1 Interpretation and properties of the gradient

11.2 Higher-order partial derivatives

12.1 The derivative as a linear map

12.2 Examples of differentiation

Quiz: Week 4: Assignment 4

Week 4 Feedback Form: Real Analysis II

Lecture materials

Week 5

Week 6

Week 7

Week 8

Week 9

Week 10

Week 11

Week 12

Download Videos

# Week 4: Assignment 4

The due date for submitting this assignment has passed.

**Due on 2021-09-01, 23:59 IST.**

As per our records you have not submitted this assignment.

 1) Which of the following functions are sublinear on  $\mathbb{R}$ ?

**0 points**

 $h$ 

 $h^2$ 

 $\log(1 + |h|)$ 

 $e^{-h} - 1$ 
**No, the answer is incorrect.**  
**Score: 0**
**Accepted Answers:**
 $h^2$ 
 $\log(1 + |h|)$ 
 $e^{-h} - 1$ 

2) Consider the function

**1 point**

$$f(x, y) = \begin{cases} y & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then

 $f$  is differentiable at 0.

 $f$  is continuous at 0.

 The partial derivatives of  $f$  exist at 0.

 $f$  is continuously differentiable at 0.

**No, the answer is incorrect.**  
**Score: 0**
**Accepted Answers:**
 $f$  is continuous at 0.

 The partial derivatives of  $f$  exist at 0.

 3) Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  be a differentiable at all points in an open ball centred at 0. Which of the following are correct interpretations of the derivative map at 0? **1 point**


 The slope of the tangent to the graph of  $f$ .

 The best linear approximation of  $f$  at 0.

 Viewing the matrix of the derivative map under the standard basis as a vector, this vector gives the direction of maximum rate of change of  $f$ .

 The vector in the previous part is normal to the graph of  $f$  at  $(0, f(x))$ .

**No, the answer is incorrect.**  
**Score: 0**
**Accepted Answers:**

 The best linear approximation of  $f$  at 0.

 Viewing the matrix of the derivative map under the standard basis as a vector, this vector gives the direction of maximum rate of change of  $f$ .

 4) Let  $U \subset \mathbb{R}^n$  be open and let  $f : U \rightarrow \mathbb{R}$  be differentiable and whose derivative map is the 0 linear functional at all points. Then **1 point**

 $f$  is identically 0.

 $f$  is identically constant.

 $f$  is identically zero if  $f$  is continuously differentiable.

 Either  $f$  is identically constant or  $U$  is disconnected.

**No, the answer is incorrect.**  
**Score: 0**
**Accepted Answers:**

 Either  $f$  is identically constant or  $U$  is disconnected.

 5) We will identify the collection of  $n \times n$  matrices as  $\mathbb{R}^{n^2}$  for this problem. Which of the following function defined on the space of matrices are differentiable (the codomains are not the same in each option)? **2 points**
 The determinant function.

 The transpose map.

 Fix a  $n \times n$  invertible matrix  $B$  and consider  $A \mapsto BAB^{-1}$ .

 The trace of the matrix.

**No, the answer is incorrect.**  
**Score: 0**
**Accepted Answers:**

The determinant function.

The transpose map.

 Fix a  $n \times n$  invertible matrix  $B$  and consider  $A \mapsto BAB^{-1}$ .

The trace of the matrix.

 6) In this problem, we identify the set of complex numbers with  $\mathbb{R}^2$  in the obvious way. Let  $f : \mathbb{C} \rightarrow \mathbb{C}$  be differentiable. Then the derivative map  $Df(0) : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  can be viewed as a map  $T : \mathbb{C} \rightarrow \mathbb{C}$  under our identification of  $\mathbb{C}$  with  $\mathbb{R}^2$ . Then **2 points**

 $T$  is automatically complex linear.

 $T$  is never complex linear unless  $T$  is the zero map.

 $T$  is complex linear if  $\det T > 0$ .

 Let the matrix of  $Df(0)$  be

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

 Then  $T$  is complex linear if  $a = d$  and  $b = -c$ .

**No, the answer is incorrect.**  
**Score: 0**
**Accepted Answers:**

 Let the matrix of  $Df(0)$  be

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

 Then  $T$  is complex linear if  $a = d$  and  $b = -c$ .