Week 2: Assignment 2

The due date for submitting this assignment has passed.

Due on 2021-08-18, 23:59 IST.

As per our records you have not submitted this assignment.

1) Which of the following sets are always compact in any metric space? *1 point*

- The empty set.
- Singleton sets.
- Countable sets.
- Finite sets.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The empty set.
Singleton sets.
Finite sets.

2) Which of the following spaces are complete? *1 point*

- The set $\mathbb{Z}$ given the metric inherited from $\mathbb{R}$.
- The space $L(E, F)$ where E and F are normed vector spaces.
- The space $B(X, \mathbb{R})$ where X is a metric space.
- The space $C(X, Y)$ where X and Y are metric spaces and X is compact.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The set $\mathbb{Z}$ given the metric inherited from $\mathbb{R}$.
The space $B(X, \mathbb{R})$ where X is a metric space.
The space $C(X, Y)$ where X and Y are metric spaces and X is compact.

3) Let X and Y be metric spaces and let $f : X \to Y$ be a continuous function. Let $E \subset X$, Which of the following properties hold true for $f(E)$ if that property holds for E *1 point*

- Being open.
- Being complete.
- Being compact.
- Being bounded.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Being compact.

4) Let X be a complete metric space and let F be a subset. Mark the true staements *1 point*

- If F is complete then F is closed.
- If F is closed then F is complete.
- If $f : F \to Y$ is uniformly continuous then f extends continuously to $\bar{F}$. Here Y is some metric space.
- If F is open the F is **not** complete.

No, the answer is incorrect.
Score: 0

Accepted Answers:
If F is complete then F is closed.
If F is closed then F is complete.

5) Which of the following are true *1 point*

- All metric spaces are normed vector spaces.
- All normed vector spaces are metric spaces.
- Any metric space can be completed.
- Any metric space can be viewed as a dense subset of a Banach space.

No, the answer is incorrect.
Score: 0

Accepted Answers:
All normed vector spaces are metric spaces.
Any metric space can be completed.
Any metric space can be viewed as a dense subset of a Banach space.

6) In an arbitrary metric space which of the following are true about compactness? *1 point*

- The Heine–Borel theorem, that is, a subset is compact iff it is closed and bouned.
- Sequential compactness and compactness are equivalent.
- A subset is compact if it has the Bolzano–Weierstrass property.
- A subset that is compact must necessarily have the Bolzano–Weierstrass property.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Sequential compactness and compactness are equivalent.
A subset is compact if it has the Bolzano–Weierstrass property.
A subset that is compact must necessarily have the Bolzano–Weierstrass property.

7) Which of the following are true about totally bounded subsets of a metric space? *1 point*

- They are automatically bounded.
- A bounded set is automatically totally bounded.
- In $\mathbb{R}^n$ with the Euclidean metric, being bounded and totally bounded are equivalent.
- If X is a metric space such that the compact subsets are precisely those that are closed and bounded then being bounded and totally bounded are equivalent.

No, the answer is incorrect.
Score: 0

Accepted Answers:
They are automatically bounded.
In $\mathbb{R}^n$ with the Euclidean metric, being bounded and totally bounded are equivalent.
If X is a metric space such that the compact subsets are precisely those that are closed and bounded then being bounded and totally bounded are equivalent.

8) Which of the following are topological properties? *1 point*

- Compactness.
- Boundedness.
- Openness.
- Completeness.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Compactness.
Openness.