Week 3: Assignment 3

The due date for submitting this assignment has passed.

Due on 2021-08-25, 23:59 IST.

As per our records you have not submitted this assignment.

1) Which of the following sets are always connected in any metric space? *1 point*

- [ ] Any connected component.
- [ ] Singleton sets.
- [ ] Any subset that is closed and open.
- [ ] Finite sets.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Any connected component.
Singleton sets.

2) Let X be a compact metric space. Which of the following spaces of continuous functions to the real numbers on X is always equicontinuous? *1 point*

- [ ] Any finite collection of continuous functions.
- [ ] Any countable collection of continuous functions.
- [ ] The space of all bounded and continuous function.
- [ ] Any compact subset of $BC(X, \mathbb{R})$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Any finite collection of continuous functions.
Any compact subset of $BC(X, \mathbb{R})$.

3) Consider the function *1 point*

$$f(x) = \begin{cases} \sin(1/x) \text{ if } x \neq 0, \\ y \text{ if } x = 0. \text{ Here } y \text{ is a real number.} \end{cases}$$

For which choices of y is the graph of the function a connected subset of $\mathbb{R}^2$?

- [ ] y = 2
- [ ] y = 1
- [ ] y = 0
- [ ] For no choice of y is the graph of f connected.

No, the answer is incorrect.
Score: 0

Accepted Answers:
y = 1
y = 0

4) Which of the following subsets of the space $C([0, 1], \mathbb{R})$ are dense? *1 point*

- [ ] The collection of differentiable functions.
- [ ] The collection of all polynomials with integer coefficents.
- [ ] The collection of all polynomials with rational coefficients.
- [ ] The collection of functions $\text{span}_{\mathbb{R}}\{1, \sin t, \sin^2 t, \ldots\}$.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The collection of differentiable functions.
The collection of all polynomials with rational coefficients.
The collection of functions $\text{span}_{\mathbb{R}}\{1, \sin t, \sin^2 t, \ldots\}$.

5) Consider the sequence sin n. Which of the following statements are true? *1 point*

- [ ] The sequence converges.
- [ ] $\limsup_{n\to\infty} \sin\ n = 1$
- [ ] $\limsup_{n\to\infty} \sin\ n = 0$
- [ ] $\liminf_{n\to\infty} \sin\ n = -1$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\limsup_{n\to\infty} \sin\ n = 1$
$\liminf_{n\to\infty} \sin\ n = -1$

6) Which of the following statements about connectedness and path-connectedness in a metric space are true? *1 point*

- [ ] Any path-connected set is always connected.
- [ ] Any path-connected component is always closed.
- [ ] Any connected component is always closed.
- [ ] In $\mathbb{R}^n$, the connected subsets are precisely the path-connected subsets.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Any path-connected set is always connected.
Any connected component is always closed.
In $\mathbb{R}^n$, the connected subsets are precisely the path-connected subsets.

7) Let $f : X \to Y$ be a continuous bijective map between metric spaces. We say f is a homeomorphism if f^{-1} is also continuous. The metric spaces X and Y are said to be homeomorphic if we can find a homeomorphism from X onto Y . Which of the following spaces are homeomoprhic? *2 points*

[Hint : Use connectedness and compactness]

- [ ] [0, 1] and $\mathbb{R}$
- [ ] The unit circle in $\mathbb{R}^2$ and [0, 1]
- [ ] The unit circle in $\mathbb{R}^2$ and (0, 1)
- [ ] (0, 1) and $\mathbb{R}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
(0, 1) and $\mathbb{R}$