Week 10 : Assignment 10

The due date for submitting this assignment has passed.

Due on 2021-10-06, 23:59 IST.

As per our records you have not submitted this assignment.

1) Let $f_n \in L(I)$ converge pointwise to $f : I \to \mathbb{R}$. Under which of the following assumptions can you conclude that $f \in L(I)$? ***3 points***

- If f_n are continuous
- If each f_n is bounded.
- If f_n is a monotone sequence
- If f_n is a monotone, non-negative and there exists $g \in L(I)$ such that $f_n \leq g \forall n \in \mathbb{N}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
If f_n is a monotone, non-negative and there exists $g \in L(I)$ such that $f_n \leq g \forall n \in \mathbb{N}$

2) In the statement of dominated convergence theorem, suppose we replace the dominating function g by a constant $M > 0$, that is, we assume that $|f(n)| < M$. Then which of the following are true? ***3 points***

- The conclusion of the theorem is still true.
- The conclusion of the theorem is true whenever I is a bounded interval
- The conclusion of the theorem never holds $unless$ I is a bounded interval.
- If I is an unbounded interval then we can always find a sequence $f_n \in L(I)$ that is bounded above by M but yet the limit function f is not Lebesgue integrable.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The conclusion of the theorem is true whenever I is a bounded interval
If I is an unbounded interval then we can always find a sequence $f_n \in L(I)$ that is bounded above by M but yet the limit function f is not Lebesgue integrable.

3) Which of the following can be deduced from the dominated convergence theorem? ***2 points***

- Any pointwise limit of Lebesgue integrable functions is Lebesgue integrable.
- If $f_n : [a, b] \to R$ are Riemann integrable, uniformly bounded and f_n converges pointwise to a Riemann integrable function f then $\int_b^a f = \lim_{n\to\infty} \int_b^a f_n$.
- Any pointwise limit of continuous functions is Lebesgue integrable
- The uniform limit of continuous functions is Lebesgue integrable.

No, the answer is incorrect.
Score: 0

Accepted Answers:
If $f_n : [a, b] \to R$ are Riemann integrable, uniformly bounded and f_n converges pointwise to a Riemann integrable function f then
$$\int_b^a f = \lim_{n\to\infty} \int_b^a f_n.$$